Exam 1
AST6112: Fall 2003

October 18, 2013

Full Marks 80

Weight towards grade: 30%

Full Name:

---

1. Consider that you are going to write a large survey proposal for transiting planet search. What are the considerations you should keep in mind about the following and why? (Total 15)

   (a) Field of view. 2

   (b) Stellar properties. 4

   (c) Duration of search. 2

   (d) Frequency of data collection. Note: Remember, you cannot collect and send infinite amount of data to Earth. 2

   (e) If you want to detect an Earth-like planet around a Sun-like star, what should be your detector precision, and duration of search? 3

   (f) If you are interested in detecting transit timing variation signals how should you modify the frequency of data collection? 2

2. Here we will derive the vertical structure of a passive circumstellar disk using the hydrostatic equilibrium condition. (Total 10)

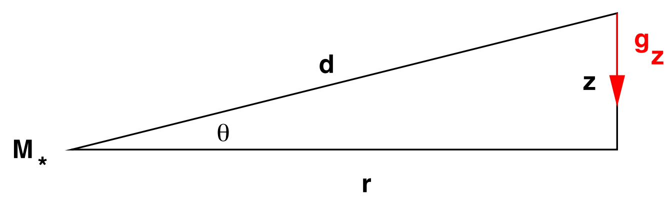

Figure 1: Geometry for calculation of the vertical hydrostatic equilibrium of a circumstellar disk.

(a) Using Figure **??** write down the vertical hydrostatic equilibrium equation for a passive, non-self-gravitating disk around a star of mass $M_\star$. 2
Hint: This equation should relate vertical pressure gradient, $\frac{dP}{dz}$, with density, ρ.

(b) Solve for the density structure in this disk. At the end of this stage you should obtain $\rho = \rho(z, h)$. Use $h \equiv \frac{c_s}{\Omega}$, where Ω is the angular velocity of the gas. 4
Hint: You may want to use the relation between pressure and density: $P = \rho c_s^2$, where c_s is the sound speed. You may use Ω is close to Keplerian. You can also use $z \ll r$.

(c) What is the physical meaning of h? 1

(d) Assume that the temperature profile in the disk is $T \sim r^{-1/2}$. Show that the disk will be flared. 3
Hint: How is T related to c_s?

3. In the core-accretion paradigm planets form by growing from dust grains to fully formed planets. Describe the dominant physical processes, growth mechanism, and size scales at the following distinct stages of planet formation. (Total 30)

(a) Dust to rocks. 3

(b) Rocks to planetesimals. 3

(c) Planetesimals to rocky planets. 3

(d) Rocky planets to gas giants. 3

(e) During the growth stage between rocks to planetesimals, a serious problem is the often referred loosely as the "meter-size barrier". Describe what this is and what physical effect is responsible for this. 3

In the following parts we will derive some important numbers regarding this effect.

(f) Write down the radial component of the force equation for gas. 3
Hint: The pressure is force per unit area. What is momentum due to radial pressure gradient $\frac{dP}{dr}$? Momentum due to pressure gradient should be a function of $\frac{dP}{dr}$ and density ρ. You should be able to write down azimuthal gas velocity $v_{\phi,\mathrm{gas}}$ as a function of star mass, $M_\star$, distance from star, r, and pressure gradient.

(g) Solve for the gas azimuthal velocity $v_{\phi,\mathrm{gas}}$ as a function of the Keplerian velocity v_K, the sound speed c_s, and n, where $P = P_0 \left(\frac{r}{r_0}\right)^{-n}$. P_0 and r_0 are normalization constants. Sound speed is related to pressure and density by $P = \rho c_s^2$. 8

(h) What is the fractional difference between $v_{\phi,\mathrm{gas}}$ and v_K assuming $c_s/v_K = 0.05$, and $n = 3$? 2

(i) What is the relative velocity between decoupled particles traveling in gas at a distance of 1 AU from a star of mass $1\,M_\odot$? 2

4. After planets are fully formed, interactions between planets and the gas disk lead to changes in the planets' orbital properties. These changes are regularly called migration since the planets move from their birth places. (Total 10)

   (a) What are the different types of disk-driven migration? 3

   (b) What are the dominant mechanisms for driving each of the above migration types? 4

   (c) What disk properties can lead to outward migration via which type of migration? 3

5. After formation, planetary orbits can change via two major ways: planet-gas disk migration, and planet-planet scattering. There are clear differences in the observable final planetary orbital properties created via either mechanism. The following questions relate to our understanding of these two very different migration mechanisms and planet-planet scattering. (Total 15)

   (a) Compare the expected orbital properties of planets if they are created via planet-disk migration vs planet-planet scattering. 4

   (b) Planet-planet scattering is a chaotic process and there are very few things that can be estimated analytically. This is one of the few. Consider a two planet system. The inner and outer planet masses are m_1 and m_2, respectively. The inner and outer planets are orbiting at a semimajor axes a_1 and a_2, respectively. Write down an expression for the maximum change in a_2 possible via scattering between the two planets. 7
   Hint: Semimajor axis determines the orbital energy. For a maximum change in semimajor axis, you need the maximum allowed change in orbital energy. Gladman stability criteria for two planets is $\delta a/a > 2.4\left(\frac{m_1+m_2}{M_\star}\right)^{1/3}$, where δa is the mutual distance between the planets, $M_\star$ is the mass of the star.

   (c) Now consider planet-planetesimal scattering. Assume that a planet of mass m_p is scattering with planetesimals of mass δm. Assume, $m_p \gg \delta m$. Show that to obtain $\frac{\delta a}{a} \sim 1$ (order one change) the total mass in the planetesimal disk must be at least $\sim m_p$. 4