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Formulating Natural Hazard Policies under Uncertainty*

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Abstract. Uncertainty issues are crucial in assessing the risk posed by natural hazards and developing strategies to mitigate their consequences for society. The challenges are illustrated by the giant earthquake that struck Japan's Tohoku coast in March, 2011, which was much larger than had been predicted by sophisticated hazard models and so caused a tsunami that overtopped 5–10 m seawalls, causing more than 15,000 deaths and $210 billion damage. Deciding whether to rebuild these defenses and more generally what strategies to employ against such rare events depends on estimating the balance between the costs and benefits of mitigation. Making such estimates is a complex challenge at the intersection of geoscience, mathematics, and economics. The major uncertainty is the probabilities of the rare, extreme events and the waiting or recurrence times between them. The probabilities of these events are difficult to estimate because the physics of earthquake recurrence is not adequately understood, and the short geologic record provides only a few observations. We present a general stochastic model in which the probabilities either are constant with time or depend on the previous history. We then develop models for two hazard policy issues facing Japan. One uses a stochastic model to select an optimum mitigation strategy against future tsunamis by minimizing the sum of the expected present value of the damage, the costs of mitigation, and a risk premium reflecting the variance of the hazard. We also consider whether new nuclear power plants should be built, using a deterministic model that does not require estimating essentially unknown probabilities. These models can be generalized to mitigation policy situations involving other natural hazards.

Key words. differential game, earthquakes, natural hazards, optimization, stochastic processes, tsunami, uncertainty

AMS subject classifications. 60Gxx, 60Hxx

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1. Choosing natural hazard mitigation policies.

In confronting the risks of natural hazards, society can be viewed as playing a high-stakes game of chance against nature. The challenges are illustrated by the complex policy issues Japanese authorities have faced after the great Tohoku earthquake of March 11, 2011. The resulting tsunami overtopped extensive coastal defenses, primarily 5–10 m seawalls, causing more than 15,000 deaths and $210 billion damage [20]. Hence, how the seawalls and other defenses should be rebuilt is a challenging question [21].

Because defenses adequate to withstanding tsunamis as large as that of March, 2011, are too expensive, those planned are about 12 m high, only a few meters higher than the older ones [3]. These would provide protection for the largest tsunamis expected every 200–300 years,

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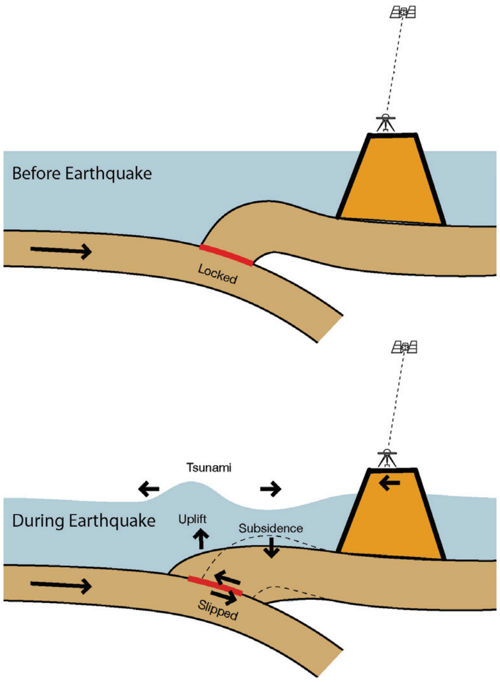

Figure 1. *Schematic illustration of how strain builds up for many years at a subduction zone (top) until it is released in a major earthquake (bottom).*

augmented with land-use planning and improved real-time warning and evacuation procedures to provide protection against much larger tsunamis like that of March, 2011. The defenses should reduce economic losses, while improved warning and evacuations should reduce loss of lives, as shown by the Tohoku experience [1]. However, critics argue that especially in many areas where populations are small and decreasing, it would be more efficient to relocate communities. Otherwise "in 30 years there might be nothing here but fancy breakwaters and empty houses" [21].

A major problem for planning is that the probabilities of such extreme events are very difficult to estimate. The tsunami hazard under consideration results from earthquakes on the plate boundaries along Japan's east coast, where the Pacific plate subducts at the Japan Trench, and the Philippine Sea plate subducts at the Nankai Trough. Although the subducting plates converge on Japan at about 80 mm per year, the plate boundary faults are locked, so strain builds up on them (Figure 1). This strain buildup causes motions of the ground that can be measured using methods including the global positioning system (GPS). Eventually the accumulated strain exceeds the frictional strength of the fault, and it slips in a great earthquake, generating seismic waves that can do great damage near the earthquake. Moreover, the overriding plate that had been dragged down since the last earthquake rebounds and displaces a great volume of water, causing a tsunami that can have devastating effects.

Because this process—known as the earthquake cycle—that gives rise to large earthquakes and tsunamis is irregular in time and space, we cannot reliably predict the timing and magnitude of events. Prior to 2011, most Japanese seismologists assumed that giant magnitude (M) 9 earthquakes would not occur here [2, 23]. The largest future earthquakes were expected to have magnitude between 7 and 8, and tsunami defenses were planned accordingly. However, the Tohoku earthquake had magnitude 9 and generated a huge tsunami that overtopped the sea walls.

Such a giant earthquake was not anticipated due to several incorrect assumptions that reinforced one another [28, 27]. First, the earthquake history from seismology, spanning the roughly 100 years since the invention of the seismometer in the 1880s, appeared to show no record of such giant earthquakes. However, in the decade prior to 2011, increasing attention was also being paid to geological and historical data showing that large tsunamis had struck the area in the years 869 [18], 1896, and 1933. Some villages had stone tablets marking the heights reached by previous tsunamis, warning "Do not build your homes below this point" [8]. However, the revised ideas about maximum earthquake and tsunami size were not yet fully appreciated or incorporated into the Japanese hazard map. Thus, as summarized by Sagiya [23], "If historical records had been more complete, and if discrepancies between data had been picked up, we might have been alert to the danger of a magnitude-9 earthquake hitting Tohoku, even though such an event was not foreseen by the Japanese government." Still, assigning a useful probability to such an event—even in hindsight—would have been very difficult, as is inferring when the next such megatsunami should be expected. As Kanamori [17] notes in discussing why "the 2011 Tohoku earthquake caught most seismologists by surprise," "even if we understand how such a big earthquake can happen, because of the nature of the process involved we cannot make definitive statements about when it will happen, or how large it could be."

Given that the 2011 tsunami was much larger than anticipated, concerns have arisen for the heavily populated Nankai Trough area to the south, where new estimates warning of tsunamis 2–5 times higher than in previous models (Figure 2) raise the question of what to do, given that the timescale on which such events may occur is unknown [4]. In one commentator's words [14], "the question is whether the bureaucratic instinct to avoid any risk of future criticism by presenting the worst case scenario is really helpful... What can (or should be) done? Thirty meter seawalls do not seem to be the answer."

The policy challenge, as posed to the authors of this paper by Japanese economist Hori [15], is "What can we, and should we, do in face of uncertainty? Some say we should rather spend our resources on the present imminent problems instead of wasting them on things whose results are uncertain. Others say that we should prepare for future unknown disasters precisely because they are uncertain."

Our paper approaches this question by exploring optimal methods for selecting strategies to best use society's limited resources, given our limited ability to estimate the occurrence and effects of future destructive rare events. The issue is to decide how much natural hazard mitigation is appropriate. More mitigation can reduce losses in possible future disasters, but at increased cost. Less mitigation reduces costs, but can increase potential losses. Typically these decisions are made politically, without explicitly considering the trade-off between costs and benefits. As an alternative, we illustrate simple models that seek optimum mitigation strategies for two cases in Japan dealing with earthquake and tsunami hazards. These models can be easily generalized to mitigation policy situations involving other natural hazards.

A point worth noting is that defending society against natural hazards is in many ways similar to defending a nation against human enemies. Hence our approach is similar to the

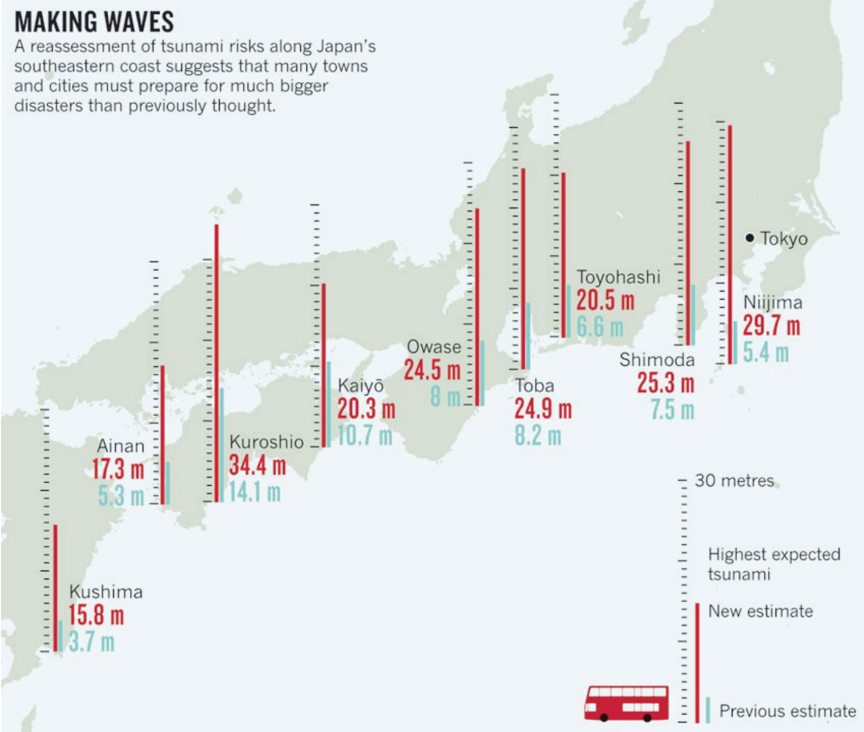

Figure 2. *Comparison of earlier and revised estimates of possible tsunami heights from a giant Nankai Trough earthquake* [4].

cost-benefit analysis approach introduced in the 1960s by Robert McNamara, U.S. Secretary of Defense, for formulating a defense budget to address possible threats [6]. The multidisciplinary systems analysis approach used "is a reasoned approach to highly complicated problems of choice in a context characterized by much uncertainty; it provides a way to deal with different values and judgments.... It is not physics, engineering, mathematics, economics, political science, statistics... yet it involves elements of all these disciplines. It is much more a frame of mind." As McNamara explained, its goal is to decide how much is enough.

2. Earthquake probability observations. Selecting an optimal mitigation strategy depends on estimating the expected present value of future damage. This in turn depends on estimating the probabilities of rare events.

To illustrate the challenge, consider the frequency of large earthquakes. The starting point in all analyses is the Gutenberg–Richter frequency-magnitude relation. As illustrated in Figure 3, using 13,000 earthquakes with magnitudes of $M \geq 5$ for the 30 years between 1968 and 1997, the number N of earthquakes with magnitude greater than or equal to M is approximately

$$N(M) = A10^{-bM} \quad \text{or} \quad \log N = a - bM.$$

The slope b is approximately equal to 1 both worldwide and in specific earthquake zones, and the constant a describes the rate of seismicity. There is an approximately tenfold increase in the number of earthquakes for successively smaller magnitudes: annually around the world

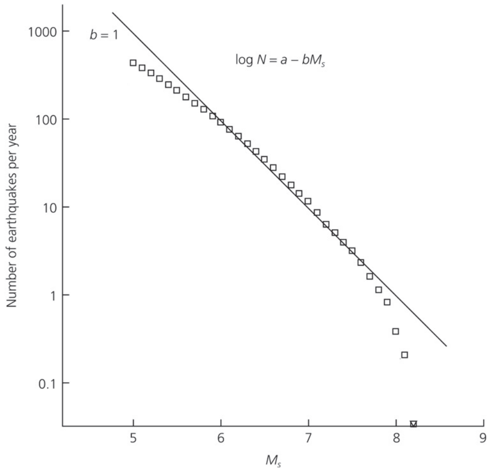

Figure 3. *Gutenberg–Richer relationship for worldwide earthquakes during* 1968–1997 [30].

there is about one magnitude 8 earthquake, 10 magnitude 7 events, 100 magnitude 6 events, and so forth.

This relation applies in individual seismic areas. For example, in a 1300 year period in Japan, about 190 M $>$ 7 and 20 M $>$ 8 earthquakes occurred. Thus Japan has had on average one M $>$ 8 every 65 years. Similarly, if magnitude 9's occur, there should be one on average somewhere in Japan about every 650 years. This analysis does not say whether there actually are magnitude 9's, which was unclear before March, 2011. Now that we know they happen, we also suspect they could happen on any portion of the subduction zone that is long enough, but it is hard to say anything useful about how often they should happen on any particular part, like the Nankai Trough.

Knowing the mean frequency of such rare events based on the historical data gives little insight into when the next event will occur, and hence this is of limited value in mitigation planning, because the present value of future loss depends dramatically on when the event occurs—it is much larger for an event 10 years from now than one 500 years away. Seismologists thus seek to extract additional information from the historical earthquake record. However, even the best earthquake records provide only limited information on earthquake probabilities.

A crucial limitation is that the earthquakes of greatest concern are infrequent, occurring hundreds of years apart on any individual fault. Because the seismometer was invented in about 1890, information about earlier earthquakes comes from historical accounts and geologic records of paleoearthquakes. A famous example is the record of paleoearthquakes at a site on the San Andreas called Pallett Creek, using sand layers that were disturbed by earthquake shaking. The most recent earthquake was in 1857, when historical records show that an earthquake about the size of the 1906 one happened. As shown in Figure 4, earlier large earthquakes can be dated approximately by radiometric dating of organic materials in the deposits.

By analogy to recent large San Andreas earthquakes, each Pallett Creek quake probably involved several meters of motion. Over time, the resulting net motion is easily visible. A classic example is a stream called Wallace Creek that crosses the San Andreas fault (Figure 4).

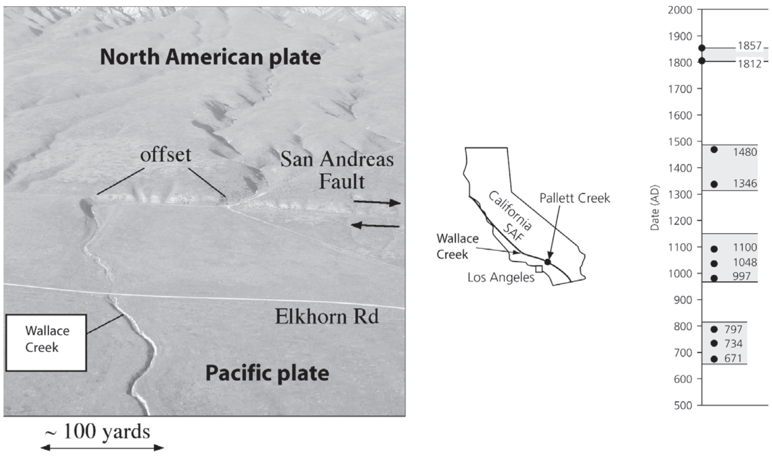

Figure 4. *Geological data for earthquake recurrence on the San Andreas fault. Left: The cumulative motion due to many earthquakes is measured using the offset of Wallace Creek. Right: Paleoearthquake history at Pallett Creek* ([26]).

In the past, the creek ran straight across. But as the Pacific plate moved to the left (north) relative to the North American plate, it shifted the creek so that the parts on either side of the fault are now about 130 m apart. Radiometric dating of charcoal in the streambed shows that this happened over the past 3700 years. Thus over this time the average speed of plate motion here is 130 m / 3700 years, or 35 millimeters per year. The motion over the past few thousand years is the same as that observed today using GPS data.

The large earthquakes appear to have occurred approximately 132 years apart. However, the interval between earthquakes varies from 45 years to 332 years, with a standard deviation of 105 years. Hence, given that the last event occurred in 1857, the simplest analysis predicts the next in 1989 $\pm$ 105, or between 1884 and 2094. This large range is of limited value in hazard mitigation planning.

3. A general earthquake probability model.

The variability shown by earthquake histories on many faults, like that just shown, has prompted the use of various probability density functions (pdfs). The basic choice is between a time-independent Poisson process with no "memory," so that a future earthquake is equally likely immediately after the past one and much later, and various pdfs for time-dependant models, in which the probability of the next large earthquake is small shortly after the past one and increases with time [30]. Pdfs that have been used include Poisson, Gaussian, lognormal, Weibull, and Browning passage time distributions. However, even long paleoseismic earthquake records often cannot resolve well between different pdfs [24, 22, 27].

We thus consider a general probability model of earthquake recurrence, in which the various models currently used can be viewed as specific cases. The history of large earthquakes on a fault is described as a stochastic process via an urn model. At initial time $t = 0$ the urn contains a number $s(0) = s$ of balls representing earthquakes, and a number f representing no-earthquakes. At each time, drawing a ball can give rise to either an earthquake state $S(t)$ or a no-earthquake state $F(t)$. The initial probability of drawing an s-ball, event $S(0)$, is

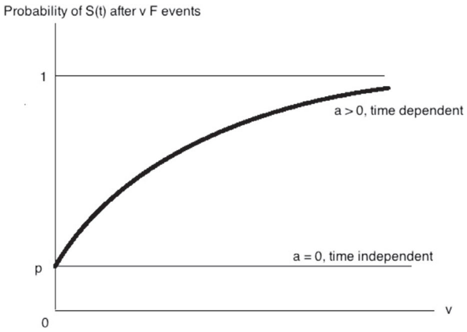

Figure 5. *Comparison of the probability of an earthquake for two cases after an interval* v *with no earthquakes.*

$p = s/(s + f)$. The no-earthquake event occurs with probability $q = 1 - p$.

The evolution of the system is described by adding and removing s-balls while the number of f-balls is constant. Because earthquakes are rare, at most times an f-ball is drawn. If the fault follows an earthquake cycle (Figure 1), stress continues to accumulate on the fault, making an earthquake more likely. We model this increase by adding a fraction $a \geq 0$ of s-balls. Because there are now $s(t) = (1+a)s(t-1)$ s-balls and f is unchanged, the conditional probability that the next draw will give an earthquake becomes

$$P[S(t)|F(t-1)] = [s(t-1)(1+a)]/[s(t-1)(1+a)+f]. \quad (1)$$

Infrequently, an s-ball is drawn and an earthquake occurs. In a cycle model, strain is released, decreasing the probability of a future earthquake. We model this decrease by removing a fraction $r \geq 0$ of s-balls. Because there are now $s(t) = (1-r)s(t-1)$ s-balls, the conditional probability that the next draw will give an earthquake is now

$$P[S(t)|S(t-1)] = [s(t-1)(1-r)]/[s(t-1)(1-r)+f]. \quad (2)$$

Equations (1) and (2) describe the earthquake history as a stochastic process. Typically, there are long intervals between earthquakes, represented as a run of f-balls. Hence a sequence of v f-balls from time $(t-v)$ to time $(t-1)$ will add a fraction $a \geq 0$ of $s-$balls at each time. Because the number of s-balls increases, then at time t the conditional probability that an s-ball will be drawn and an earthquake occur is

$$P[S(t)|F(t-1), F(t-2), \ldots, F(t-v)] = s(1+a)^v/[s(1+a)^v + f]. \quad (3)$$

This time series is described by the concave function in Figure 5. As the length of the run increases, the probability of drawing an s-ball converges to unity. The longer the run of f-balls, the more likely is it that we are due for an earthquake. This situation corresponds to the time-dependant models of earthquake recurrence.

A commonly used case is sampling with replacement, independence, and no memory, such that the probability of drawing an s-ball is constant at its initial value of p. In our model, this corresponds to $a = r = 0$. The straight line in Figure 5 shows the probability of an earthquake at time t after a series of v successive failures/no-earthquakes. As v increases, the probability does not change, and an earthquake is never due. This situation corresponds to time-independent models of earthquake recurrence.

The urn model is thus a general model that can replicate the various models used in earthquake studies, by choosing parameters a and r that describe the processes of strain accumulation between large earthquakes and strain release during these earthquakes. A similar approach can be used for the recurrence of other natural hazard events.

4. Stochastic model for optimal tsunami hazard mitigation.

We first illustrate our approach to inferring optimal policy for natural hazard mitigation by considering the question of how to rebuild Tohoku's tsunami defenses, using a formulation summarized in Stein and Stein [29] and developed further here.

At some point on the coast, we denote the cost of defense construction by $C(n)$, where n is the height of a seawall, which we use as our example, or a measure of mitigation in another method that increases resilience [7], such as the width of a no-construction zone. For a tsunami of height h, the economic loss is $L(h-n)$, where $h-n$ is the height at which a tsunami will overtop a seawall or otherwise exceed a design parameter. $L(h-n)$ is zero for a tsunami smaller than the design value n and increases for larger tsunamis. L includes both the damage and the resulting indirect economic losses, such as those resulting from the destruction of the Fukushima nuclear power plant [19], including the relocation of population and loss of income. The probability of a tsunami overtop of height $h-n$ is $p(h-n)$, so the expected present value of the loss from a number of possible tsunamis over the life of the tsunami wall is

$$Q(n) = E\{L(n)\} = \Sigma_h p(h-n)L(h-n), \tag{4}$$

the expected present value E or sum of losses from tsunamis of different heights weighted by their probabilities.

Thus $p(h-n)$ describes the hazard, the occurrence of tsunamis of a certain size, and $Q(n)$ reflects the present value of the resulting risk, which also depends on the mitigation level n. The expected loss as a function of $p(h-n)L(h-n)$ increases less rapidly with tsunami height than the loss itself for the largest events, because these events are rarer.

The optimum level of mitigation n^* minimizes the total cost $K(n)$, the sum of the expected loss $Q(n)$ and mitigation cost $C(n)$:

$$K(n^*) = \min_n [Q(n) + C(n)]. \tag{5}$$

Because increasingly high levels of mitigation are progressively more costly, the first and second derivatives, $C'(n)$ and $C''(n)$, are positive. Conversely, because increasing mitigation reduces expected loss, the derivative $Q'(n)$ is negative. $K(n)$ illustrates the trade-off between mitigation and damage, because it has a positive minimum at $n = n^*$, the optimum mitigation level (Figure 6(top)). More mitigation gives less expected damage but higher total cost, whereas less mitigation decreases construction costs but increases the expected damage and thus total cost.

Because the expected loss $Q(n)$ and mitigation cost $C(n)$ vary along the coast, the optimal mitigation level also varies. For sparsely populated areas, n^* shifts leftward, implying lower

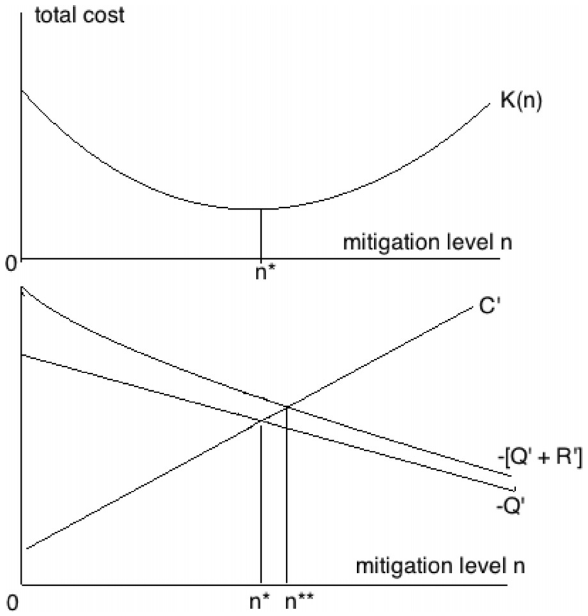

Figure 6. *Top: Variation in total cost, the sum of expected loss and mitigation cost, as a function of mitigation level. The optimal level of mitigation, n^*, minimizes the total cost. Bottom: The same analysis shown by the derivatives. The optimal level of mitigation, n^*, occurs when the gain in mitigation, $-Q'(n)$, equals the incremental construction costs, $C'(n)$. Including the effect of uncertainty and risk aversion, the optimal wall height, n^{**}, occurs when the incremental cost equals the sum of the incremental mitigation and incremental decline in risk $R'(n)$.*

mitigation levels. Where expected losses are greater, such as urban areas or sites of critical facilities, n^* shifts rightward, justifying higher mitigation. How this works can be seen by considering the derivatives of the functions (Figure 6(bottom)). The solution to (5) is

$$C'(n^*) = -[Q'(n^*)], \tag{6}$$

where $n^* > 0$ is the optimum mitigation level.

Because increasingly high levels of mitigation are progressively more costly, the marginal cost function $C'(n)$ increases with wall height. Similarly, $-Q'(n)$ is the marginal benefit mitigation from a small increment of wall size. As the wall size rises, the gain in mitigation decreases. The lines intersect at the optimal point n^*, the highest level to which it pays to build the wall. If the intersection occurs where n^* is positive, it pays to build a wall. However, if even when the wall height is zero the incremental cost of a wall $C'(0)$ is greater than the incremental gain in mitigation $-Q'(0)$, it does not pay to build a wall.

Applying this approach requires calculating $C(n), L(h - n)$, and $p(h - n)$ The first, the mitigation cost, is straightforward. The second requires tsunami models [13] and the present value of predicted losses. Some uncertainty arises because, as Tohoku showed, the effectiveness of the wall constructed can be much less than planned [31]. The largest uncertainty involves estimating the probability of a tsunami of a certain height. The fact that the Tohoku tsunami was much greater than predicted based on the Japanese hazard map [12] showed that earthquake and tsunami hazard models that predict the future occurrences of these events have large uncertainties [27].

The cost $K(n)$ reflects the mean value, the expected loss. For a given mitigation n, the total cost could be higher or lower than $K(n)$ because the tsunami loss can be higher or lower than its expected/predicted value, due to the uncertainty in the hazard model and loss estimate. If the height of the tsunami is lower than predicted, there is a windfall gain. If it is higher than predicted, there is a windfall loss. If the tsunami height is much higher than the predicted/expected value, enormous damage can occur, as in the Indonesian and Japanese tsunamis.

In general one is risk averse in the face of uncertainty. The actual total loss can be far greater than its expected value, as occurred with the Tohoku tsunami. In terms of our model, one does not want a value of mitigation n that minimizes $K(n)$, the expected total costs. Instead, the aim should be a value n^{**} that minimizes the expectation of a convex function of the expected total cost $F[K(n)]$, where the first two derivatives are positive, $F' > 0, F'' > 0$. (See Stein [25, Chapter 4] for a derivation and application of this concept to optimal debt in finance, using stochastic optimal control techniques.)

In this case, losses are more heavily penalized than gains are rewarded. For example, if the wall height/mitigation is based upon the expected value of the tsunami height, the marginal cost will be equal to the *expected* marginal gain. But if the tsunami height is sufficiently above the expectation, disaster strikes and the area is wiped out. Thus focusing upon means/expectations does not address the issue of catastrophes. We thus introduce a term $R(n)$, which is the product of risk aversion α and "risk," defined as the variance $\sigma^2(h-n)$. The variance concerns the range of overtopping $(h-n)$. It has three components: the variance of the height of the tsunami, the variance of the effectiveness of the wall height (quality of its construction), and the covariance of the two. Because the higher the tsunami is above the mean, the less effective the wall is relative to its mean, the covariance is negative and thus increases the total variance

(7) $$\sigma^2(h-n) = \sigma^2(h) + \sigma^2(n) - 2cov(h,n).$$

Let $K(n)$ be, as before, the mean/expected loss $Q(n)$ plus the presumed-known construction costs $C(n)$. We add the new term $R(n) = \alpha\sigma^2(n)$ to the optimization equation and seek a mitigation level that minimizes $K(n) + R(n)$. This is the minimization of the expectation of the convex function F of total loss. Then the optimal mitigation level n^{**} is

(8) $$\min_n [K(n) + R(n)] = \min_n [Q(n) + R(n) + C(n)].$$

In the tsunami example, greater risk aversion and uncertainty increase $R(n)$. The wall height should be increased as long as the marginal mitigation benefit and decline in the risk term $-[Q'(n) + R'(n)]$ exceed the incremental cost of the wall, $C'(n)$. Hence the optimum height increases from n^* to n^{**} in Figure 6(bottom). In this figure, the risk premium decreases as the mitigation level, the wall height, increases. This is because the higher the wall, the smaller the variance of the damage: the upper tail of the hazard is cut off.

In terms of Figure 6(bottom), equation (8) implies that the marginal cost $C'(n^{**})$ is equal to the benefit from mitigation $[-Q'(n^{**})]$ plus the reduction in the product of variance and risk aversion $[-R'(n^{**})]$:

(9) $$C'(n^{**}) = -[Q'(n^{**}) + R'(n^{**})].$$

Therefore, risk and risk aversion lead to a higher mitigation level, wall height: $n^{**} > n^*$.

One can calculate the cost of mitigation $C(n)$ and estimate the mean and variance of the loss $L(h, n)$ for a predicted hazard. However, the risk aversion coefficient is subjective. Several people in the same situation, facing the same risks, will demand different risk premia.

However, as the Tohoku earthquake illustrates, it is very difficult to estimate the probabilities $p(h)$ of the extreme events posing the highest hazards. There are few observations of these events, such as earthquakes of M > 8 or tsunamis greater than 10 m. Hence, it is often unclear how to describe their occurrence via a pdf. We thus consider next a way to address this situation.

5. Deterministic model for nuclear power plant construction. The destruction of the Fukushima nuclear power plant has prompted intense debate in Japan about whether to continue using nuclear power. On the one hand, there are clear economic and societal benefits to using nuclear power rather than more expensive alternatives. However, on the other hand, there is obvious danger in operating nuclear plants in a nation with widespread earthquake and tsunami risks. Although nonnuclear plants would also be vulnerable, the loss of such plants does not pose the potential dangers associated with the destruction of a nuclear plant.

The question is to optimally balance the costs and benefits to Japan of building nuclear plants. The challenge involved in comparing the costs and benefits is the uncertainty in estimating the likelihood of great earthquakes and megatsunamis. As discussed, this is difficult for the Tohoku coast; we know even less about the Nankai coast to the south, where we have no modern, historical, or geologic observations of megatsunamis, but the Tohoku tsunami suggests that they might occur. Because the stochastic model in the past section requires probability estimates, we consider an alternative deterministic model.

The benefit of nuclear power is its effect upon GDP (gross domestic product, a measure of real national income) and its growth, described by the net return on the capital invested less its cost. The net return on capital is the difference between two components. The first is the direct return and societal gain, including the fact that the nuclear plants produce less pollution and carbon emissions [16]. The second is the losses due to large earthquakes or tsunamis, which for simplicity we term "shocks." The direct and indirect losses due to shocks increase as more capital is invested in the nuclear plants.

Building a power plant involves costs. One has to either borrow funds, which involves future interest payments, or use funds that could have been invested elsewhere at a rate of return. These costs are reflected in the interest payments on the capital. Operating costs are further deductions from the beneficial effects of invested capital.

Combining the benefits and costs, we derive an equation for the growth of real income. The level of real income is the integral of the growth terms. If the net return exceeds the interest costs, investing capital in nuclear plants will increase growth and the level of real income. Otherwise, it is not desirable to invest in nuclear plants, and one would be better off using conventional energy sources.

The real GDP, $X(t)$, grows at a rate

$$(1/X(t))dX(t)/dt = (b - r - vs)k(t), \tag{10}$$

where $k(t)$ is the capital and $(b - r)$ is the return on capital b less the interest rate r. The return on capital is also reduced by shocks, parameterized by s, times a vulnerability factor v. For example, the Fukushima plant would have been much less vulnerable if the auxiliary power supply had been higher above sea level.

The integral of the growth rate is the real GDP. Defining the initial level of real GDP as unity implies that its logarithm is zero. For $X(0) = 1, \log X(0) = 0$. It is convenient to work with constant levels of capital $k(t) = k$. Thus integrating (10) gives

$$\log X(t) = \log X(0) + [(b - r - vs)k]t. \tag{11}$$

Risks involved in building nuclear plants arise from shocks that reduce the growth of real income. Not all shocks are equally likely, because the larger shocks are rarer. We thus estimate the "expected" or "risk adjusted" growth by weighting the shock term by a "likelihood" term reflecting the relative risk of events. This term is in effect an inverse measure of probability, even though we cannot precisely specify the probabilities. Following Fleming [9, 10, 11], we use a mathematically tractable quadratic increasing function of s:

$$q(s) = \exp[(1/2)s^2 t], \qquad \ln q = [(1/2)s^2 t]. \tag{12}$$

The expected real GDP is the product z of $X(t)$ and its "expectation" or "likelihood," where $q(t)$ is an inverse measure of likelihood:

$$z = qX. \tag{13}$$

Z, the logarithm of z, is

$$Z = [(b - r - vs)k] + (1/2)s^2, \tag{14}$$

where we have suppressed the time variable because we are dealing with constant levels of capital.

Our strategy to determine the optimum investment in nuclear plants has two stages. In the first, we ask what is the worst expectation or likelihood of the loss due to shocks. This is not the actual worst outcome, but the likely or expected worst outcome given the quadratic risk function. In the second stage we determine a scale of nuclear plant construction that maximizes the minimum expected real income. This two-stage approach gives the optimal scale of the nuclear plants conditional on the expected worst outcome. In other words, given the harm that nature is most likely to do, this is the optimal level of investment in nuclear plants. It is positively related to the net return on capital invested (less the interest rate) and negatively related to the vulnerability of the plant to the shocks.

The first stage of this optimization is to derive the worst case of *expected* loss in real income. This is the value of the shock parameter s that produces the minimum value of $Z, \min_s Z$. Figure 7 is a graph of (14) as a function of the shock parameter s. The minimum occurs for $s = s^*$:

$$s^* = vk. \tag{15}$$

Here s^* is proportional to k, the value of capital invested, times its vulnerability v.

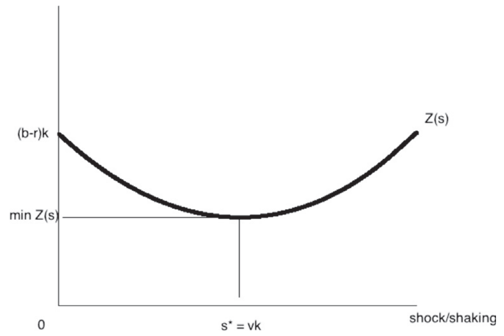

Figure 7. *Z has a minimum,* $\min_s Z$*, at* $s = s^*$*.*

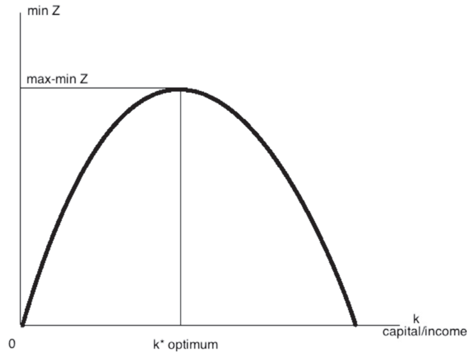

Figure 8. *The optimum capital/income* k^* *maximizes the minimum value of the expected real income.*

The second stage of optimization is to select a size k of nuclear facility that maximizes the minimum expected real income. We derive the optimum value of capital by substituting $s^* = vk$ into (14) for the expected GDP:

$$\max_k [Z = (b - r)k - (vk)^2 + (1/2)(vk)^2] = \max_k [(b - r)k - (1/2)(vk)^2]. \tag{16}$$

The optimal value of capital $k = k^*$ that maximizes $\min_s Z$ is

$$k^* = (b - r)/v^2, \tag{17}$$

as shown in Figure 8.

This is the optimal level of investment in nuclear plants, given the harm that nature is most likely to do to them. If there were no danger of a large tsunami or earthquake, then more capital should be invested, insofar as the return exceeds the interest rate to borrow. If the net return $(b - r)$ were not positive, then there should be no investment in nuclear plants. However, insofar as there is a vulnerability to hazards, there should be less investment in nuclear plants in proportion to the square of vulnerability. Similarly, this result calls for less building of nuclear plants in highly vulnerable areas and more building in less vulnerable areas. A similar analysis applies to other major facilities and could be used for other natural hazards.

6. Conclusions. The uncertainty involved in estimating the probability of the rarest and most damaging events is the major challenge in assessing the risk posed by natural hazards and developing strategies to mitigate their consequences for society. The probabilities of these events are difficult to estimate because the physics of their recurrence is not adequately understood and the short geologic record provides only a few observations. The various probability models used can be treated as specific cases of a general stochastic model in which the probabilities are either constant with time or depend on the previous history.

Formulating strategies to defend society against these hazards involves recognizing and incorporating the uncertainties. We have illustrated two approaches by considering hazard policy issues facing Japan. One involves using a stochastic model to select an optimum mitigation strategy against future tsunamis by minimizing the sum of the expected present value of the damage, the costs of mitigation, and a risk premium reflecting the variance of the hazard. We have also considered the question of whether new nuclear power plants should be built, using a deterministic model that does not require estimating essentially unknown probabilities.

Both models are schematic in that they illustrate approaches rather than detailed implementations. One simplification is that they focus on property losses and do not explicitly address life safety issues. The tsunami mitigation analysis assumes, as is increasingly recognized to be the case, that life safety is better addressed by tsunami warning systems that allow evacuations [1]. The nuclear plant example implicitly includes life safety in the indirect costs of a disaster.

Although our discussion has been in terms of earthquakes and tsunamis, similar analyses could be used for other natural hazards, including river flooding and hurricanes. For example, the flooding of areas of New Orleans in 2005 by hurricane Katrina caused about 1800 deaths and damage estimated at $108 billion, making Katrina the costliest hurricane in U.S. history. Whether the flood defenses that failed should be rebuilt to withstand a similar hurricane or the expected much larger ones could be explored. Similarly, our approach could be used to explore policies to mitigate the effects of global warming. Although the magnitudes of these effects are uncertain, our formulation can be used to develop strategies by exploring the range of possible effects [16], including the increased threat to coastal communities from hurricanes [5] and rising sea level.

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