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**SEISMOLOGICAL**  
 RESEARCH LETTERS



# ***Could $M_{\max}$ Be the Same for All Stable Continental Regions?***

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## **ABSTRACT**

In probabilistic seismic-hazard assessment for stable continental regions (SCRs), the maximum magnitude  $M_{\max}$  truncating the earthquake magnitude–frequency distribution is commonly based on Bayesian updating of a global prior distribution derived from the distribution of observed  $M_{\max}$  in superdomains (groups of tectonically similar domains). We use randomly simulated earthquake catalogs to test if this observed superdomain  $M_{\max}$  distribution could also be explained by a global uniform  $M_{\max}$  value in SCRs, given our limited periods of observation. Using published average recurrence parameters per continent, catalog completeness thresholds for different regions within each continent, and assuming a Poisson temporal occurrence model, we simulate 10,000 random catalogs for each SCR domain, combine them into superdomain catalogs, and determine the largest sampled magnitude and the number of sampled earthquakes in each. Imposing an  $M_{\max}$  of 7.9, the largest magnitude observed in SCRs to date, and catalog lengths similar to those presently available, we obtain superdomain  $M_{\max}$  distributions similar to that observed. Hence, we cannot presently distinguish whether  $M_{\max}$  in SCRs is spatially variable or uniform. As a result, using a single value of  $M_{\max}$  in seismic-hazard analyses for all SCRs might make sense. Simulations with larger  $M_{\max}$  and longer catalogs confirm that catalog length is the limiting factor in our knowledge of  $M_{\max}$ .

*Online Material:* Table of data available for the 255 stable continental region domains.

## **INTRODUCTION**

$M_{\max}$ , the largest earthquake magnitude assumed to be possible in a given region, is an important parameter in probabilistic seismic-hazard assessment (PSHA). For the short return periods and high response frequencies considered in standard building

codes, its influence may still be modest in stable continental regions (SCRs), but  $M_{\max}$  becomes increasingly more important for longer return periods and lower response frequencies, which are relevant for nuclear installations. Despite its importance,  $M_{\max}$  remains an elusive quantity. Although this is true for seismically active regions, the degree of epistemic uncertainty is even larger in SCRs, where seismicity is often sparse and information on active faults is lacking. In these regions, PSHA is usually conducted using area sources in which seismicity is assumed to be homogeneously distributed and represented by a Gutenberg–Richter magnitude–frequency distribution (MFD) truncated at the  $M_{\max}$  assumed for that region. Various methods have been proposed to estimate  $M_{\max}$  for such distributed-seismicity sources, which can be mainly categorized as empirical (see overview in [Wheeler, 2009](#)) and statistical approaches (e.g., [Kijko, 2004](#)). The latter are based on extrapolation of MFDs from earthquake catalogs for a particular region. For this category, it has been formally demonstrated ([Holschneider et al., 2011](#)) that even with large observational records, it is essentially impossible to infer  $M_{\max}$ , or to determine meaningful confidence intervals, without additional information. Given two alternative hypotheses for  $M_{\max}$ , it even appears impossible to test which is more consistent with the data ([Holschneider et al., 2014](#)). In this study, we look at the limitations of one of the empirical approaches.

The most widely used empirical approach for estimating  $M_{\max}$  relies on the principle of ergodicity, in which a larger spatial extent is used to overcome the limited temporal extent of our observations, by assuming that  $M_{\max}$  should be similar in tectonically similar regions around the world. This approach was proposed in a report by the Electric Power Research Institute (EPRI) on the earthquakes of SCRs ([Johnston et al., 1994](#)), and updated in a report on seismic-source characterization for nuclear facilities in the central and eastern United States ([Electric Power Research Institute \[EPRI\] et al., 2012](#)), hereafter referred to as NUREG-2115. The SCRs around the world were divided into 255 tectonic domains based on crustal

age, crustal type, age of most recent extension, stress state, and the orientation of major tectonic structures relative to the maximum horizontal stress axis. Using a catalog of SCR earthquakes with revised moment magnitudes, the largest observed magnitude (determined instrumentally or inferred from historical records) and the number of observed earthquakes were compiled for each domain. Because the sample size in most domains appeared insufficient to infer  $M_{\max}$  from the largest observed magnitude with confidence (Johnston *et al.*, 1994), domains with similar characteristics (age of most recent extension, state of stress, source-stress angle, and optionally type of crust) were combined into so-called superdomains. These superdomains are not contiguous, but consist of domains scattered across different continents (Fig. 1). The implicit assumption behind combining domains into superdomains is that their common characteristics correlate with a common value of  $M_{\max}$  (EPRI *et al.*, 2012). The resulting distribution of largest observed magnitudes, which we refer to as the observed superdomain  $M_{\max}$  distribution, is shown for a set of superdomains in Figure 2, as a histogram and as a map.

Here, we explore whether the differences in observed maximum magnitude between SCR superdomains reflect spatial variability of the real  $M_{\max}$  or could just represent samples from the same parent distribution due to the short time span covered by current earthquake catalogs. In other words, should we combine all superdomains into one to infer  $M_{\max}$  in all SCRs?

The observed  $M_{\max}$  distribution for a specific set of superdomains is commonly used as a global prior distribution, which is updated using local observations following a Bayesian procedure, to obtain a posterior distribution on  $M_{\max}$  for a particular area source (Coppersmith, 1994). This posterior distribution is taken to represent the epistemic uncertainty on  $M_{\max}$  in PSHA. In many SCR areas, the number of local observations is small, making the posterior distribution strongly dependent on the assumed prior, so the choice of prior distribution is critical in estimating  $M_{\max}$  (Johnston *et al.*, 1994).

In the EPRI report, global prior  $M_{\max}$  distributions were developed for extended and nonextended superdomains. In NUREG-2115, a separation between Mesozoic or younger extended superdomains (MESE), and nonextended or older extended superdomains (NMESE) was found to be statistically more significant. However, because the statistical significance was still low, an alternative pooling into superdomains was also considered that ignores crustal type, resulting in a so-called composite prior. Superdomains with unknown stress classification, consisting of a single domain or containing only one earthquake, were excluded. The histogram in Figure 2a corresponds to the largest observed magnitudes in the 16 superdomains that were retained for the composite prior in NUREG-2115. The mean value of this observed superdomain  $M_{\max}$  distribution was adjusted for bias due to limited sample size using the average number of earthquakes in the superdomains. The bias-adjusted mean and standard deviation were then used to define the global  $M_{\max}$  prior, assuming a normal distribution (although, given the low number of data, a uniform distribution might be appropri-

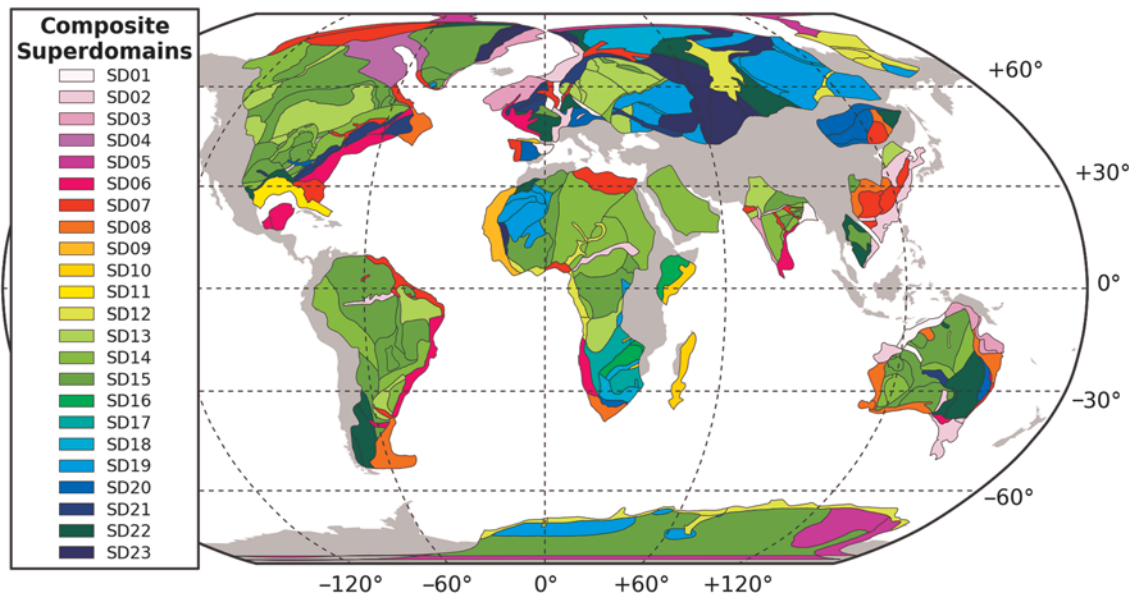
ate as well). For the composite prior in Figure 2a, the mean and standard deviation were reported as 6.88 (bias-adjusted to 7.2) and 0.64 (table 5.2.1-4 in NUREG-2115). The two largest values in the histogram correspond to preinstrumental earthquakes with magnitudes inferred from historical accounts: one earthquake in eastern China in 1668 ( $M_w$  7.9) and the largest 1812 New Madrid earthquake ( $M_w$  7.8). It should be noted that the SCR status of the domain containing the earthquake in eastern China may be questioned when compared with more detailed plate-boundary models (e.g., Bird, 2003), and the magnitude of the largest New Madrid earthquake is being revised downward (Hough, 2008; Hough and Page, 2011). However, because reassessment of the SCR earthquake catalog compiled in NUREG-2115 is beyond the scope of this article, we use this information as is for consistency with the earlier work.

## OBJECTIVE AND METHODS

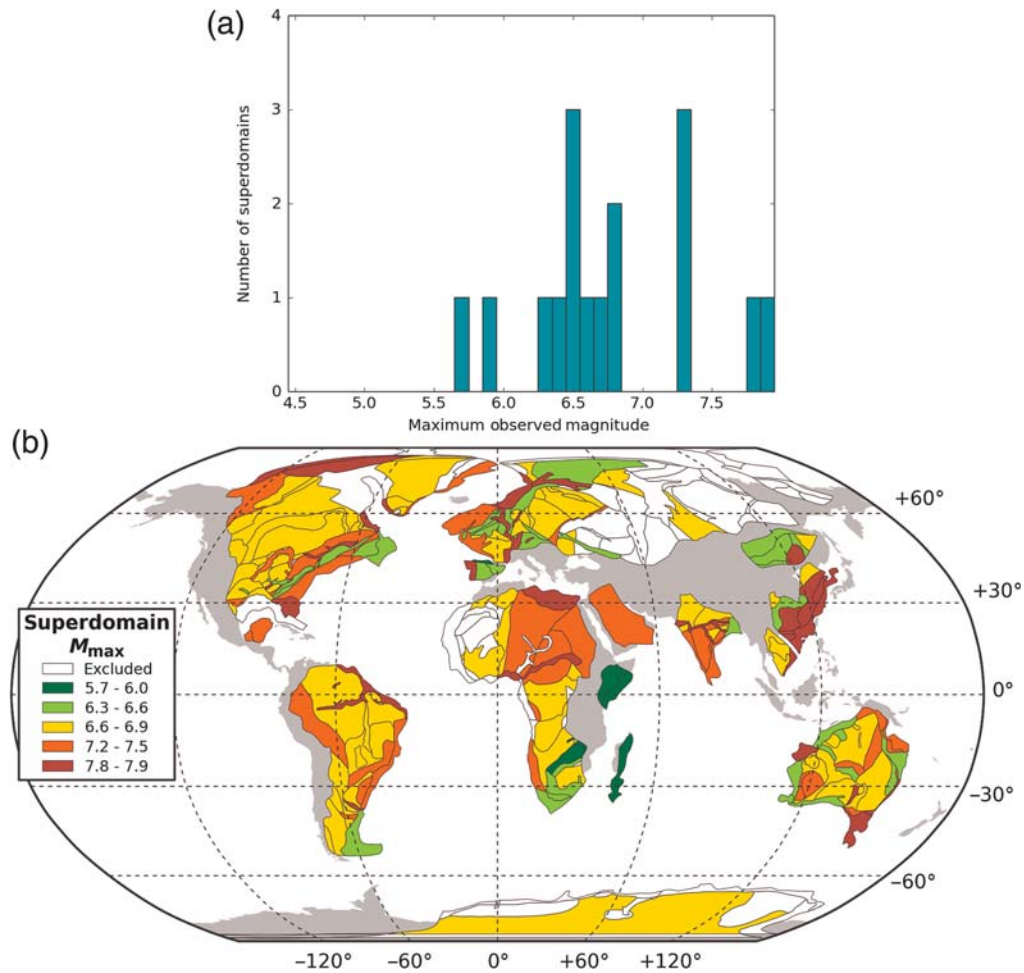
The assumption that common characteristics of superdomains correlate with a common value of  $M_{\max}$  (EPRI *et al.*, 2012) suggests that the observed superdomain  $M_{\max}$  distribution (Fig. 2a) reflects real variability of  $M_{\max}$  between SCRs. Considering the limited periods of observation (catalog lengths) at our disposal in all SCRs, it is natural to ask whether the apparent differences in  $M_{\max}$  (Fig. 2a) are real or just reflect a short time sample. Would we observe the same pattern if we could collect longer earthquake records, or would  $M_{\max}$  turn out to be the same in all continental interiors?

In this study, we explore whether the observed  $M_{\max}$  distribution could be explained by a uniform global  $M_{\max}$  value by simulating earthquake catalogs that assume a global uniform  $M_{\max}$  value in SCRs and then comparing the resulting superdomain  $M_{\max}$  distributions to the one observed. We investigate only the composite prior, because it is based on more data (195 domains compared to 71 and 119 for the MESE and NMESE priors, respectively) and is thus likely more robust.

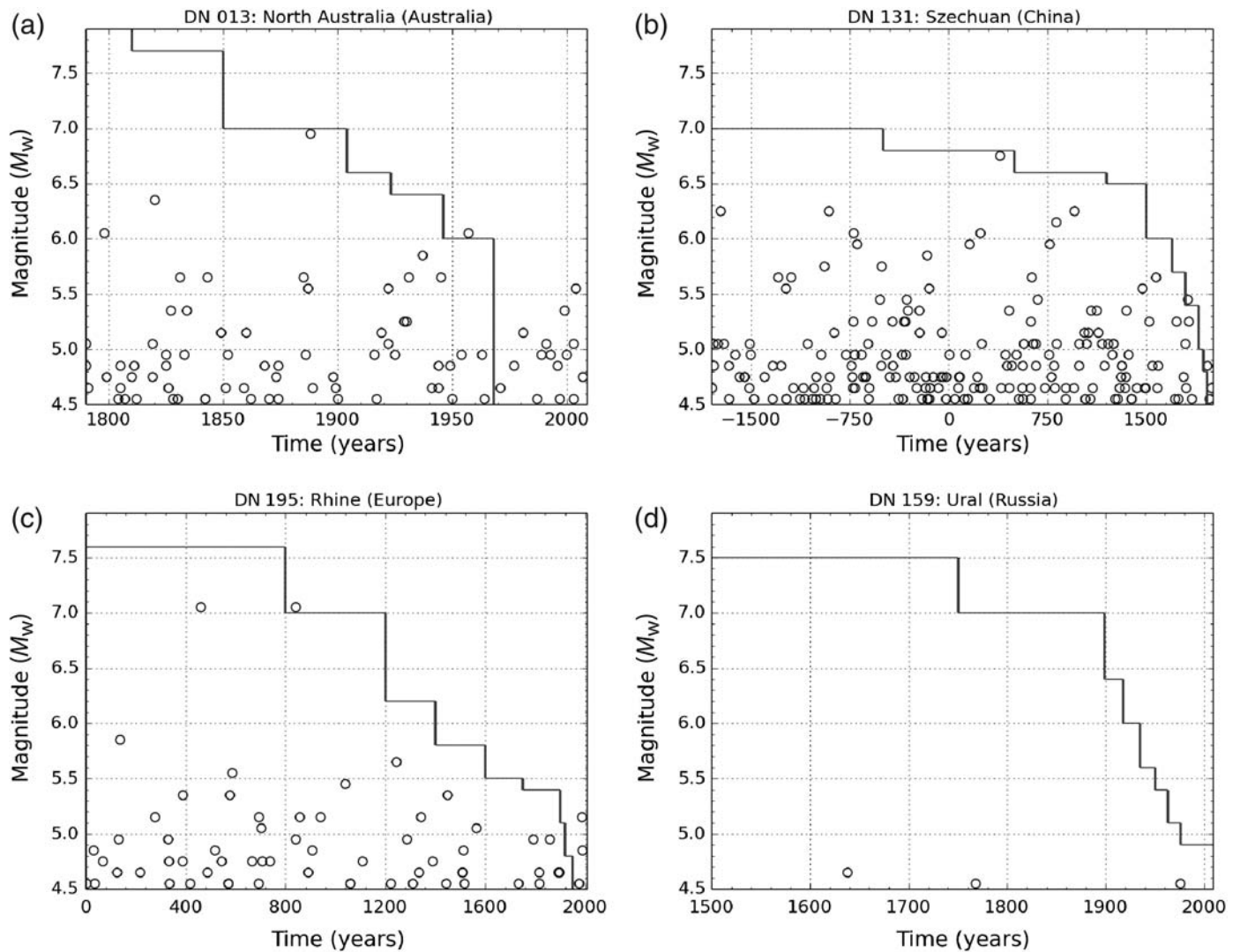
We simulated random catalogs, assuming that seismicity in an SCR domain is represented by a doubly truncated Gutenberg–Richter MFD and follows a Poisson distribution. Similar assumptions are commonly considered for distributed-seismicity sources in PSHA (Cornell, 1968; McGuire, 1976). For each SCR domain, we constructed an MFD with a lower bound of  $M_w$  4.5 and an upper bound equal to a particular parent  $M_{\max}$ . Lacking robust domain-specific recurrence parameters, we used the average recurrence parameters reported for the respective continent (tables 4–7 in Johnston *et al.*, 1994) but scaled by domain area. The MFDs were discretized with a bin width of 0.1 magnitude units, and synthetic catalogs were generated following the procedure outlined in the Appendix, which we implemented by extending code from the open-source hazard engine OpenQuake (Pagani *et al.*, 2014). To determine the start year and completeness thresholds of each catalog, we assigned each domain to a completeness region (tables 3–6 in Johnston *et al.*, 1994). The start year was taken as the earliest year for which a completeness magnitude was reported. © The completeness region, catalog start year, surface



▲ **Figure 1.** Composite superdomains according to NUREG-2115 (EPRI *et al.*, 2012).



▲ **Figure 2.** Distribution of largest observed magnitudes in composite superdomains reproduced using table K-2 in NUREG-2115 (EPRI *et al.*, 2012): (a) histogram and (b) map of apparent  $M_{\max}$  differences between stable continental region (SCR) domains, based on the largest observed magnitude in corresponding composite superdomains. Superdomains with unknown stress classification, consisting of a single domain or containing only one observed earthquake (SD04, SD05, SD09, SD11, SD12, SD19, and SD23) were excluded and do not contribute to the histogram in (a).



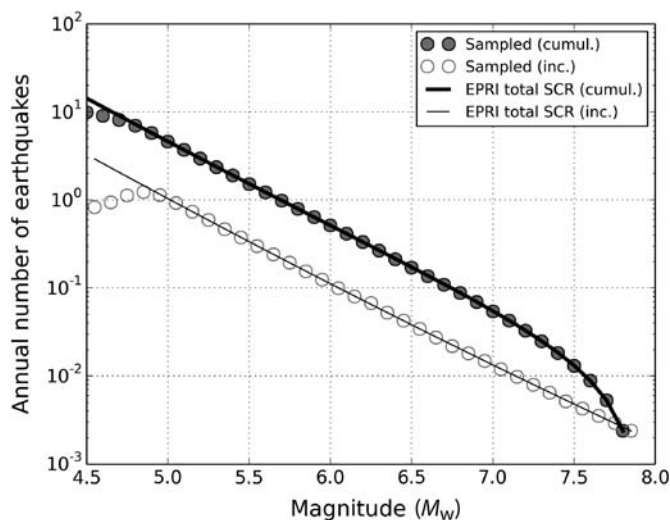
▲ **Figure 3.** Some examples of random domain catalogs (first sample for case 1), illustrating the variability of catalog length, completeness, activity rate (depending on surface area and continent), and maximum sampled magnitude: (a) Domain 13, North Australia; (b) domain 131, Szechuan, China; (c) domain 195, Rhine, Europe; and (d) domain 159: Ural, Russia. The bold line shows the completeness threshold (from Johnston *et al.*, 1994). Only magnitudes above this line are taken into account if completeness filtering is applied.

area, and  $a$ - and  $b$ -values for each domain are listed in Table S1, available in the electronic supplement to this article.

For each SCR domain, we generated 10,000 earthquake catalogs for seven cases in which we varied parent  $M_{\max}$ , catalog end year (and hence catalog length), and application of the completeness filter (i.e., the removal of earthquakes occurring in periods when the catalog is considered incomplete for their magnitude, as shown in Fig. 3). In case 1, parameters were chosen as close as possible to those in the NUREG-2115 SCR catalog (table K-1 in EPRI *et al.*, 2012): parent  $M_{\max}$  was set to 7.9 (the highest magnitude in the catalog), catalog end year was set to A.D. 2008, and completeness filtering was applied. Catalog lengths for the different SCR domains thus ranged between 111 and 3809 years.

Some domain catalogs sampled for this case are shown in Figure 3. The normalized activity rates of all domains within a continent are the same, so modeled activity rates depend only on continent and domain area. As a result, the activity rate of a particular domain may not correspond to the actually observed activity rate, which is variable across each continent. However, this is compensated by higher-than-observed activity in other domains.

To check the sampling procedure, we computed the average total SCR MFD (Fig. 4) from all catalogs sampled in case 1. The binned annual frequencies are in very good agreement with the total SCR MFD (sum of continent MFDs in Johnston *et al.*, 1994): discrepancies are below 0.5%, except for lower magnitudes ( $M_w < 4.9$ ), which are under-represented. Tests



▲ **Figure 4.** Average magnitude–frequency distribution (MFD) computed from 10,000 random catalogs for each SCR domain in case 1 compared with total SCR MFD (sum of continent MFDs in Johnston *et al.*, 1994). cumul., cumulative frequencies; inc., incremental (binned) frequencies.

show that the latter is entirely due to the completeness filter. Because the discrepancy is restricted to lower magnitudes, it has little or no impact on sampled  $M_{\max}$  values.

## RESULTS

### Case 1

Each of the 10,000 samples for case 1 consists of 255 domain catalogs, which are combined into superdomain catalogs. We determine the largest sampled magnitude for each superdomain and assemble the values for all superdomains in a histogram similar to Figure 2a. We use the final selection of composite superdomains in NUREG-2115, but, instead of excluding superdomains containing only one observed earthquake, we exclude superdomains containing only one sampled earthquake. Histograms obtained for the first four samples are shown in Figure 5.

We compute the mean value from these histograms, yielding 10,000 estimates of the mean sampled  $M_{\max}$ , which we compare with the mean observed  $M_{\max}$ . The histogram of mean sampled  $M_{\max}$  is shown in Figure 6a, along with the histograms of minimum and maximum sampled  $M_{\max}$ . The overall mean sampled  $M_{\max}$  is 6.82, which is very close to the observed mean  $M_{\max}$  (6.88, not corrected for bias). The average standard deviation of the sampled distributions (0.731) is also comparable to that of the observed distribution (0.64). This is also the case for the average sampled and observed minimum  $M_{\max}$  values (5.44 vs. 5.7). The histogram of maximum sampled  $M_{\max}$  values further indicates that the parent  $M_{\max}$  is recovered in more than 50% of the samples. The overall good agreement of these summary statistics between sampled and observed superdomain  $M_{\max}$  distributions indicates that a global uniform  $M_{\max}$  of 7.9 yields synthetic  $M_{\max}$  distributions that are consistent with that observed, if we take into account

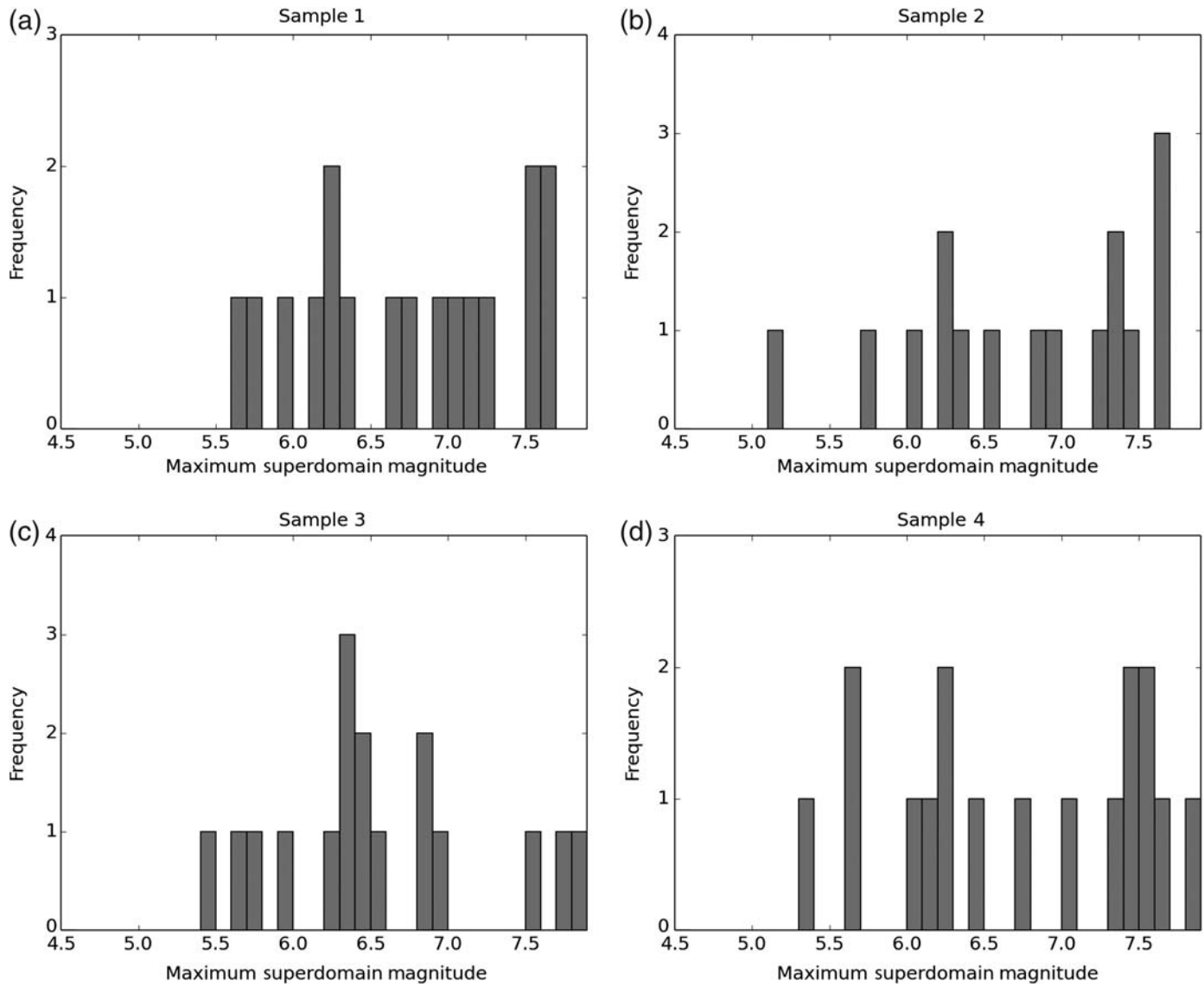
the limited lengths of our catalogs. A global uniform  $M_{\max}$  of 7.9 in SCRs can thus not be ruled out *a priori*.

### Other Cases

We repeated the sampling procedure for six other cases, in which we varied parent  $M_{\max}$ , catalog length, and completeness filtering. The results are summarized and compared with the first case in Table 1, and histograms of minimum, mean, and maximum values of the sampled superdomain  $M_{\max}$  distributions are shown in Figures 6b–d and 7. In case 2 (Fig. 7b), we consider the same parent  $M_{\max}$  and catalog lengths but do not apply completeness filtering (i.e., all sampled earthquakes are retained regardless of the completeness thresholds). It can be appreciated from Figure 3 that the total number of earthquakes in each sample is dramatically larger than in case 1. As a result, the mean value of the sampled  $M_{\max}$  distribution means increases to 7.05, approaching the bias-corrected mean  $M_{\max}$  in NUREG-2115 (7.2). The most important difference with case 1 is that the histogram of minimum sampled  $M_{\max}$  values becomes narrower and shifts to higher values, resulting in a smaller standard deviation.

In cases 3 and 4 (Fig. 7c,d), we consider higher parent  $M_{\max}$  values, while keeping the same catalog lengths and application of the completeness filter as in case 1. The overall mean sampled  $M_{\max}$  increases only slightly to 6.90 and 6.96 for parent  $M_{\max}$  of 8.2 and 8.5, respectively. These values are smaller than in case 2, with a parent  $M_{\max}$  of 7.9 and complete catalogs. The histogram of maximum sampled  $M_{\max}$  broadens, whereas the histogram of minimum sampled  $M_{\max}$  remains essentially constant, increasing the average standard deviation. Thus, the observed superdomain  $M_{\max}$  distribution, while becoming less compatible with the sampled distributions, remains well within the range of possibilities. Hence, a uniform  $M_{\max}$  higher than presently observed cannot be ruled out entirely.

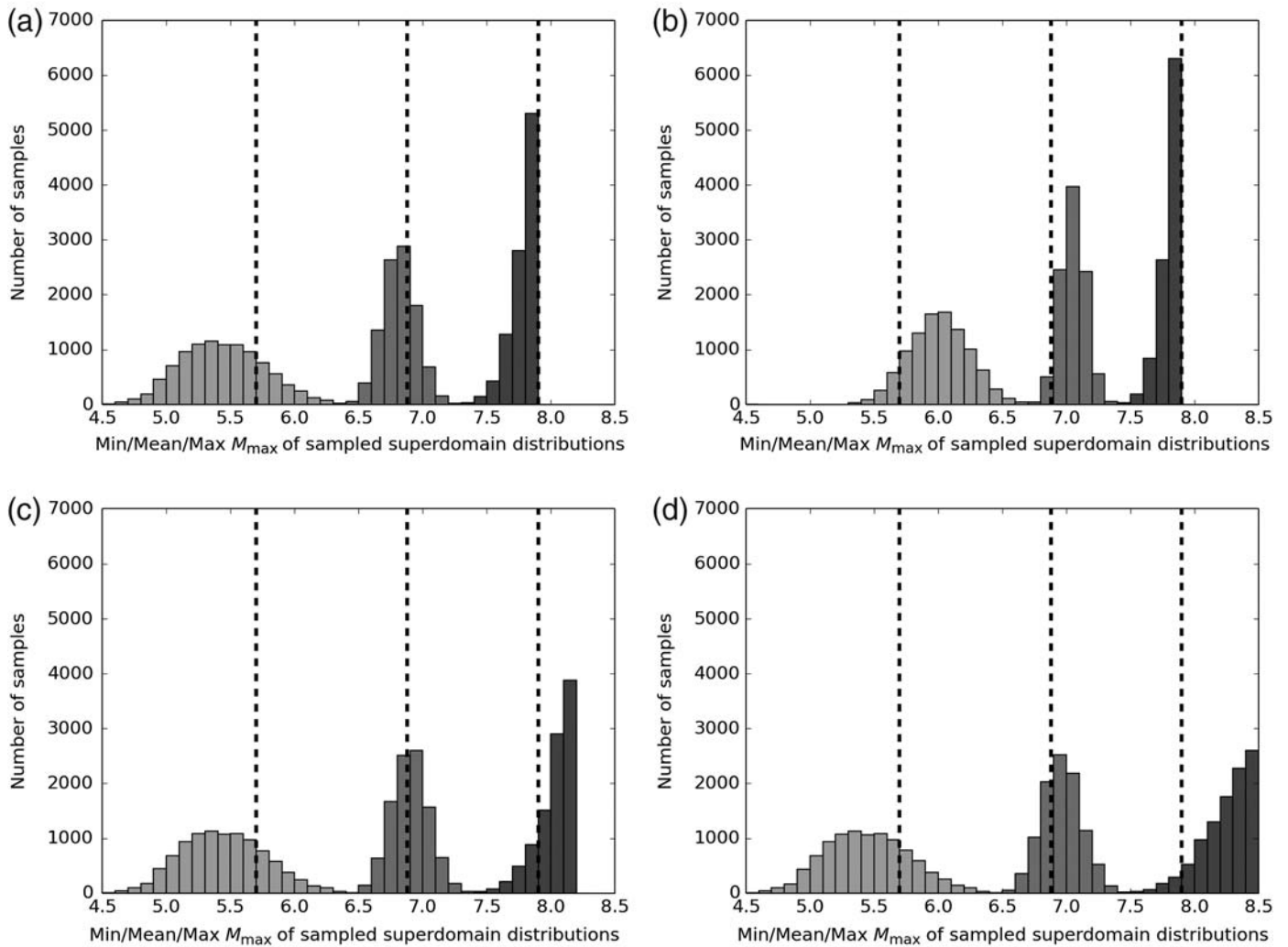
In cases 5–7 (Fig. 7a–c), we consider catalogs ending in A.D. 2258, A.D. 2508, and A.D. 3008, keeping the same parent  $M_{\max}$  and application of the completeness filter as in case 1. The overall mean sampled  $M_{\max}$  now increases more significantly to values of 7.10, 7.23, and 7.37 for catalogs that are, respectively, 250, 500, and 1000 years longer than at present. In parallel, the minimum sampled  $M_{\max}$  values increase, whereas the maximum sampled  $M_{\max}$  values converge to the parent  $M_{\max}$ , significantly reducing the average standard deviation. The mean and standard deviation for the 250-yr-longer catalog are very similar to those for case 2 with complete catalogs. It thus appears that we would need another ~250 yrs of data before the effect of catalog incompleteness is undone. With increasing catalog length, the sampled superdomain  $M_{\max}$  distributions become less compatible with the one observed. For 250-yr-longer catalogs, only 2% of the sampled distribution means fall below the mean observed  $M_{\max}$ . For longer catalogs, this fraction is reduced to zero. This indicates that a significantly longer period of observation (at least 250 yrs) would be required before we could reject the hypothesis of a uniform global  $M_{\max}$  for SCRs, provided the observed  $M_{\max}$  distribution did not increase as well.



▲ **Figure 5.** Superdomain  $M_{\max}$  distributions corresponding to first four samples of case 1.

Case	Parent $M_{\max}$	Catalog End Year	Completeness Filtering	Mean of $M_{\max}$ Distribution Means	Mean of $M_{\max}$ Distribution St. Dev.*	Mean of $M_{\max}$ Distribution Minima	Mean of $M_{\max}$ Distribution Mean neq. <sup>†</sup>	Mean of $M_{\max}$ Distribution Total neq. <sup>†</sup>
1	7.9	2008	Yes	6.82	0.731	5.44	32.5	542.9
2	7.9	2008	No	7.05	0.562	6.01	468.2	7959.1
3	8.2	2008	Yes	6.90	0.799	5.45	32.6	544.7
4	8.5	2008	Yes	6.96	0.857	5.45	32.6	545.8
5	7.9	2258	Yes	7.10	0.560	6.04	166.9	2836.8
6	7.9	2508	Yes	7.23	0.494	6.27	301.8	5129.9
7	7.9	3008	Yes	7.37	0.422	6.52	571.7	9719.1

\*st. dev., standard deviations ( $n - 3$  degrees of freedom).  
<sup>†</sup>neq, number of earthquakes.



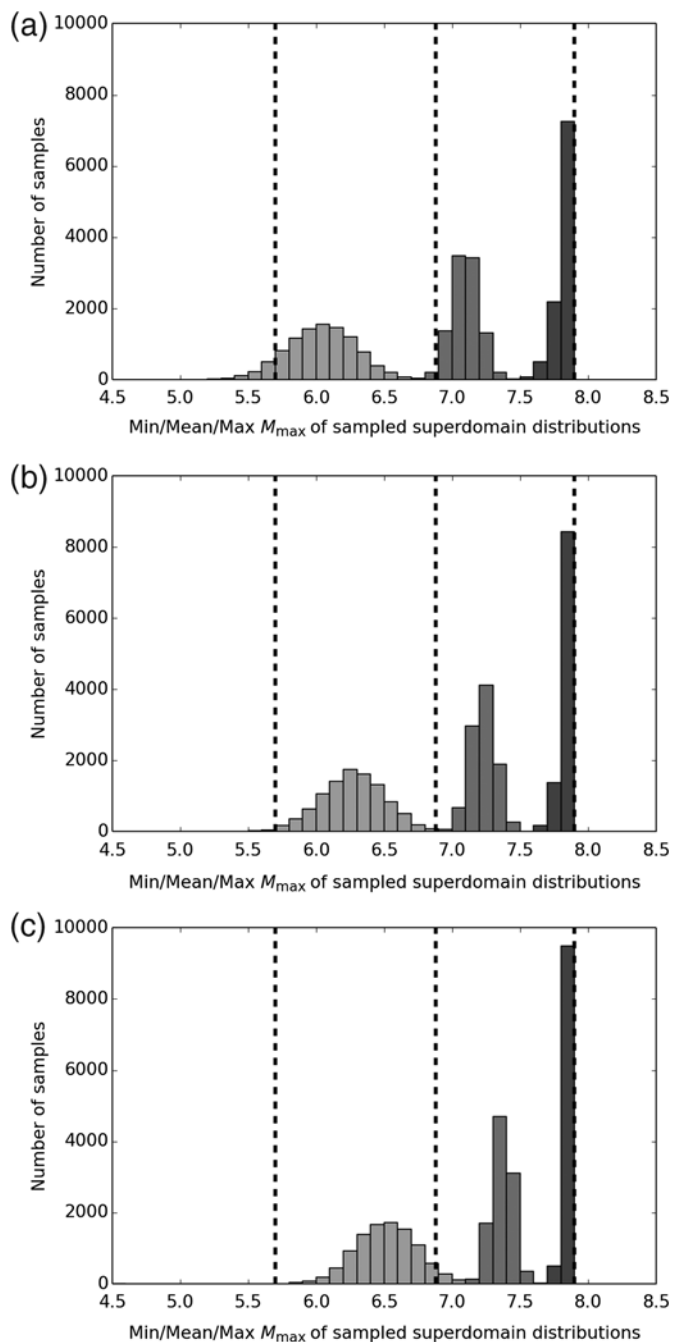
▲ **Figure 6.** Superdomain  $M_{\max}$  distribution minima (light shading), means (intermediate shading), and maxima (dark shading) for different combinations of parent  $M_{\max}$  and completeness filtering: (a) case 1, parent  $M_{\max} = 7.9$ , completeness applied; (b) case 2, parent  $M_{\max} = 7.9$ , no completeness; (c) case 3, parent  $M_{\max} = 8.2$ , completeness applied; and (d) case 4, parent  $M_{\max} = 8.5$ , completeness applied. In each case, catalog end year was 2008. The dashed lines indicate corresponding statistics for observed superdomain  $M_{\max}$  distribution (from [EPRI et al., 2012](#)).

## DISCUSSION

Our simulations indicate that (1) randomly sampled SCR earthquake catalogs generated assuming a global uniform  $M_{\max}$  of 7.9 yield superdomain  $M_{\max}$  distributions very similar to that observed; (2) higher parent  $M_{\max}$  values yield superdomain  $M_{\max}$  distributions that remain more or less compatible with the observed distribution; and (3) for longer simulated catalogs, the sampled superdomain  $M_{\max}$  distributions are affected more significantly, becoming incompatible with the observed distribution for catalogs a few hundreds of years longer than present catalogs.

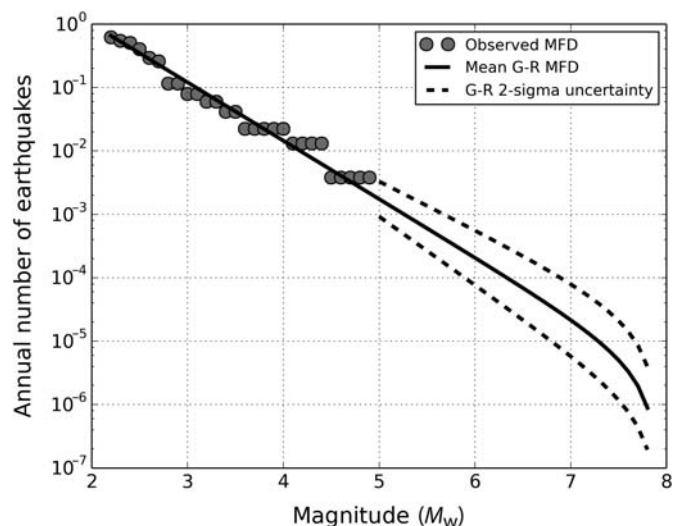
Addressing the question posed in the article title, it is not inconceivable that, given the short earthquake records,  $M_{\max}$  could indeed be the same in all continental interiors. Our sim-

ulations used an  $M_{\max}$  of 7.9 (or higher), but a lower  $M_{\max}$  value may be possible too, considering that the two largest events in the NUREG-2115 catalog are subject to debate, as discussed in the [Introduction](#). Hence, we conclude in more general terms that the current state of knowledge does not allow refuting a global uniform  $M_{\max}$  in SCRs equal to the largest magnitude observed to date. This result is a viable alternative to, but does not necessarily invalidate, the common assumption of superdomains in which  $M_{\max}$  varies with crustal properties (age of most recent extension, state of stress, source-stress angle). Our results furthermore indicate that catalog length is the limiting factor in our knowledge of  $M_{\max}$ . Longer periods of observation should eventually allow discriminating between a global uniform and spatially variable  $M_{\max}$ . If the assumption of stationary seismicity following a Gutenberg–Richter MFD



▲ **Figure 7.** Superdomain  $M_{\max}$  distribution minima (light shading), means (intermediate shading) and maxima (dark shading) for different catalog lengths: (a) case 5, catalogs ending in A.D. 2258; (b) case 6, catalogs ending in A.D. 2508; and (c) case 7, catalogs ending in A.D. 3008. In each case, parent  $M_{\max}$  was 7.9, and completeness was applied. The dashed lines indicate corresponding statistics for observed superdomain  $M_{\max}$  distribution (from [EPRI et al., 2012](#)).

in SCRs is correct and there is indeed a global uniform  $M_{\max}$ , the observed  $M_{\max}$  distribution should show a systematic shift to higher values after  $\sim 250$  more years of observation. If we do not observe such a shift, the hypothesis of a global uniform  $M_{\max}$  value can be rejected. However, this may still not suffice



▲ **Figure 8.** Example illustrating impact on occurrence rate of  $M_{\max}$  due to poorly constrained  $b$ -value ( $\sigma = 0.08$ ) for a small area source in northwest Europe. If the activity rates of all such sources in a probabilistic seismic-hazard assessment source model are summed, the total uncertainty on the frequency of  $M_{\max}$  will be raised to an extent that it essentially drives the predicted hazard at long return periods.

to tell us the true  $M_{\max}$  value (in either or both cases) or, if the latter case holds true, to establish whether or not  $M_{\max}$  correlates with the crustal properties that are currently adopted.

Waiting until our earthquake catalogs are long enough is clearly not a practical solution. One way to improve our knowledge of  $M_{\max}$  lies in identifying and characterizing active faults based on geologic, geomorphic, and paleoseismic methods. However, active fault studies in SCRs are still in their infancy: finding faults is not always straightforward, resulting in incomplete inventory, and, even for known faults, large uncertainties remain in our estimates of the largest magnitude they can generate. Empirical relations between moment magnitude and rupture dimensions or fault slip (e.g., [Wells and Coppersmith, 1994](#)) typically have a standard deviation of  $\sim 0.3$  magnitude units that is rarely fully taken into account. Furthermore, the maximum magnitude assigned to a fault depends crucially on the assumed segment boundaries, which are often poorly constrained in SCRs. Estimates based on rupturing of a single fault segment may therefore represent only a minimum  $M_{\max}$  value. A more prudent approach could involve estimating the largest physically possible magnitude, for example, by considering a worst-case scenario involving rupture spanning discontinuous, colinear fault segments ([Ward, 1997](#)). Hence, it is likely that events larger than observed to date may occur, as experience often shows, and the larger the area in question, the more likely it is.

What are the implications for the method to estimate  $M_{\max}$  based on Bayesian updating of the global prior distributions of  $M_{\max}$  ([Johnston et al., 1994](#))? It is an attractive method that has been applied in many PSHA studies and yields


seemingly objective probability distributions for  $M_{\max}$ . However, there is no rule for the size of the zones to which it should be applied, and this can strongly affect the final distribution. Furthermore, if the premise behind the observed superdomain  $M_{\max}$  distributions used to derive global priors is correct, it might be more appropriate to assign the  $M_{\max}$  value corresponding to the superdomain the source belongs to according to a particular prior model and to restrict the Bayesian procedure to sources situated in superdomains not represented in the histogram in Figure 2a. Single  $M_{\max}$  values based on tectonic analogy have been used in the 2002 seismic-hazard map of the United States (Frankel *et al.*, 2002)—but abandoned again in later updates—and in Canada's fourth-generation seismic-hazard map (Adams and Halchuk, 2003).

Because our results indicate that the observed superdomain  $M_{\max}$  distribution may also have emerged from a global uniform  $M_{\max}$  value (in which case it would not constitute a good prior), one could take this replacement of the Bayesian posterior distribution with a single value one step further by considering a single  $M_{\max}$  value for all SCRs. This has the benefit of simplicity, because it can be applied in any region regardless of the level of information available. It need not be overly conservative if measures are taken to avoid raising the uncertainty on the occurrence rate of the largest magnitudes to unrealistic levels. Earthquakes with magnitude equal to  $M_{\max}$  are rare: the recurrence interval for  $M_w$  7.9 in an area of 1,000,000 km<sup>2</sup> characterized by average SCR activity is ~70,000 yrs. However, care should be taken when extending Gutenberg–Richter MFDs of individual distributed-seismicity sources to high  $M_{\max}$  values, a condition that applies wherever there is a large gap between the largest observed magnitude and the assumed  $M_{\max}$ . Commonly, the range of possible  $a$ - and  $b$ -values has to be estimated from catalogs with only few events. This results in large uncertainties that severely impact the predicted frequency of events with magnitude  $M_{\max}$  (Fig. 8), essentially driving the seismic hazard at long return periods. This can be avoided either by increasing the size of area sources or, if one seeks to capture small-scale spatial variability of seismic activity for short return periods, by decoupling the statistics of the largest magnitudes from the small-to-moderate events. The latter procedure is used for the new seismic-hazard map for Canada, in which activity rates up to a threshold magnitude of  $M_w$  6.75 are based on fairly small source zones, whereas the activity rates of earthquakes with magnitudes between 6.75 and  $M_{\max}$  are computed for larger regional zones (Adams, 2011). This is equivalent to saying that the largest earthquakes could occur anywhere with equal probability in these larger zones. Because of the larger number of earthquakes, the frequencies of the highest magnitudes can be better constrained. A single, high  $M_{\max}$  value applied this way would not necessarily increase predicted hazard to higher levels than applying the Bayesian method to individual area sources that are too small.

Our simulations indicate that  $M_{\max}$  cannot presently be reliably estimated using the empirical approach based on tectonic analogy. This is similar to the limitations of  $M_{\max}$  estimation based on statistical extrapolation of earthquake catalogs, for

which it has been shown (Holschneider *et al.*, 2011) that it is impossible to derive confidence intervals for a particular  $M_{\max}$  value unless there is additional information, such as an upper bound accounting for the physical limitation of earthquake size. However, because the posterior distribution strongly depends on this unknown quantity, they conclude that  $M_{\max}$  as used in seismic-hazard assessment is essentially meaningless. Instead, they propose replacing it with the maximum expected magnitude in a particular time window, for which confidence intervals can be computed from an earthquake catalog in the framework of Gutenberg–Richter statistics. The confidence interval and time window should be selected in accordance with the requirements of the hazard assessment (Zöller *et al.*, 2013). Future efforts on the  $M_{\max}$  issue for PSHA could perhaps apply this statistical method at the scale of superdomains, based on a global SCR earthquake catalog. From the empirical side, it would be useful to include paleoseismic data for deriving the observed superdomain  $M_{\max}$  distribution.

## DATA AND RESOURCES

The main characteristics and superdomain assignment for the stable continental region (SCR) domains are from table K-2 in EPRI *et al.* (2012). The SCR recurrence parameters (Gutenberg–Richter  $a$ - and  $b$ -values) per continent are from tables 4–7 in Johnston *et al.* (1994). The completeness magnitudes and years for different SCRs are from tables 3–6 in Johnston *et al.* (1994). Completeness regions are defined in general geographic terms; their precise limits are not indicated. The NUREG-2115 SCR earthquake catalog is located in table K-1 in EPRI *et al.* (2012). 

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## APPENDIX

Random earthquake catalogs following a Poisson process can be generated by sampling interevent times (IET) from an exponential distribution with mean  $1/\lambda$  (compare with [Knuth, 1981](#)):

$$\text{IET} = -\frac{\ln U}{\lambda},$$

in which rate parameter  $\lambda$  corresponds to the mean annual frequency of a particular magnitude, and  $U$  is a random variable drawn from a uniform distribution between 0 and 1. This process is continued until the cumulated interevent time exceeds the desired catalog length and is repeated for each magnitude bin of the magnitude–frequency distribution.

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