Robust Contracting for Search

with Théo Durandard (UIUC) and Boli Xu (Iowa)

November 21, 2023

A Tale of Two Distilleries

(a) Mystic Farm & Distillery

(b) Buffalo Trace

- “We will pay you for a call option with strike price \$X” - Buffalo Trace
- “No. Why try new things if there’s no upside?” - Mystic

Question and Preview of Results

- Q: How to finance / incentivize innovation with little knowledge of the alternatives
- Results focus on debt contracts
  - Properties: minimal distortion, co-monotonicity of payoffs
  - Provide the principal's best payoff guarantee

Literature

- Finance-oriented literature: Townsend (1979), Innes (1990), Chang (1990), Krasa and Villamil (2000), Attar and Campioni (2001), Hebert (2018), Min (2020)...
- Robust contracting: Hurwicz and Shapiro (1978), Bergemann and Morris (2005), Chassang (2013), Carroll (2015, 2019), Kambhampati et al (2023), ...
- Contribution: linking dynamic moral hazard (search) to debt contracts

Model Setup

- Principal (Investor) and Agent (Entrepreneur)
- Principal controls access to a “room” (production or distribution capability)
- Agent enters the room… does something…
  and exits the room holding a prize (expected profits from an idea)
- Prior to entry, principal writes a contract which describes how the prize is split
- Goal: write a contract that maximizes the principal’s take of the prize

The Room Where It Happens: Weitzman Search

- Inside the room are boxes (potential ideas or projects)
- Agent performs unmonitored Weitzman search (with recall)
- Box: (c, F)
  - Cost $c \in \mathbb{R}^+$, unverifiable and privately borne by the Agent
  - Prize $x \in X := [0, X^{max}]$
  - $F \in \Delta(X)$, atomless and full support

Weitzman Search

- Agent's optimal strategy:
  - Order boxes from highest to lowest index
  - Stop if prize exceeds remaining indices
- Index r (reservation value) of (c, F) solves

$$c = \int [x - r]^+ dF(x).$$

- with contract w, the index r^w solves

$$c = \int [w(x) - r^w]^+ dF(x).$$

$\Rightarrow$ Generates distribution over X

Timing and Information

- Principal knows $\mathcal{A}_0 = (c_0, F_0)$
- Wage contract $w : X \to \mathbb{R}^+$
  - Limited liability: $0 \leq x \leq w(x)$
- Agent learns set of projects $\mathcal{A} = \{(c_i, F_i)_{i=0}^k\}$
  - $\mathcal{A} \supseteq \mathcal{A}_0$
- Agent searches over $\mathcal{A}$, presents one prize x to principal
- Principal wants a payoff guarantee against all $\mathcal{A} \supseteq \mathcal{A}_0$

Principal's Robust Objective

Given contract w and realized boxes $\mathcal{A}$, Principal's payoff is

$$V_P(w \mid \mathcal{A}) := \mathbf{E}_{\sigma(\mathcal{A},w)}[x - w(x)]$$

$\sigma(\mathcal{A}, w)$: agent-optimal search given $(\mathcal{A}, w)$

Principal evaluates contracts based on their payoff guarantee

$$V_P := \sup_{w} \inf_{\mathcal{A} \supseteq \mathcal{A}_0} V_P(w \mid \mathcal{A})$$

Desirable Features of Contracts

- What leads a contract to perform well regardless of $\mathcal{A}$?
- Some potentially desirable features:
  - no incentive for the Agent to stop early – “minimal distortions”
  - if agent prefers $\mathcal{A}$ to $\mathcal{A}'$, then principal does too – “co-monotonicity”

Two Robustness Properties

- w is order-preserving if for all (c_1, F_1) and (c_2, F_2),

$$r_1 \geq r_2 > 0 \quad \& \quad r_2^w > 0 \implies r_1^w \geq r_2^w$$

- w does not change the order of projects (so long as they are still profitable)

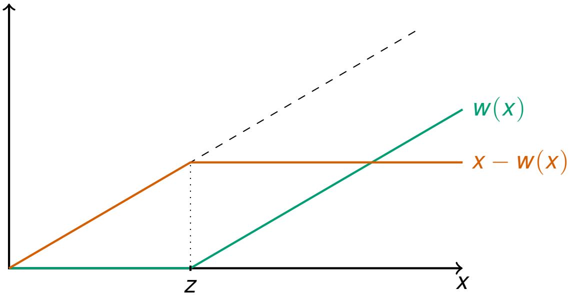
- w is aligned if for all $\mathcal{A}_0$ where $\mathbf{E}_{F_0}[w(x)] - c_0 \geq 0$,

$$\mathcal{A} \supseteq \mathcal{A}_0 \implies V_P(w \mid \mathcal{A}) \geq V_P(w \mid \mathcal{A}_0)$$

- Enlarging the set of projects always benefits the principal

Debt Contracts

- A z-debt contract is the contract where $w(x) = [x - z]^+$
- Below z, the Principal takes everything
- Above z, the Agent gets $x - z$ and is the “residual claimant”



Equivalence

Proposition: TFAE

1. w is order-preserving
2. w is aligned
3. w is a debt contract

Key Observation

- Reservation value: $c = \int [x - r]^+ dF(x)$
- z-Debt contract: $w(x) = [x - z]^+$
- Debt contract exactly mirrors the index!
- If w is a $z-$debt contract, then for any box, $r^w = r - z$

Intuition for Equivalence

Proposition: TFAE

1. w is order-preserving
2. w is aligned
3. w is a debt contract

with a $z-$debt contract, $r^w = r - z$

Proof sketches:

- (3) $\implies$ (1) immediate
- (3) $\implies$ (2) coupling argument; what if $\mathcal{A}_0$ not searched?
- Converses: by construction; a new project $(c', \delta_{x'})$ crowds out existing ones

Back to Robust Optimization

$$V_P := \sup_{w} \inf_{\mathcal{A} \supseteq \mathcal{A}_0} \mathbf{E}_\sigma[x - w(x)]$$

σ: agent-optimal search given $(\mathcal{A}, w)$

Optimality of Debt Contracts

- Theorem: Let r_0 be the index of (c_0, F_0).

  The r_0-debt contract is robustly optimal.

- Proof sketch:
  - By alignment, the worst-case is $\mathcal{A} = \mathcal{A}_0$
  - When $\mathcal{A} = \mathcal{A}_0$, the Principal gets first-best because $r_0^w = 0$

Comments

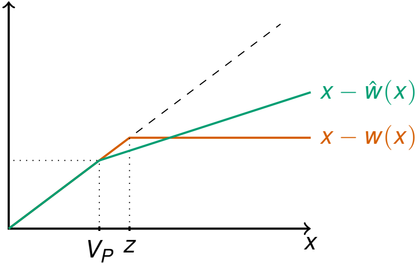
- Debt contract $=$ giving Agent a call option with strike price z
- Result holds regardless of principal's allowable mechanisms
  - e.g., screening the agent, multiple disclosures
- Debt contract weakly dominates selling the firm
  - Limited liability not so important for this result

Linear Contracts?

- Consider $w(x) = \alpha x$, where $\alpha = \frac{c_0}{\mathbf{E}_{F_0}[x]}$
- When $\mathcal{A} = \mathcal{A}_0$, principal gets first best...but worst-case payoff is 0
- Agent strictly prefers $(0, \epsilon)$ and then *stopping* for any $\epsilon > 0$
- Linear contracts are strictly suboptimal
- In this environment, worry about low-value *safe* projects crowding out risky ones
- Remark: Principal's payoff guarantee is higher when the agent can search

Uniqueness?

- Not quite.
- Recall, V_P is guarantee from the optimal debt contract
- Proposition: A contract w is robustly optimal if and only if
  1. $w(x) = 0$ for all $x \leq V_P$
  2. $\mathbf{E}_{F_0}[w(x)] = c_0$
- *Every* optimal contract has a minimal debt level



Takeaways & Extensions

- Ongoing work and extensions:
  - When $|\mathcal{A}_0| > 1$, a mixture of debt and equity may be optimal
  - Agent's strategic disclosure of $\mathcal{A}_0$
- Takeaways
  - Debt contracts provide payoff guarantees when moral hazard is *dynamic*
  - ! Optimal contract resembles index - natural extensions to other settings

THANK YOU!