AST6112: Homework Set 2

Deadline for submission: Nov. 28, 2013
Total marks 100.
Weight to grade 30%.

1. Consider a planet formation region at 1 AU from a 1 solar mass star. Following Armitage, estimate the following timescales and place them in increasing order: 20

   (a) timescale for ~ 10 cm rocks settling to mid-plane,

   (b) timescale for a ~ 10 cm rocks to migrate into the star,

   (c) timescale for a $\sim$ 6 km planetesimal to migrate into the star, (you can assume $\eta \sim 2 \times 10^{-5}$, if you havent already estimated it yourself)

   (d) timescale for planetessimal formation via Goldreich-Ward mechanism,

   (e) timescale for type II migration,

   (f) timescale to form $1\,M_{\oplus}$ planet assuming in situ ordered growth with $F_g \sim 10$, (Here in situ just means that the planet formed in its current location, rather than forming elsewhere and migrating to its present location.)

   (g) timescale to form a $10\,M_{\oplus}$ core assuming in situ ordered growth with $F_g \sim 10$,

   (h) timescale for dispersal of gas from protoplanetary disk,

   (i) observation-based timescale for final assembly of the Earth (~ 100 Myr),

   (j) observation-based timescale for dispersal of the gas in a protoplanetary disk, (See dePater & Lissauer Sec 13.4.4).

2. (a) Will convective heat transfer work in interstellar space? Why? If not, what other advection process can work? 3

   (b) Assume that a planet's atmosphere is in hydrostatic equilibrium. Using this, derive the height (z)-dependent pressure and density structures. 10

   (c) Under what condition the pressure (H_p) and density (H_ρ) scale heights are equal? 2

3. (a) Derive the wet adiabatic lapse rate $\frac{dT}{dz}$ assuming a single volatile in the atmosphere (you can follow the treatment in dP&L Chapter 4). 5

   (b) dP&L 4.14 10

   (c) Read Section 3.2 - 3.3 in dP&L. Derive equations for

      i. $\left(\frac{dT}{dz}\right)_{\text{rad}}$ 5

      and

      ii. $\left(\frac{dT}{dP}\right)_{\text{rad}}$ 5

   for an atmosphere in radiative equilibrium.

4. dP&L 3.26. 10

5. (a) How will you modify the dry adiabatic lapse rate equation if g is z-dependent? Find an expression for T as a function of z. You can assume that the atmosphere has no component gradient (c_p is constant). 8

   (b) Comment on the necessity of this consideration on Earth and on Titan (You can find the altitude of the tropopause in dP&L Figure 4.1). 2

6. dP&L 10.9 5

7. dP&L 8.5 5

8. dP&L 6.2 10

Other suggested problems to solve (no need to submit, won't be graded).

9. dP&L 6.3

10. dP&L 9.10

11. dP&L 9.15

12. dP&L 3.5 part (a) only.

13. dP&L 4.33 for Earth, Mars, and Titan.

14. Consider planet formation around a 0.3 solar mass star.

    (a) Estimate the distance to the habitable zone by calculating the semi-major axis for which a planet would receive the same stellar irradiation as Earth. (You can assume $L_\star \sim M_\star^4$).

    (b) What is the isolation mass at this separation, assuming that the disk mass and the disk surface density of planetesimals both scale with the stellar mass (i.e., disk surface density of planetesimals of $\sim 3.3\,\mathrm{gcm}^{-2}$ at 1 AU for a 0.3 solar mass star)?

    (c) What is the timescale to form such a protoplanet, assuming in situ ordered growth as a function of F_g?

    (d) Does reaching isolation mass within 10 Myr require strong gravitational focusing?

15. dP&L 8.16