1. a) $t_{settle} = \frac{2}{\pi} \frac{\Sigma}{\Omega_K \rho_d a} e^{-z^2/h^2}$ (eq. 110)

also can use eq. 116

Then integrate to get t_{sett}

Use $z \sim h$ $\Sigma \propto$

List of typical properties at 1AU
$\Sigma \propto 10^3 \, g \, cm^{-2}$
$\rho_d \sim 3 \, g \, cm^{-3}$
$\rho \sim 5 \times 10^{-10} \, g \, cm^{-3}$ (off midplane)
$\lambda \sim 1 \, cm$.
$c_s \sim 1.5 \times 10^5 \, cm \, s^{-1}$

For $a \sim 10$ cm

$t_{settle} \sim 3$ yr.

b) Epstein & $\boxed{Re < 1 \quad \text{Stokes regime}}$ make sure

η ~~$\sim$ yr~~

$\sim n \frac{c_s^2}{v_k^2}$

Using $n \sim 3$

$\frac{h}{r} \sim \frac{c_s}{v_k} \sim 0.05$ $\eta \sim 0.008$

$\frac{v_r}{v_K} \sim \frac{-\eta}{\tau + \tau^{-1}}$ (128)

For 10 cm $\tau \sim 1$ maximal drift speed

$v_r = -\frac{\eta}{2} v_K \sim 100 \, m/s$

timescale for drifting to the star $t_d \sim 50$ yr.

[Note: depending on what you assume in the values it can be few 10s to 100 yr].

c) Reynolds number $Re = \frac{2av}{\lambda c_s}$ [(102) & (72)].

For 6 km $Re \sim \frac{2a\eta^{1/2} v_K}{\lambda c_s}$ $\eta \sim 10^{-5} \times 2$

$\sim$ Large > 800 $C_D = 0.44$

$$F_D = -\frac{1}{2} C_D \pi a^2 \rho v^2$$

$$= \frac{1}{2} \cdot 0.44 \cdot \pi \cdot (8\,km)^2 \cdot (5e^{-10}) \cdot \eta v_k^2$$

You can thus calculate t_{fric} using

$$t_{fric} = \frac{mv}{|F_D|} \qquad (104)$$

~~I will simply use eq. (105) to avoid doing all of this myself.~~

[Note: All this is necessary since (105) is not valid in this size range].

$$t_{fric} = \frac{mv}{|F_D|}$$

$$= \frac{\frac{4}{3}\cancel{\pi} a^{\cancel{3}} \rho_d \cancel{v}}{\frac{1}{2} C_D \cancel{\pi} \cancel{a^2} \rho v^{\cancel{2}}}$$

$$= \frac{8}{3} \cdot \frac{1}{C_D} \cdot \frac{\rho_d}{\rho} \cdot \frac{a}{v}$$

$$\tau \equiv \Omega t_{fric} = r v_k \cdot t_{fric}$$

$$= r \cancel{v_k} \cdot \frac{8}{3} \frac{1}{C_D} \frac{\rho_d}{\rho} \cdot \frac{a}{\eta^{1/2} \cancel{v_k}}$$

$$= \frac{8}{3 C_D} \cdot \frac{\rho_d}{\rho} \cdot \frac{a r}{\eta^{1/2}}$$

Check values may not be correct. The method & equations should be correct.

Putting values

$\rho_d = 3\ g\,cm^{-3}$

$\rho = 5e^{-10}\ g\,cm^{-3}$

$C_D = 0.44$

$a = 8\,km$

$r = 1AU$

$\eta \sim 2e^5$

$\tau \sim 10^{27}$

$v_r \sim -v_k \cdot 10^{-22}$

$t_{d,8km} \sim 10^{21}\ yr.$

d) This happens on a free fall timescale

$$t_{ff} = \sqrt{\frac{\lambda_{crit}^3}{2Gm}}, \qquad \lambda_{crit} = \frac{2c_s^2}{G\Sigma_{solid}}$$

$$= \sqrt{\frac{4\cancel{8}\, c_s^6}{\cancel{2}\, G^4 \Sigma^3 m}} \qquad \sim 3\times10^{16}\ \text{cm}.$$

Using $\Sigma \sim 10^3\ \text{g cm}^{-2}$, $c_s \sim 10^5\ \text{cm s}^{-1}$, $m \sim 10^{18}\ \text{g}$

$\Sigma_{solid} \sim 10\ \text{g cm}^{-2}$

$t_{ff} \sim 1$ hour.

d) $Q = \dfrac{\sigma\Omega}{\pi G \Sigma}$

For this use $\Sigma = \Sigma_{dust} \sim 10^{-2}\Sigma \sim 10\ \text{g cm}^{-2}$

Setting $Q = 1$ you get $\sigma = 10\ \text{cm s}^{-1}$

$\lambda_{crit} \equiv$ the most unstable wavelength $= \dfrac{2\sigma^2}{G\Sigma_{dust}}$

$\simeq 3\times10^{8}\ \text{cm}.$

$m \sim \pi \Sigma_{dust} \lambda_{crit}^2 \sim 3\times10^{18}\ \text{g}.$

$$t_{ff} = \sqrt{\frac{\lambda_{crit}^3}{2Gm}} = 0.3\ \text{yr}.$$

e) $t_{type\,II} \sim \frac{2}{3\alpha}\left(\frac{h}{r}\right)^{-2}\Omega^{-1}$

$\alpha \sim 10^{-2}$

$\frac{h}{r} \sim 0.05$

$\Omega_{1AU} \sim 2e^{-7}\ s^{-1}$

$\sim 4e^3$ yr.

f) ~~dM~~ This is growth by core accretion

$$\frac{dM}{dt} = \frac{1}{2}\Sigma_p \Omega \pi R_s^2 F_g.$$

Assume $F_g = 10$.

$$M = \frac{4}{3}\pi \rho_d R_s^3$$

$$dM = \frac{4}{\cancel{3}}\pi \rho_d \cancel{3} R_s^2\, dR_s = 4\pi \rho_d R_s^2\, dR_s.$$

$$4\cancel{\pi} \rho_d \cancel{R_s^2} \frac{dR_s}{dt} = \frac{1}{2}\Sigma_p \Omega \cancel{\pi} \cancel{R_s^2} F_g$$

$$\frac{dR_s}{dt} = \frac{1}{8\rho_d}\Sigma_p \Omega F_g$$

$1M_\oplus$ equivalent to $1R_\oplus$

$$\int dR_s = \frac{\Sigma_p \Omega F_g}{8\rho_d}\int dt$$

$$\tau_{form,\,1M_\oplus} \sim \frac{8\rho_d}{\Sigma_p \Omega F_g} \cdot R_\oplus \sim 24\times10^6\ \text{yr}.$$

Note: This is long. Indicates F_g must be very high $\sim 10^3$ or higher

g) For $10M_\oplus$ with $F_g = 10$. $10M_\oplus \sim 10^{1/3} R_\oplus$

$$\tau_{form,\,10M_\oplus} \sim 10^{1/3}\,\tau_{form,\,1M_\oplus} \sim 52\ \text{Myr}.$$

h) ~~for to~~ For SS disk $\alpha \sim 10^{-2}$ & $\left(\frac{h}{r}\right) \sim 0.05$

Viscous timescale

$$\tau \sim \left(\frac{h}{r}\right)^{-2} \frac{1}{\alpha \Omega} \sim 1\,\mathrm{Myr}$$

i) 100 Myr (given)

j) $10^6 - 10^7$ yr.

2. a) No.

There is no gravity. Convection needs bouyancy forces. Diffusion will work since it just depends on the random thermal motion of molecules.

b). Hydrostatic equilibrium

$$dP = -g \rho \, dz' \qquad\qquad P = NkT = \frac{\rho k T}{\mu_a m_{amu}}$$

$$\frac{dP}{dz'} = -g \frac{P \mu_a m_{amu}}{kT}$$

$$\frac{dP}{P} = -g \frac{\mu_a m_{amu}}{kT} dz'$$

$$P = P(0) \exp\left\{ -\int_0^z dz'/H(z') \right\} \qquad \text{where, } H_p(z) = \frac{kT}{g \mu_a m_{amu}}$$

Similarly, by replacing P by ρ you will find

$$H_\rho(z) = \left[\frac{1}{T(z)} \cdot \frac{dT}{dz} + \frac{g \mu_a m_{amu}}{kT} \right]^{-1}$$

c) If in a region $\frac{dT}{dz} = 0$, $H_p = H_\rho$.

3, Remember from simple thermodynamics we derived
(a)

good idea to make sure you understand how this is derived (in class)

$$\frac{dT}{dP} = \frac{1}{\rho C_p}$$ For dry case. For wet case, $C_p\, dT = \frac{1}{\rho} dP - L_s\, d\omega_s$

$$\Rightarrow \frac{dP}{dT} = \rho\left[C_p + L_s\, d\omega_s/dT\right] \quad (1)$$

From hydrostatic equilibrium, $\frac{1}{\rho}\frac{dP}{dz} = -g$ (2)

$$\frac{dT}{dP} = \frac{dT}{dz}\cdot\frac{dz}{dP} = -\frac{dT}{dz}\cdot\frac{1}{\rho g} \quad \text{using (2)}$$

Using (1)

$$\frac{dT}{dP} = \frac{1}{\cancel{\rho}\left[C_p + L_s\, d\omega_s/dT\right]} = -\frac{dT}{dz}\,\frac{1}{\cancel{\rho} g}$$

$$\frac{dT}{dz} = -\frac{g}{C_p + L_s \frac{d\omega_s}{dT}}$$

b) dP(a) $\frac{dT}{dz} \sim 10\ \mathrm{K\,km^{-1}} = -\frac{g}{C_p}$

$$C_p \approx \frac{g}{dT/dz} \approx$$

$$\sim \frac{\cancel{10}\times 10^2\ (\mathrm{cm\ s^{-2}})}{\cancel{10}\times 10^{-5}} \sim 10^7\ \mathrm{erg/K/g}$$

dPL(b) Saturation vapor pressure

$P = q\, e^{-L/R_{gas}T}$

$$= 3\times 10^7\ (\mathrm{bar})\ e^{-\frac{5.1\times 10^{11}\ \mathrm{erg\ mole^{-1}}}{280\mathrm{K}\cdot 18}}$$

$R_{gas} \sim \mathrm{erg/g/K}$

$R_g \sim 8.3\times 10^7\ \mathrm{erg/K/mole}$

dPL (b) $P_{sat} = C_1 \, e^{-\frac{L_s}{R_g T}}$

$R_g = 8.31 \times 10^7$ erg/K/mole
$T = 280$ K
$L_s = 5.1 \times 10^{11}$ erg/mole

$$\frac{L_s}{R_g T} = \frac{5.1 \times 10^{11}}{8.31 \times 10^7 \cdot 280} \frac{\cancel{erg}\ \cancel{K}\cdot \cancel{mole}}{\cancel{mole}\cdot \cancel{erg}\ K}$$

≈ 21.92 (make sure this is dimensionless).

$$P_{sat} = 3 \times 10^7 \cdot e^{-21.92} \text{ bar} \simeq 9 \times 10^{-3} \text{ bar}$$

So the partial vapor pressure $\sim P_{sat}/p \sim 9 \times 10^{-3}$.

dPL (c) Mean molecular mass of air assuming 20% O_2 & 80% N_2

$$= \left[.2 \times \underset{O_2}{32} + .8 \times \underset{N_2}{28} \right] \text{ g/mole}$$

$$= 28.8 \text{ g/mole}$$

Water = 18 g/mole

$$\therefore \omega_s = 9 \times 10^{-3} \cdot \frac{18}{28.8} = 5.6 \times 10^{-3}$$

dPL (d) $L_s = 5.1 \times 10^{11}$ erg/mole $= \frac{5.1 \times 10^{11}}{18}$ erg/g $= 2.8 \times 10^{10}$ erg/g

$$\frac{dT}{dz} = \frac{g}{C_p + L_s \, d\omega_s/dT} \sim \frac{980}{10^7 + \frac{2.8 \times 10^{10} \times 5.6 \times 10^{-3}}{280}}$$

$$= 9.3 \times 10^{-5} \text{ K/cm}$$

$$= 9.3 \text{ K/km}.$$

Note: Things will be more complicated if g is not constant.

c) (i) Done in Sec. 3.3.1 Convince yourself.

~~Yours~~

(ii) From hydrostatic equilibrium

$$\frac{dT}{dp} = -\frac{1}{\rho g}\frac{dT}{dz}$$

4. dPL (3.26)

(a) $F = \sigma T^4_{effective}$

$T_e = \left(\frac{F}{\sigma}\right)^{1/4}$

$F = \frac{L_\odot}{4\pi r^2} \pi R^2 (1-A_b)$ incident flux

Rapidly rotating means T variation is low as a function of longitude.

Equilibrium temperature.

$P_{in} = (1-A_b)\ \frac{L_\odot}{4\pi r^2} \pi R^2$

$P_{out} = 4\pi R^2 \epsilon \sigma T_{eq}^4$

$P_{in} = P_{out}$ in equilibrium

$$4\cancel{\pi R^2} \epsilon\sigma T_{eq}^4 = (1-A_b)\frac{L_\odot}{4\pi r^2}\cancel{\pi R^2}$$

$$T_{eq} = \left[\frac{(1-A_b)\ L_\odot}{16\pi\epsilon\sigma r^2}\right]^{1/4}$$

Effective ~~T~~ is = equilibrium T.

b) Using 3.78 & 3.79.

$$T_e^4 = 2T_0^4 \Rightarrow T_0 = \left(\frac{1}{2} T_e^4\right)^{1/4} = 0.84\, T_e.$$

c) $$T^4 = T_0^4\left(1+\frac{3}{2}\tau\right)$$

$$= \frac{1}{2} T_e^4 \left(1+\frac{3}{2}\tau\right).$$

$T = T_e$ if $\tau = \frac{2}{3}$.

d) $$T_g^4 = T_e^4\left(1+\frac{3}{4}\tau_g\right) \qquad \tau_g = 10$$

$$= T_e^4\left(1+\frac{30}{4}\right)$$

$$= 8.5\, T_e^4$$

5\. a) $$\frac{dT}{dz} = -\frac{g(z)}{c_p}$$

$$g(z) = \frac{GM}{(R+z)^2}$$

$$= \frac{\frac{4}{3}\pi R^3 \rho G}{(R+z)^2}$$

$$= \underbrace{\frac{4}{3}\pi \rho R G}_{C} \frac{1}{\left(1+\frac{z}{R}\right)^2}$$

$$= C\left(1-\frac{2z}{R} + \text{higer order terms}\right)$$

~~At the top of troposphere (tropopause)~~

b) For Earth $\frac{z}{R} \approx 0.003$

For Titan $\frac{z}{R} \sim 0.02$

For Earth error $\sim 0.6\%$

For Titan error $\sim 4\%$

More Important to consider This on Titan.

6. dPL 10.9. Asteroid $B_{ast} \propto \frac{1}{r_\odot^2 r_\Delta^2} \Rightarrow B_{ast} = C_1 \frac{1}{r_\odot^2 r_\Delta^2}$

$B_{comet} \propto \frac{1}{r_\odot^\xi r_\Delta^2} \rightarrow B_{comet} = C_2 \frac{1}{r_\odot^\xi r_\Delta^2}$

At $r_\odot = 3\,AU$ & $r_\Delta = 2\,AU$ $B_{ast} = B_{comet}$

$$\frac{B_{ast}}{B_{comet}} = \frac{C_1}{C_2} \cdot r_\odot^{(\xi-2)} = 1$$

$$\frac{C_1}{C_2} = r_\odot^{2-\xi} = 3^{(2-\xi)}$$

At $r_\odot = 2\,AU$ & $r_\Delta = 2\,AU$

$$\frac{B_{ast}}{B_{comet}} = \frac{C_1}{C_2} r_\odot^{(\xi-2)} = 3^{(2-\xi)} \cdot 2^{(\xi-2)} = \left(\frac{2}{3}\right)^{(\xi-2)}$$

$$= \frac{3^2}{2^2} \cdot \left(\frac{2}{3}\right)^\xi$$

$$= \frac{9}{4} \cdot \left(\frac{2}{3}\right)^\xi$$

If $\xi = 2$ Then they are the same.

for $\xi > 2$, $B_{ast} > B_{comet}$

7. dPL 8.5

$$v_\infty = \sqrt{\frac{2g_p\, m}{C_D\, \rho_g\, A}}$$

$$\frac{2g \cdot \frac{4}{3}\cancel{\pi} R^{\cancel{3}}\rho}{C_D\, \rho_g \cdot \cancel{\pi R^2}}$$

$$= \frac{8}{3}\,\frac{gR}{C_D}\left(\frac{\rho}{\rho_g}\right).$$

$$= \sqrt{\frac{8}{3}\, g\, C_D^{-1}\, \rho_g^{-1}} \cdot (R\rho)^{1/2}$$

Assume $\rho_g = 10^{-3}\ \mathrm{g\,cm^{-3}}$.

Then just put values & get answers for each part.
Make sure to remember how to derive.

8. dPL 6.2

Pressure exerted by self-gravity. Assume hydrostatic equilibrium either by gravity or by material stregth.

$$\frac{dP}{dr} = -g\rho$$

$$dP = -\frac{GM_r}{r^2}\,\rho\, dr$$

$$= -G\,\frac{\frac{4}{3}\pi r^{\cancel{3}}\rho}{\cancel{r^2}}\,\rho\, dr$$

$$= -\frac{4\pi}{3}G\rho^2 r \cdot dr.$$

$$\int_{P_c}^{0} P = -\int_0^R \frac{4\pi}{3}G\rho^2 r\, dr$$

$$P_c = \frac{2\pi}{3}G\rho^2 R^2$$

a) $P_c = \frac{2\pi}{3} G \rho^2 R^2$

if $P_c \geq S_m$ center is compressed.

$\therefore R_{min,c} = \left[\frac{3 S_m}{2\pi G \rho^2}\right]^{1/2}$

b) $P_{1/2} =$ $\qquad \frac{1}{2} \frac{4}{3} \pi R^3 \rho = \frac{4}{3} \pi R_{1/2}^3 \rho$

$= \frac{2\pi}{3} G \rho^2 R^2 - \frac{2\pi}{3} G \rho^2 R_{1/2}^2$ $\qquad R_{1/2} = \left(\frac{1}{2}\right)^{1/3} R$

$= \frac{2\pi}{3} G \rho^2 R^2 \left[1 - \left(\frac{1}{2}\right)^{1/3}\right].$

$R_{min,\frac{1}{2}} = \left[\frac{3 S_m}{2\pi G \rho^2 (1 - (\frac{1}{2})^{1/3})}\right]^{1/2}.$

c) Just change values.

d) Change values.