1. (a) Field of View:

Large FOV with many stars — transit is low prob. event, so need a lot of targets.

(b) Must be inactive & stable — To reduce noise

Bright enough — To reduce photon limited noise

(c) at least 3 years — 1 year period, 3 transits to confirm

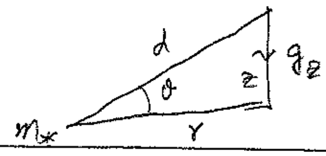
(d) Few hours — Kepler data shows a lot of few day orbits. So some fraction of a few days would be good.

(e) $\left(\frac{R_\oplus}{R_\odot}\right)^2 > 5$ Duration 3 years

(f) Must have higher freq. data. TTV signal is ~ minutes.

So $\lesssim$ minutes.

2.



(a) $\frac{dP}{dz} = -\rho g_z$

(b) $= -\rho \frac{G m_*}{d^2} \sin\theta = -\rho \frac{G m_*}{d^2} \cdot \frac{z}{d} = -\rho \frac{G m_*}{d^3} z.$

For $z \ll r$

$$d^2 = (r^2 + z^2) = r^2\left(1 + \left(\frac{z}{r}\right)^2\right) \simeq r^2$$

Replacing,

$$\frac{dP}{dz} \simeq -\rho \frac{G m_*}{r^3} z = -\rho \Omega^2 z$$

Using, $P = \rho c_s^2$ & using $\frac{dc_s^2}{dz} = 0$ we find

$$\frac{dP}{dz} = c_s^2 \frac{d\rho}{dz} = -\Omega^2 \rho z$$

$$\frac{d\rho}{dz} = -\frac{\Omega^2}{c_s^2} \rho z$$

$$\frac{d\rho}{\rho} = -\frac{\Omega^2}{c_s^2} z\, dz$$

Integrating, $\ln \rho \Big|_{z=0}^{z} = -\frac{\Omega^2}{2c_s^2} z^2 \Big|_{z=0}^{z}$

$$\ln\left[\frac{\rho}{\rho(z=0)}\right] = -\frac{\Omega^2}{2c_s^2} z^2$$

$$\rho = \rho_{z=0}\, e^{-z^2 \frac{\Omega^2}{2c_s^2}} \qquad (1)$$

$h \equiv \frac{c_s}{\Omega} \Rightarrow \frac{2c_s^2}{\Omega^2} = 2h^2$

Replacing in (1) we get

$$\rho = \rho_{z=0}\, e^{-\frac{z^2}{2h^2}}$$

(c) Physical meaning of h is the disk scale height.

(d) A flaring disk will have

$$\frac{h}{r} \sim r^{+ve} \quad \text{exponent}$$

$T \sim c_s^2$

hence, $c_s \sim r^{-1/4}$ $\quad h \equiv \frac{c_s}{\Omega}$

$\Omega \sim r^{-3/2}$

Hence, $h \sim r^{-1/4 + \frac{3}{2}} = r^{\frac{5}{4}}$

$\therefore \frac{h}{r} \sim r^{1/4}$ $\quad$ hence $\frac{h}{r}$ increases with increasing r

$\rightarrow$ Flaired disk.

~~Size~~

3. a) Dust → Rock

Size: $\mu m \rightarrow cm - m$

Physical Processes: Coupled with gas, ~~[illegible]~~

Growth: Collisions & sticking

b) Rocks to planetesimals

Size: $cm-m \longrightarrow \sim 10\ km$

Physical Processes: Decoupled from gas

high radial drift

Not easy to grow via collisions only

Growth: Likely gravitational ~~collap~~ instability

Shear instability, streaming instability are a few proposed ways to increase the surface densities to become Toomre unstable.

Must grow fast.

(c) Planetesimal to rocky planets:

Size: $\sim 10\ km \rightarrow$ Several Thousand km

Physical Processes: Core accretion, isolation mass, Type I.

Growth: Gravitationally focused collisions/accretion of small bodies onto a dominant body

d) Rocky planets to gas giants:

Size: several thousand $\longrightarrow \sim 10^{4.5}$ km.

Physical processes: Type II migration, planet-planet scattering

Growth: Collapse of hydrostatically unstable massive gas envelope on the core.

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(e) Rocks have a high radial drift speed inwards $T_{drift} \sim 100$ yr. Since in MMSN the fastest moving rock size ~ 1m, at 1AU it is often loosely called the m-size barrier.
The problem refered to in the name here is that if all rocks fall onto the star at $T \sim 10-100$ yr. Then how can planets grow beyond that on a $\sim$ Myr timescale?

(f)
$$\frac{v_{gas}^2}{r} = \frac{Gm_*}{r^2} + \frac{1}{\rho}\frac{dP}{dr} \qquad (1)$$

(g)
$$\Rightarrow \frac{v_g^2}{r} = \frac{Gm_*}{r^2} + \frac{c_s^2}{P}\frac{dP}{dr}$$

For a power law P. profile $\quad P = P_0\left(\frac{r}{r_0}\right)^{-n}$

$$\frac{dP}{dr} = -n P_0 r_0^{+n} r^{-(n+1)}$$

$$\frac{1}{P}\frac{dP}{dr} = \frac{-n \cancel{P_0} \cancel{r_0^n} r^{-(n+1)}}{\cancel{P_0} \cdot r^{-n} \cancel{r_0^n}} = -n r^{-1}$$

Replacing $\frac{1}{P}\frac{dP}{dr}$ in equation (1)

$$\frac{v_g^2}{\cancel{r}} = \frac{Gm_*}{r^{\cancel{2}}} - \frac{n c_s^2}{\cancel{r}}$$

$$\Rightarrow v_g^2 = \frac{Gm_*}{r} - n c_s^2 \qquad (2)$$

the keplerian speed

Note that $v_k = r\Omega_k = \left(\frac{G m_*}{r}\right)^{1/2}$. Replacing this identity in eqn 2 we get

$$v_g^2 = v_k^2 - n c_s^2 = v_k^2\left[1 - n\frac{c_s^2}{v_k^2}\right].$$

(h) $\frac{v_g^2}{v_k^2} = \left[1 - n\frac{c_s^2}{v_k^2}\right].$

For $\frac{c_s}{v_k} = 0.05$ & $n = 3$ we get

$\frac{v_g^2}{v_k^2} = 0.9925 \rightarrow \frac{v_g}{v_k} = 0.9962.$

~~$v_g - v_k$~~ $\frac{v_k - v_g}{v_k} = 0.004.$

(i) Keplerial velocity $\left(\frac{Gm_*}{r}\right)^{1/2}$ at 1AU

$\simeq 30$ km/s

Hence the magnitude of relative velocity of gas and rock will be $\simeq$ 120 m/s.

4. a) They are Types I, II, & III.

b) Type: | Mechanism

Type:	Mechanism
I	Gravitational torques exerted by the disk on the planet, no gap
II	A gap is opened. The planet moves with the local disk embedded in the gap.
III	Asymmetries in the coorbital torques can lead to very fast runaway migration

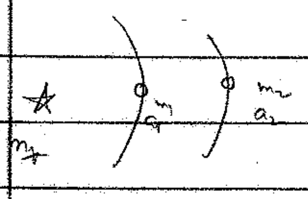
c) Type I : Temperature gradient & magnetic fields under certain conditions can lead to outward type I

Type II : If for some reason the disk is locally moving outward it can also shepherd a planet in its gap outward.

5. a) Disk Migration | Planet Planet Scattering

	Disk Migration	Planet Planet Scattering
$\langle e^2 \rangle$:	low	high
$\langle I^2 \rangle$:	low	high
$\langle \# \text{ of MMR} \rangle$:	high	low.

(b) Total orbital energy $E_{orb} = -\frac{G m_* m}{2a}$ (1)



Largest energy change of an orbit is equivalent to the largest a change.

Due to the structure of the problem the ~~inner~~ outer planet can get at most the energy in the inner orbit. The smaller the a_1, the higher is the energy content that it can donate to planet 2. If a_1 is further from a_2 than δ where δ is the Gladman criteria then they do not strongly interact. So, the largest energy change possible for planet 2 is at that separation

Take log on both sides of (1) & differentiate

$$\frac{dE_{orb}}{E_{orb}} = \frac{da}{a} - \frac{dm}{m}. \qquad dE_{orb} = -\frac{G m_* m_1}{(a_2 - \delta)}$$

$$\therefore \frac{da}{a} = \frac{dE_{orb}}{E_{orb}} + \frac{dm}{m}.$$

In this case the max $dm \sim m_1$ & max $dE_{orb} \sim -\frac{G m_* m_1}{(a_2-\delta)}$

$$\Rightarrow \left.\frac{da_2}{a_2}\right|_{max} = \frac{m_1}{m_2}\frac{a_2}{(a_2-\delta)} + \frac{m_1}{m_2} = \frac{m_1}{m_2}\left[1 + \frac{a_2}{(a_2-\delta)}\right].$$

(c) From the solution in (b)

$$\frac{da_2}{a_2} \approx \frac{m_1}{m_2}\left[1 + \frac{a_2}{(a_2-\delta)}\right].$$

In this case $m_1 \rightsquigarrow \delta m$

$m_2 \rightsquigarrow m_p$. $a_2 \rightsquigarrow a$

~~For each interaction, the maximum~~

$$\frac{\delta a}{a} \simeq \frac{\delta m}{m_p} \cdot 2$$

For $\delta \ll a_2$

$\rightarrow$ always the case for star-planet systems.

$\therefore$ For $\frac{\delta a}{a} \sim 1$

$\sum_{\delta m} \frac{\delta m}{m_p}$ must be ~ 1

$\therefore$ $\sum \delta m \sim 0.5\, m_p \sim m_p$.