Ex-Post Information and Project Choice

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Motivating Example

- Consider a career-concerned manager of a firm
- The manager's ultimate goal is to be hired as a CEO of a major corporation, so she takes actions that maximize the market's perception of her ability, rather than the firm's profits
- The manager must decide whether to introduce a new product or not, and better quality managers have a better sense of whether the product is profitable or not
- If the manager introduces the product, its profitability will be commonly known
- If not, it's true profitability will never be learned to observers
- Market can only infer manager's ability through the action and any observed information

General Question

- How does the amount of generated information affect the project choice of a career-concerned agent?
- How can a firm design a review policy (experiment) to align the manager's preferences?
- Preview of results
    - Low-quality managers exhibit obfuscation bias
    - Reducing information (Blackwell) following an action makes it more attractive to low-quality managers
    - One optimal information design is through a "simple" experiment, and it implements the first-best whenever the conflict of interest is not too great

Related Literature

- Reputation: Holmstrom, 1999 ; Prendergast and Stole, 1996; Brandenburger and Polak, 1996
- Reputational Cheap talk: Ottaviani and Sørensen, 2006
- Prior Bias: Gentzkow and Shapiro, 2006
- Transparency and monitoring: Prat, 2005
- Mechanism Design: Deb et al., 2018 ; Clark, 2020
- Career Concerns and Experimentation: Halac and Kremer, 2020

Model

- 3 players: manager (agent, she), firm (principal, it), market (they)
- Two states of the world: $\theta \in \{0, 1\}$, uniformly likely
- Manager has private type $t \sim U[\frac{1}{2}, 1]$, drawn independently of θ
- Manager of type t receives a signal $s \in \{0, 1\}$ about θ with accuracy t.

$$\Pr[s = 0|\theta = 0, t] = t$$
$$\Pr[s = 1|\theta = 1, t] = t$$

- Manager decides to take a safe or risky action $a \in \{0, 1\}$, which will generate profits for her firm

Information, Preferences

- Following the manager's action a, a public experiment $\mathcal{P}_a : \Theta \to \Delta(P)$ for some set of signals P. We fix $\mathcal{P}_1$ to be an experiment that fully reveals θ.

- The market observes a and the ex-post information p generated by $\mathcal{P}_a$, and pays a competitive wage $w^a(p) \in [0, 1]$

- The manager chooses a to maximize $\mathbf{E}[w^a(p)|t, s]$

- Firm payoff: for some $r \in (0, 1)$

$$v(a, \theta) = \begin{cases} 0 & a = 0 \\ r & a = 1, \theta = 1 \\ -1 & a = 1, \theta = 0 \end{cases}$$

- Note that since $r < 1$, ex-ante the firm prefers $a = 0$

Strategies

- Denote a behavioral strategy for the manager as

$$\sigma_t(s) = \Pr[a = 1 | s, t]$$

- $\sigma_t(s)$ is the probability a manager of type t takes the risky action $(a = 1)$ after observing the signal $s \in \{0, 1\}$

- For a given $\mathcal{P}_0$, we say that the strategies $(\sigma_t(s))$ and $(w^a(p))$ are IC if

$$\sigma_t(s) \in \operatorname*{argmax}_{\sigma} \mathbf{E}\left[(1-\sigma)w^0(p) + \sigma w^1(p) | s, t\right] \quad \forall s, t$$

$$w^a(p) = \mathbf{E}[t | a, p] \quad \forall a, p$$

- Firm problem: choose $\mathcal{P}_0, \sigma, w$ to maximize $\mathbf{E}[v(a, \theta)]$ s.t. IC

Benchmark: First Best

- Suppose that the firm could choose the manager's actions directly
- First best policy: there exists $t^* \in (\frac{1}{2}, 1)$ such that
  - if $t < t^*, \sigma_t(s) = 0$
  - if $t \geq t^*, \sigma_t(s) = s$
- The firm wants those sufficiently uninformed types to take the safe action, while the sufficiently informed types take the action that corresponds to their signal

Ex: Choice of $\mathcal{P}_0$ and IC

- Note that the first-best policy is *not* IC if, for example, $\mathcal{P}_0$ is fully revealing of θ
- As a numerical example, suppose $r = \frac{1}{3}$, which implies $t^* = \frac{3}{4}$
- Recall that $t \sim U[\frac{1}{2}, 1]$, so $\mathbf{E}[t] = 0.75$
- According to the first-best policy, we get the following wage functions for $w^a(p)$:

$$w^0(\theta = 0) = 0.74 \qquad w^0(\theta = 1) = 0.65$$
$$w^1(\theta = 0) = 0.83 \qquad w^1(\theta = 1) = 0.88$$

- However, note that the wage is always higher following the risky action than the safe action, so in fact no agent would want to take the safe action regardless of their signal

IC : Threshold Strategies

- Recall the IC constraint: $\sigma_t(s) \in \text{argmax}_\sigma \, \mathbf{E}\left[(1-\sigma)w^0(p) + \sigma w^1(p)|s,t\right] \quad \forall s,t$

- Suppose $s = 0$. We can re-write the IC constraint using the accuracy of the type-t manager:

$$\begin{aligned}\sigma_t(0) \in \underset{\sigma}{\text{argmax}} \quad & (1-\sigma)\left(\mathbf{E}[w^0(p)|\theta = 0]t + \mathbf{E}[w^0(p)|\theta = 1](1-t)\right) \\ & +\sigma\left(\mathbf{E}[w^1(p)|\theta = 0]t + \mathbf{E}[w^1(p)|\theta = 1](1-t)\right)\end{aligned}$$

- If $E[w^0(p)|\theta = 0] \geq \mathbf{E}[w^1(p)|\theta = 0]$ and $E[w^0(p)|\theta = 1] \leq \mathbf{E}[w^1(p)|\theta = 1]$, then the objective function is sub-modular in σ, t

- Thus, a necessary condition for IC is that we must have threshold strategies: $\sigma_t(0)$ is decreasing in t, and similarly $\sigma_t(1)$ is increasing in t

IC: Threshold Strategies

- $\sigma_t(0)$ is decreasing in t, and $\sigma_t(1)$ is increasing in t
- Similarly, we can show that the objective function is supermodular in (σ, s), which means that $\sigma_t(1) \geq \sigma_t(0)$ for all t
- Intuitively, these conditions mean that more informed types are more likely to take the action matching their signal, and that each type is more likely to take the risky action when they receive a high signal than when they receive a low signal
- Together, these imply that if there exists some $\tilde{t}$ such that $\sigma_{\tilde{t}}(1) = 0$, then $\sigma_t(0) = 0$ for all t

Ex: Threshold Strategies

- The threshold strategies will depend on $\mathcal{P}_0$
- For example, if $\mathcal{P}_0$ is fully informative of θ, then the unique (non-babbling) equilibrium has $\sigma_t(s) = s$ for all t, s
  - i.e. all managers follow their private signal, regardless of how strong
- For example, if $\mathcal{P}_0$ is completely *uninformative*, then the equilibrium strategies are

$$\sigma_t(0) = 0$$
$$\sigma_t(1) = \begin{cases} 0 & t < \hat{t} \\ 1 & t \geq \hat{t} \end{cases}$$

where $\hat{t}$ is indifferent between taking the two actions after receiving a high signal:

$$\mathbf{E}[t|a = 0] = \hat{t}\mathbf{E}[t|a = 1, \theta = 1, t > \hat{t}] + (1 - \hat{t})\mathbf{E}[t|a = 1, \theta = 0, t > \hat{t}]$$

Comparative Statics in Informativeness

- For experiments $\mathcal{Q}$, $\mathcal{Q}'$, I use $\mathcal{Q} < \mathcal{Q}'$ to mean $\mathcal{Q}'$ is Blackwell more informative than $\mathcal{Q}$

- Let $\bar{t}(\mathcal{P}_0) := \sup\{t : \sigma_t(1) = 0\}$

- Proposition: if $\mathcal{P}_0 < \mathcal{P}_0'$, then $\bar{t}(\mathcal{P}_0) > \bar{t}(\mathcal{P}_0')$

- Intuition:
    - A 1-to-1 mapping between posterior beliefs of θ and posterior expectations of the expert's type

    - A less informative signal leads to a mean-preserving decrease in risk in the posterior beliefs following the safe action

    - Tends to benefit those with low t, who are more likely to be incorrect

    - Makes the less informative option *more* attractive to the low types, and raises the threshold type

Optimal Information Policy

- In light of the previous proposition, it is easy to discuss the optimal information policy
- Let $\mathcal{P}_0^{\varnothing}$ denote the uninformative experiment
- Let t^* denote the first-best threshold for the firm
- Case 1: $\bar{t}(\mathcal{P}_0^{\varnothing}) < t^*$
  - Even with no information revealed following the safe action, too many types are following their signal
  - $\mathcal{P}_0^{\varnothing}$ is optimal, and the firm is strictly below its first-best

Optimal Information Policy

- Let $\mathcal{P}_0^{\varnothing}$ denote the uninformative experiment
- Let t^* denote the first-best threshold for the firm
- Case 2: $\bar{t}(\mathcal{P}_0^{\varnothing}) > t^*$
    - If there is no state-information following the safe action, too many managers would disregard their signal
    - Let $\mathcal{P}(q)$ denote a binary signal of θ with accuracy q:

$$\Pr[\tilde{s} = 1|\theta = 1] = \Pr[\tilde{s} = 0|\theta = 0] = q$$

    - Note that if $q < q'$, $\mathcal{P}(q) < \mathcal{P}(q')$, and $\mathcal{P}^{\varnothing} < \mathcal{P}(q) < \mathcal{P}_1$
    - By continuity, there exists q^* such that $\bar{t}(\mathcal{P}_0(q^*)) = t^*$, and the firm achieves its first-best payoff

Concluding Observations and Extensions

- The optimal experiment is *not* unique – we could have done this same argument with a different class of information structures that are Blackwell-ranked and continuously vary from uninformative to completely informative

- Result also means that any threshold equilibrium is implementable by a binary experiment (can actually prove this without the main result)

- If firm could choose $\mathcal{P}_1$, it would be optimal to have it be perfectly informative as long as $r < 1$

- Extensions:
  - Communication game between manager (consultant) and firm
  - Additional public signals about θ and t
  - Comparative statics in the type distribution of t

Takeaways

- “Obfuscation Bias” can lead to “status-quo bias” when counterfactuals are imprecise
- A principal maker may be able to align incentives via appropriately selecting the amount of ex-post information gathered
- The best information structure for the principal depends critically on what action she prefers ex-ante. If the decision maker prefers to take the “less informative” action, there are many senses in which more information harms the principal.
- Providing the right incentives means it is actually optimal to review “risky” decisions more thoroughly than “safe” ones