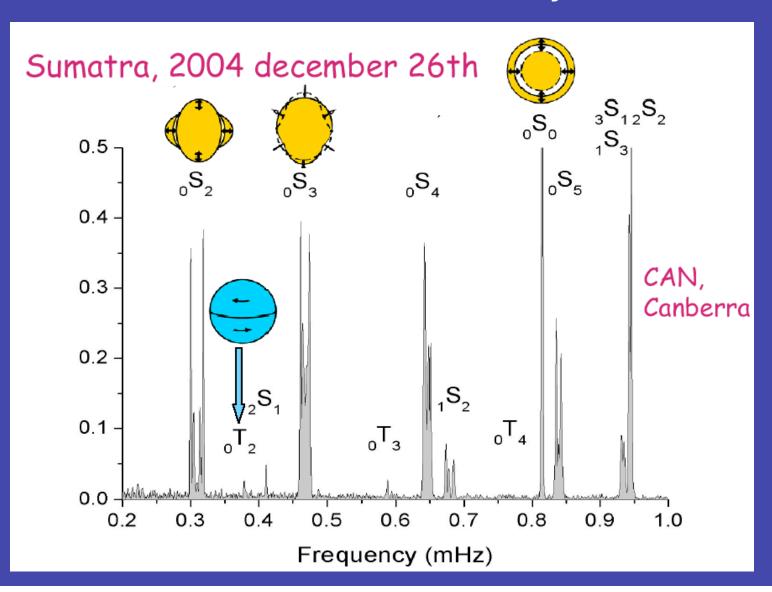
NORMAL MODES AND BEACHFRONT HOTELS

Seth Stein & Emile Okal Northwestern University



NORMAL MODES OF A STRING

Traveling wave built up as sum (Fourier series) of standing waves (modes)

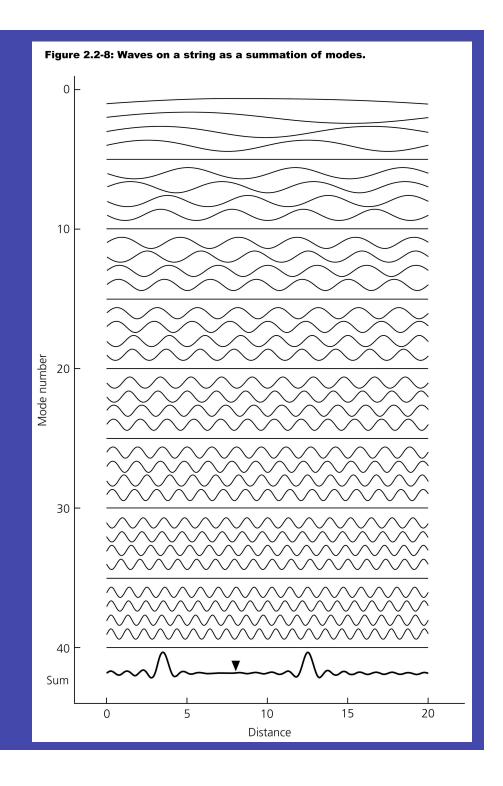
Each mode has an integral number of half wavelengths along the string's length L

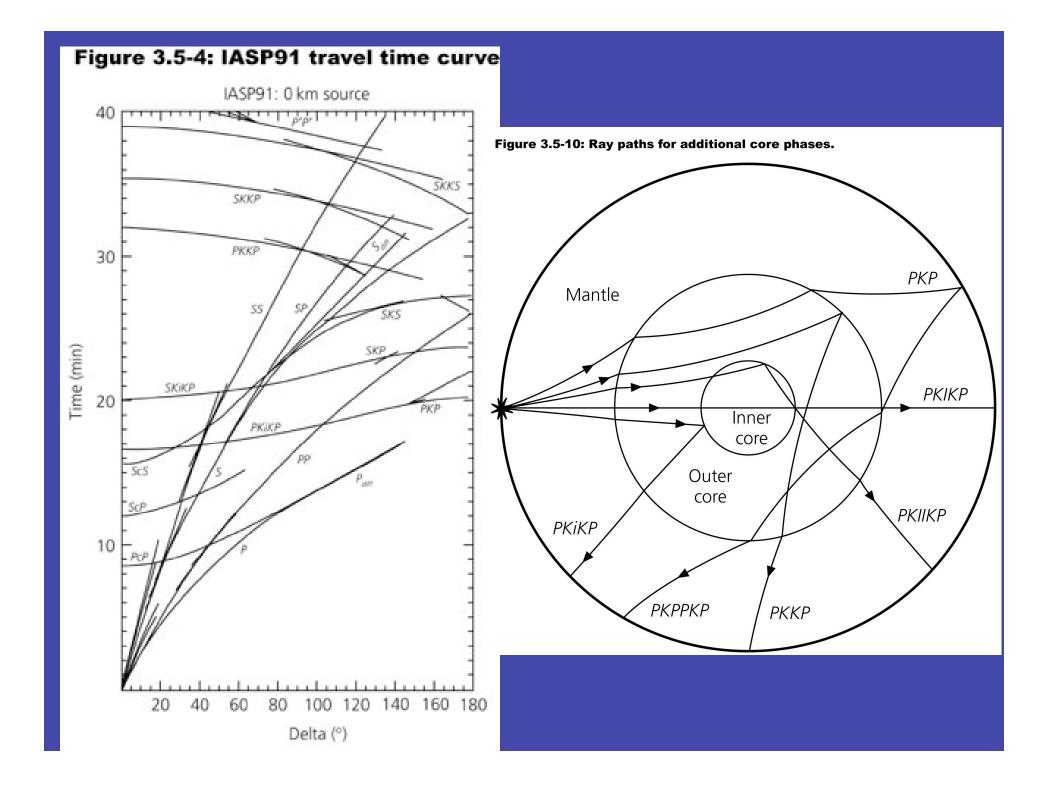
 $L = n\lambda/2$

Because $\lambda = vT$, each has a period = 2L / nv where v is the velocity

n=1 is the fundamental or longest period mode

Physically, what's it's period?





DECEMBER 26, 2004

JANUARY 7, 2005





"I actually called our news desk on Saturday and said: 'I know that we have this tsunami going on, and -- and all these people, but is it true that they broke up?'"

Katie Couric, CBS "Today" show

Harvard CMT found pure thrust faulting in expected geometry with M_w 9

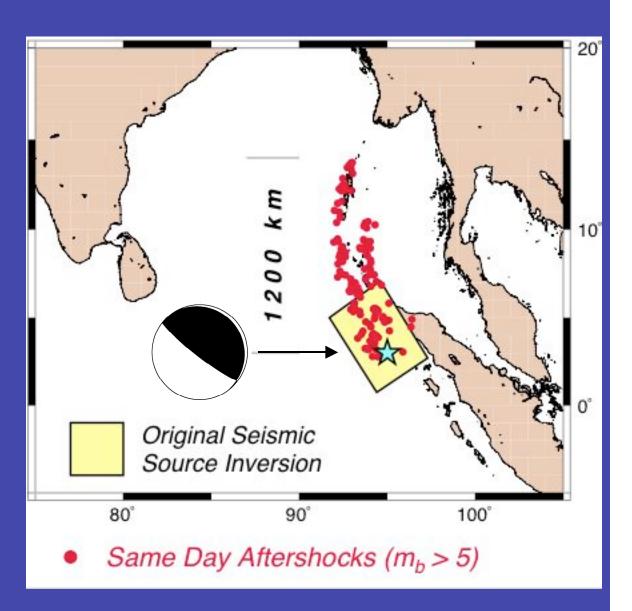
First day's aftershock zone extended from the epicenter off Sumatra where rupture started north 1200 km to the Andamans

Modeling seismic waves found most slip in southern 1/3

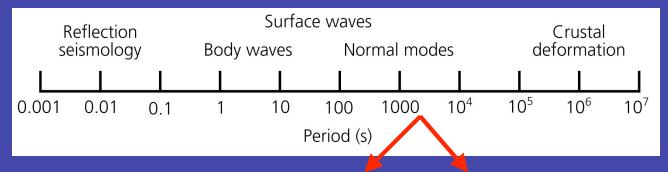
If the northern 2/3 hadn't slipped, was it about to?

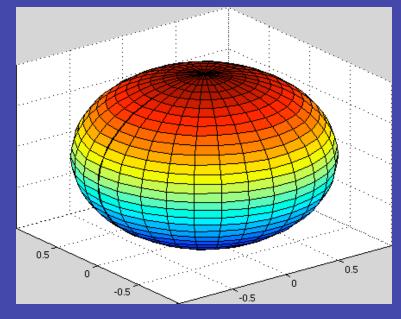
Could have even bigger event & tsunami

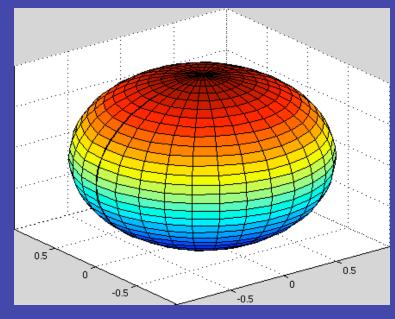
DECEMBER 2004: COULD AN EVEN BIGGER DISASTER BE IMMINENT?



TO SEE ALL FAULT: USE LONGEST PERIOD WAVES (NORMAL MODES) IN WHICH WHOLE EARTH "RINGS LIKE A BELL" - ACTUALLY RATTLES LIKE GARBAGE CAN







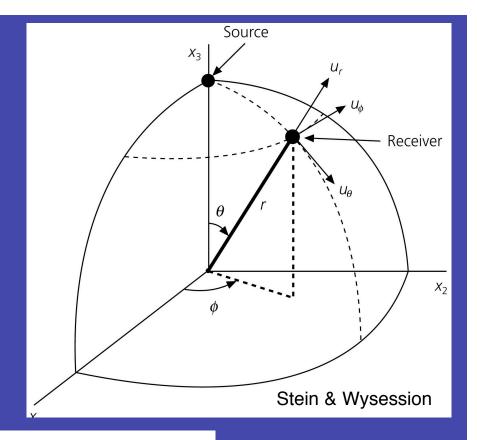
R. Aster

₀S₂ - period 3230 sec (54 minutes) "football mode"

₀S₀- period 1227 sec (20 minutes) "breathing mode"

NORMAL MODES OF SPHERICAL EARTH

Displacement (traveling seismic waves)
represented by 3-D sum in spherical coordinates of normal modes



$$\mathbf{u}(r,\,\theta,\,\phi) = \sum_{n} \sum_{l} \sum_{m} {}_{n} A_{l}^{m} {}_{n} y_{l}(r) \mathbf{x}_{l}^{m}(\theta,\,\phi) e^{n\omega_{l}^{m}t}$$

n, l, m - radial, angular, and azimuthal orders

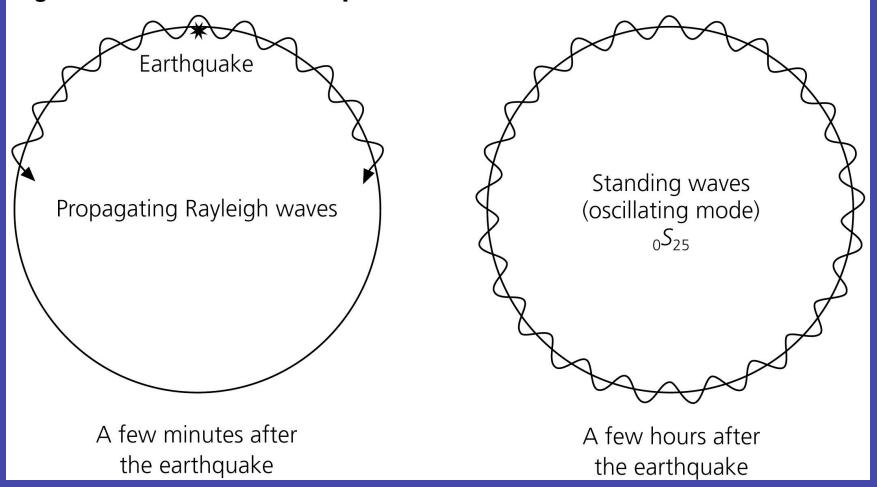
 $_{n}y_{l}(r)$ - scalar radial eigenfunction

 $\mathbf{x}_{l}^{m}(\theta, \phi)$ - vector surface eigenfunction

 $_{n}A_{l}^{m}$ - excitation amplitudes (weights for eigenfunctions) that depend on the seismic source.

Surface eigenfunctions are vector spherical harmonics: derivatives of $Y_{lm}(\theta, \phi)$

Figure 2.9-8: Cartoon of the equivalence of surface waves and normal modes.



The mode with angular order l and frequency ${}_{n}\omega_{l}$ corresponds to a traveling wave with horizontal wavelength $\lambda_{x} = 2\pi/|\mathbf{k}_{x}| = 2\pi a/(l+1/2)$ that has l+1/2 wavelengths around the earth.

These waves travel at a horizontal phase velocity $c_x = {}_n\omega_l/|\mathbf{k}_x| = {}_n\omega_la/(l+1/2)$

Velocity of equivalent wave gives eigenfrequency

MULTIPLET SPLIT INTO SINGLETS

For radial order n and angular order I: 2l+1 modes of different azimuthal orders -l ≤ m ≤ l are singlets, and the group of singlets is a multiplet

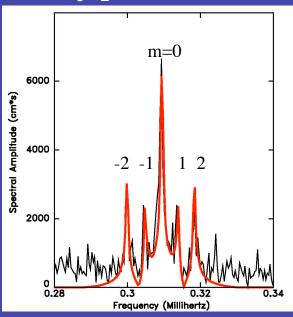
If the earth were perfectly spherically symmetric and not rotating, all singlets would have the same eigenfrequency

In real earth, singlet frequencies vary, which is called splitting

Splitting occurs because waves on different paths have different velocities & thus eigenfrequencies

Similar effects for waves in rotating fluid and atomic spectra (Zeeman splitting)

₀S₂ multiplet



Stein & Okal, 2005

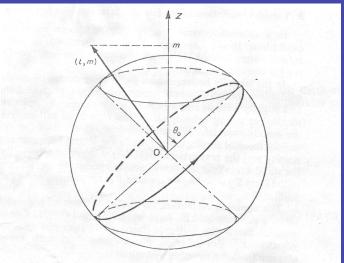
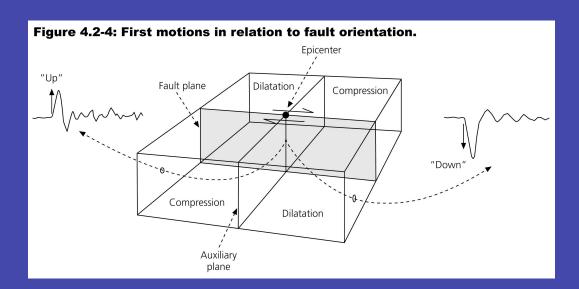
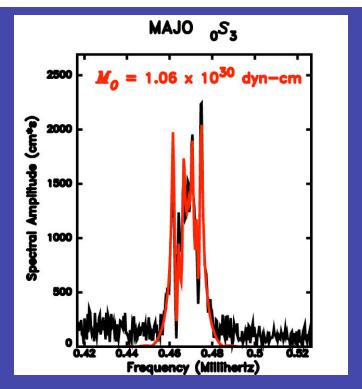


Fig. 1. The Great Circle paths of the surface waves associated with $Y_l^m(\theta, \phi)$. Each complex spherical harmonic represents a wave propagating in the counterclockwise direction around the sphere. The ray paths associated with these waves are Great circles (l, m) such that a vector of length $l+\frac{1}{2}$ along the axis has a projection of length m along the z-axis.

Madariaga, 1972

SPLIT MODE AMPLITUDES





In coordinate system with pole at earthquake the radiated energy has strong symmetry, so only m = 0, ±1, and ±2 singlets are excited

For split modes all 2l+1 singlets are excited

Transform spherical harmonics from earthquake frame of reference to geographic coordinate system that is symmetric about the rotation axis (Stein & Geller, 1977)

AMPLITUDES OF THE EARTH'S SPLIT NORMAL MODES

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VOL. 83, NO. B5

JOURNAL OF GEOPHYSICAL RESEARCH

MAY 10, 1978

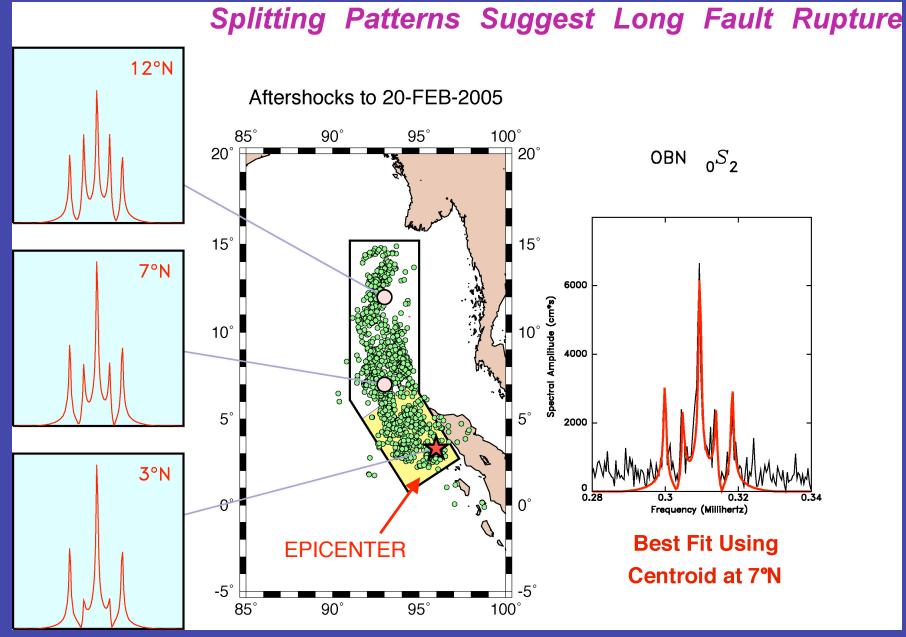
960

SEISMICITY AND TECTONICS OF THE NINETYEAST RIDGE AREA:
EVIDENCE FOR INTERNAL DEFORMATION OF THE NDIAN PLATE

Seth Stein and Emile A. Okal

Seismological Laboratory, California Institute of Technology, Pasadena, California 91125

Seth Stein was supported by a fellowship from the Fannie and John Hertz Foundation.



Best fit for center of rupture north of epicenter, in center of aftershock zone

SPLIT MODE AMPLITUDES ALSO SHOW MUCH BIGGER EARTHQUAKE

Moment 3x bigger than found by Harvard Centroid Moment Tensor (CMT)

Magnitude 9.3 instead of 9.0

Fault 3 times longer than first thought

Important implications for tsunami generation & recurrence

The New Hork Times

PRINTER-FRIENDLY FORMAT Melinda and Melinda
SPONSORED BY NOW PLAYING IN THEATERS

March 31, 2005

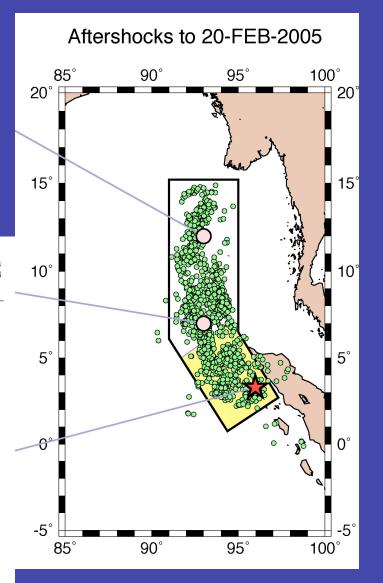
Scientists Reexamine December Earthquake

By THE ASSOCIATED PRESS

Filed at 5:49 a.m. ET

DENVER (AP) -- Researchers at a U.S. university have recrunched data from December's devastating earthquake in Sumatra and found it was even more powerful than previously believed, but other scientists suggest their findings are premature.

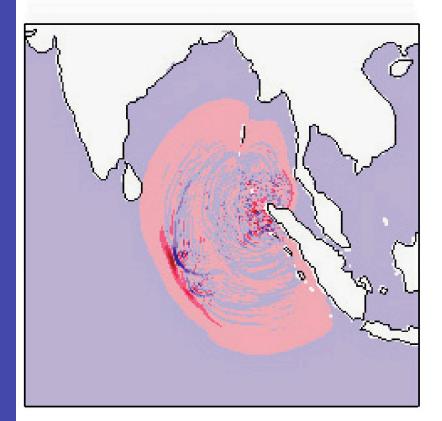
A study by seismologists at Northwestern University increases the intensity of the Dec. 26 earthquake from a magnitude 9.0 to a whopping magnitude 9.3. That's about three times more powerful than initial estimates.



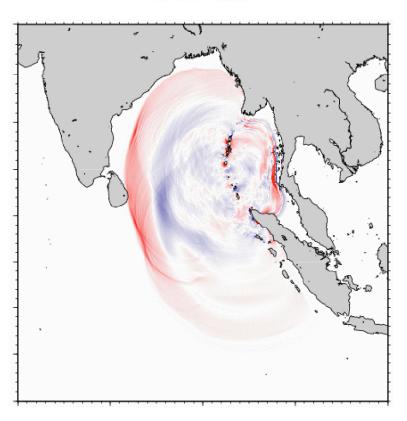
LONG RUPTURE EXPLAINED MAJOR DAMAGE IN THAILAND, SRI LANKA, AND INDIA

TSUNAMI GENERATION

South segment only



Entire fault



Snapshots from simulations 100 minutes after the earthquake

http://www.nio.org/jsp/tsu_simu.htm

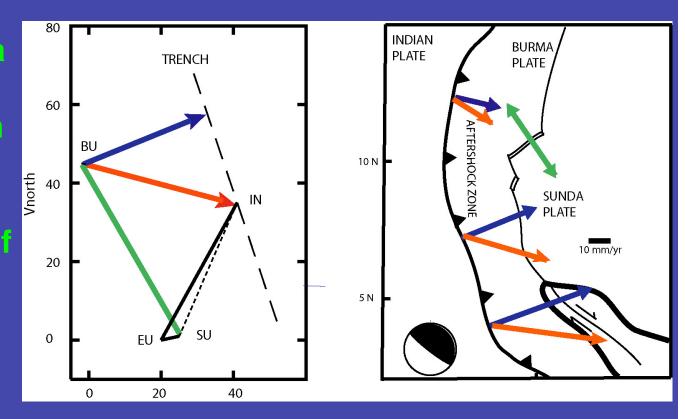
http://staff.aist.go.jp/kenji.satake/animation.gif

TO ESTIMATE RECURRENCE TIME FIND PLATE MOTIONS

Combining
motions of
India and Sunda
with respect to
Eurasia, known
from GPS data

with estimates of
Burma motion
with respect to
Sunda inferred
from back-arc
spreading

yields estimated India motion with respect to Burma



Thrust focal mechanism reflects arc-normal component of convergence, 20-50 mm/yr

ESTIMATE RECURRENCE TIME FROM PLATE MOTIONS & SLIP

12/26/04 slip ~ 10 m --> 10000 mm/ 35 mm/yr ~ 300 yr

Longer if some slip aseismic

Could be longer, given no cultural memory of such events

Recent paleotsunami study in Thailand finds event 600 years ago and earlier ones



Kruawun Jankaew, Chulalongkorn University, manuscript submitted to Nature, 3/08 (B. Atwater)

Fact that whole aftershock area slipped is good news for Thailand, India, Sri Lanka

2004 tsunami probably won't recur for very long time

Good news for all, including beachfront hotels & tourist industry

Tourism 7% Thai GDP

If the entire aftershock zone slipped, then strain accumulated on the northern part of the rupture has been released. There is therefore no immediate threat of an oceanwide tsunami being generated by slip on this segment of the plate boundary, because such earthquakes should be at least 400 years apart. However, the danger of a large tsunami resulting from a great earthquake on segments to the south remains.

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31 March 2005

