Screening and Information-Sharing Externalities*

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Abstract

In many settings, multiple uninformed agents bargain simultaneously with a single informed agent in each of multiple periods. For example, workers and firms negotiate each year over salaries, and the firm has private information about the value of workers' output. I study the effects of transparency in these settings; uninformed agents may observe others' past bargaining outcomes, e.g. wages. I show that in equilibrium, each uninformed agent will choose in each period whether to try to separate the informed agent's types (screen) or receive the same outcome regardless of type (pool). In other words, the agents engage in a form of experimentation via their bargaining strategies. There are two main theoretical insights. First, there is a complementary screening effect: the more agents screen in equilibrium, the lower the information rents that each will have to pay. Second, the payoff of the informed agent will have a certain supermodularity property, which implies that equilibria with screening are "fragile" to deviations by uninformed agents. I apply the results to study pay-secrecy regulations and anti-discrimination policy. I show that, surprisingly, penalties for pay discrimination may have no impact on bargaining outcomes. I discuss how this result depends on the legal framework for discrimination cases, and suggest changes to enhance the efficacy of anti-discrimination regulations. In particular, anti-discrimination law should preclude the so-called "salary negotiation defense".

Many ongoing policy debates relate to transparency in negotiations. These include pay transparency among employees, the disclosure of payer-negotiated rates by hospitals, and

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transparency in debt relief negotiations between countries and bilateral lenders. These settings generally feature asymmetric information. Insurers that negotiate rates with a hospital have only partial knowledge of the hospital's costs for performing different services. Workers are generally unsure about how much revenue they will generate for a potential employer. When there is one-sided asymmetric information and the informed party, say a firm, is long-lived, the firm will have incentives to establish a reputation which will keep its future costs down. For example, a firm may be reluctant to increase wages for current workers, since future workers will infer that their value to the firm is high, and demand higher wages as well. On the other hand, when the uninformed party, say a worker, is also long-lived, they may engage in strategic experimentation to try to learn about the firm's private information. For example, early in their career the worker may take a hard-line position in wage negotiations, in the hopes of learning how valuable they are to the firm. This information will be useful to the worker in future negotiations. When there are multiple workers, each one may learn something about their own value to the firm by observing the wages of others. This gives rise to information externalities.

The goal of this paper is to understand the interaction between reputation effects and information externalities. In particular, I am interested in the implications of this interaction for policies that affect the transparency of negotiations, i.e. the amount of information shared between the (initially) uninformed parties. I consider a model in which multiple uninformed parties screen a single agent with an unknown type. I will call the uninformed parties "workers" and the agent the "firm". The firm's type determines the output generated by each worker.

Before describing the model that will be studied in this paper, it will be helpful to establish some basic intuition regarding reputation effects. Reputation effects alone can be understood in a model in which workers arrive sequentially and bargain with the firm over their wage. Consider such a model, in which before negotiating a worker may observe the wages paid to some previous workers. Reputational bargaining games of this sort are studied by Kreps and Wilson (1982), Fudenberg and Levine (1989), and Schmidt (1993). In general, workers who observe that previous workers were paid a high wage will infer that the firm is the high-value type, and demand high wages themselves. Thus the firm will be unwilling to pay high wages, for fear of damaging its reputation. Increases in negotiation transparency, for example example an increase in the probability that each worker observes the wages of those

¹The use of the terms "worker" and "firm" is for exposition only. The model applies equally well to any setting in which multiple principals simultaneously screen an informed agent, such as hospital-insurer negotiations.

that came before, strengthen the firms reputational incentives, and lead to lower wages.² In other words, the information rents that the workers must pay in order to separate the firm types increase in the degree of information sharing. When these rents become too large, workers will give up trying to screen the firm, and settle for a fixed wage independent of type.

While the effect of observing past bargaining outcomes in the sequential setting is well understood, less is known about the information externalities that arise in situations in which negotiations take place with multiple uninformed parties each period. This is the case in many prominent settings. In the worker-firm example, the firm recruits a new class of junior employees, and negotiate wages with each simultaneously. Similarly, hospitals negotiate with multiple insurers over the the rates to be paid by the insurers for different services, where the hospital has private information about its costs. Information sharing between workers features prominently in ongoing policy debates in these settings. For workers and firms, the costs and benefits of "pay secrecy" policies, in which firms prevent employees from sharing or discussing their compensation, are the subject of lively debate.³ Regulation in place since the National Labor Relations Act of 1935 aims to prevent employers from imposing pay-secrecy policies. Transparency of the payer-negotiated rates payed by insurers to hospitals is the subject of new regulations from the Centers for Medicaid and Medicare Services. The new regulation would require hospitals to publicly disclose negotiated rates, and facilitate access to this information by the general public. These requirements are being vigorously resisted by both insurers and hospitals.⁴

I first introduce a general model of screening with multiple uninformed agents in a dynamic interaction. Many of the key insights of the paper hold within this general setting. For clarity, I describe the insights here in the context of a simple special case, which I study in greater detail in later sections.

Consider a simple model in which multiple workers negotiate with the firm in each period. Both the firm and workers live for two periods. Before the start of the second period, workers may with some exogenous probability observe the wages paid to other workers in the first period. I show that in equilibrium, each worker will adopt one of two types of strategies; a screening strategy, in which the worker structures their offers so as to learn the firm's type in the first period, or a pooling strategy, in which they are paid the same first-period wage regardless of the firm's type.

The key insight is that in the first period the firm is less averse to being screened by

²Greater information-sharing of this form is analogous to an increase in the discount factor of the firm.

³Bradford (2018).

⁴Wilson Pecci (2020).

any individual worker when many of the workers are screening, as opposed to only a few screening and the rest pooling. I refer to this as the complementary screening effect. To see why this is the case, consider the payoff to the firm from agreeing to a high first-period wage for worker i which reveals the firm's type, i.e. allowing i to screen. Compare the firm's payoff from doing so in two different scenarios: when the only worker, aside from i, who screens is j, or when workers j and k both screen. There are two factors to consider when comparing these two scenarios: the information flowing to i and the information flowing from i. In terms of information flowing from i, there are two differences between these scenarios. First, note that information sharing only has an effect on workers that did not screen in the first period (and thus do not know the firm's type when bargaining in the second period). Thus when both j and k are screening, the firm does not need to worry about k observing that i received a high wage, as it would if only i is screening. Second, some workers other than i may observe k's wage. When k is screening these workers learn the firm's type, and so any information revealed to them by observing i's wage is redundant. In other words, the information revealed by i's wage is less likely to be pivotal. There is, moreover, one difference regarding the information flowing to i. Should the firm reject i's offer, i.e. prevent i from screening, i is more likely to learn the firm's type anyway when both j and k are screening then when j alone is screening. Thus in the scenario with both j and k screening, the firm's payoff from rejecting i's screening offer is lower. All three of these differences point in the same direction: the firm is more willing to be screened by i when j and k are also screening than when j is screening and k is not. Because of this complementary screening effect, there is an externality from screening that operates through the information rents that must be paid to the high-type firm in order to screen; which are lower the more workers screen. This is in addition to the purely informational externality: workers who do not screen may benefit from the information generated by a screening worker.

Formally, complementary screening manifests as supermodularity in the payoffs of the high-type firm, as a function of the set of workers that it allows to screen in the first period. This has a number of interesting implications. First, screening is fragile; if any worker who is expected to screen in the first period deviates, either by demanding too high of a wage or switching to a pooling strategy, the high type firm will mimic the low type firm with all workers. In other words, screening breaks down completely. Second, screening by some workers will only be possible in equilibrium if enough workers engage in screening. If only a small number of workers attempt to screen, the information rents needed to induce the high-type firm to reveal itself would be too high.

I also consider a model in which workers differ in how costly it is for them to screen the

firm. I show how the distribution of worker types affects wages. I discuss how policies that increase wage transparency may have the adverse effect of destroying screening, and thus reducing the amount of information about the firm type that is generated. These negative effects can be offset however by policies that encourage more workers to screen. Importantly, there are increasing returns to such interventions; the more workers engage in screening, the less costly it is to induce them to screen.

Finally, I incorporate discrimination into the model of pay secrecy. Reducing pay discrimination is one of the primary motivations for increasing pay transparency. I show how the legal framework within which discrimination cases are tried affects the efficacy of penalties for discrimination. An important factor is whether the law admits the so-called "salary negotiation defense". If the law admits such a defense, a worker in a protected group must demonstrate not only that they were paid less than a coworker, but also that the firm rebuffed their attempt to negotiate a higher wage. Surprisingly, penalties for discrimination cannot eliminate discriminatory equilibria. This is due to the supermodularity in firm payoffs discussed above: if a firm rejects an equilibrium screening offer, or if it rejects a screening offer from a pooling worker who deviates to screening, then it will reject all screening initial offers. As a result, there will never be verifiable cases of pay discrimination, and hence the penalty in such cases is irrelevant. This result holds regardless of the degree of transparency. However, penalties for discrimination can make possible additional equilibria in which the discriminated-against group receives higher wages. Moreover, high penalties and high transparency help prevent discrimination by low-type firms. I discuss how these findings relate to empirical evidence. The results suggest ways in which discrimination should be defined in order to restore the efficacy of anti-discrimination penalties.

Related literature

This paper combines elements from the strategic experimentation and reputational bargaining literatures. Conceptually, this paper is closely related to the large literature on strategic experimentation with multiple agents, in which information externalities also arise. Screening by workers in the first period can be thought of as costly experimentation. In their seminal paper, Bolton and Harris (1999) study a multi-agent version of the multi-armed bandit problem. When there are multiple agents experimenting and observing each other's signals, free riding and encouragement effects arise. Keller et al. (2005) study a version of this problem with exponential bandits. Other papers, such as Murto and Välimäki (2011), study similar problems in which only actions are observed. One key difference between this literature and the current paper is that in my model the cost of experimentation is endogenously determined

by the information rents that must be paid to the firm. This is the source of the complementary screening affect, and its interesting implications for transparency policy. These indirect externalities are distinct from the direct payoff externalities in some experimentation models, such as those of Strulovici (2010) and Thomas (2020). Moreover, the fact that information is revealed by the strategic firm means that each agents actions can have contemporaneous effects on the information received by other workers. For example, when a screening worker i deviates to pooling, they anticipate that the high type firm will reject the screening offers of all other screening workers (Lemma 11). This affects both the information that i expects to receive from other workers and the payoffs of other workers.

There is also a close connection between this paper and the reputational bargaining literature (early contributions include Fudenberg and Levine (1989), Schmidt (1993)). In both cases, a long-lived principal, in this case the firm, has an incentive to take actions which influence the beliefs of short-lived agents (workers) regarding its type. Chaves (2019) studies the role of privacy in reputation games. Much of this literature assumes the existence of so called "commitment types" of the informed player. More recent papers, such as Pei (2020), study reputation without commitment types. In contrast to most of this literature, I study a situation in which i) both the informed and uninformed players are long-lived, and ii) there are multiple uninformed agents who negotiate simultaneously with the informed player. Combining these elements yields the novel insights on informational externalities and complementary screening. Additionally, there are no direct payoff externalities. Fudenberg and Kreps (1987) and Ghosh (2014) share some of these features. However these papers focus on infinite time horizons and conditions under which the informed player can obtain their Stackleberg payoff.

This paper is also related to the common agency literature. Martimort and Stole (2002) discuss the revelation principal in the context of static common agency games. In such games, the agent's private information consists not only of their primitive type, but also of the set of mechanisms offered by the principals. Martimort and Stole (2002) introduce the delegation principal in response to the failure of the standard revelation principal, in which agent's report only their primitive type. Pavan and Calzolari (2009) prove related "menu theorems" when principals contract sequentially with the agent. The current paper differs from this literature in that it restricts the set of mechanisms available to the principals. In the general model of Section 1, the principals are restricted to direct-revelation mechanisms in which the agent reports their type. This structure is shown to arise naturally when contracting is governed by a bargaining game, as in Section 2. This restriction implies that principals cannot respond directly do deviations by others. My focus is on the role of transparency in

shaping contracting in this constrained environment.

This paper also contributes to the literature on pay transparency and discrimination. Cullen and Pakzad-Hurson (2019) also study the effect of transparency on wage negotiations in a dynamic model. Their model is similar to sequential arrival models, such as Kreps and Wilson (1982). The dominant effect is the firm's reputation motive, and greater transparency leads to lower wages. The information-rent externalities studied in the current paper are not present. Halac et al. (2020) study transparency and discrimination in a moral hazard model. They show that pay discrimination does not occur under the firm's optimal effort-inducing mechanism when pay is private. This is in contrast to the finding of Winter (2004), who shows that pay discrimination is a feature of optimal effort-inducing mechanisms when wage contracts are public.

This paper is also related to the literature on bargaining with incomplete information, including the important contributions of Grossman and Perry (1986). For an overview of this literature see Ausubel et al. (2002). It also contains elements of bilateral bargaining games, such as those studied by Stole and Zwiebel (1996) and Collard-Wexler et al. (2019). Krasteva and Yildirim (2012) study a dynamic model with sequential, rather than simultaneous, bilateral negotiations.

The remainder of this paper is organized as follows. Section 1 presents the general model, and derives the key results within this framework. Section 2 introduces a simple special case, and characterizes equilibrium play. In Section 3 I examine some implications of worker heterogeneity. Section 4 presents the application to pay secrecy policy with discriminatory firms. Appendix C discusses an extension to a model in which bargaining occurs via an alternating offers game à la Rubinstein (1982).

1 General screening problem

Consider a general version of a screening problem with two periods, a single informed party (firm), and a set W of uninformed parties (workers). The firm may be one of two types, which I refer to as high and low. Workers have a common prior belief p that the firm is the high type.

In the first period each worker offers the firm a menu of contracts from which to choose. These proposals are made simultaneously. The firm then chooses a contract from the menu offered by each worker, which governs its relationship with that worker, or chooses not to contract with the worker in question. Throughout, I assume that there are no direct payoff externalities across workers: a worker's payoff in a given period is independent of what other

workers do in that period, and the firm's payoffs are additively separable across workers.

The second period payoffs of the firm from its interaction with worker i depend on i's belief about the firm's type. Let U(q, s) be the second-period payoff to a type s firm from a worker who believes that the firm is the high type with probability q.

Information sharing is as follows. Workers observe nothing of the negotiations between the firm and other workers within each period. However after the first period, but before the second period wage negotiations, worker i observes worker j's first period wage with probability ρ_{ij} (so i's observations are independent across workers). Wage observations are hard evidence; the worker can prove to the firm that they have observed a given wage.⁵ The solution concept is PBE.

At this stage it is not necessary to fully specify the game. Rather, I will focus on the first-period strategies available to workers. In the first period, workers can either screen or pool. Worker i screens by offering a menu of contracts $\kappa^i = (\kappa^i(l), \kappa^i(h))$ to the firm. The first-period payoff that a type s firm derives from its interaction with a worker with whom the firm agrees to contract κ is given by $u(\kappa|s)$, and the first-period payoff to the worker in this situation is $v(\kappa|s)$. A worker pools by offering a single contract κ_0 which it expects both types to accept (or equivalently a menu where $\kappa^i(l) = \kappa^i(h)$). The set of feasible contracts is κ . The outside option for both types of firms is 0.

1.1 Features of equilibria

Consider first the incentives of workers who screen in the first period, and of the high-type firm confronted with screening offers. I will restrict attention to pure strategies.

The defining feature of an equilibrium will be the set C of workers who screen in the first period, with the remainder pooling. Let $P^{j}(C)$, the observability of C to $j \notin C$, be the probability that j observes the period 1 wage of at least one worker in C. This is given by

$$P^{j}(C) = 1 - \prod_{k \in C} (1 - \rho_{jk}).$$

Define $\bar{P}(C)$ to be the expected number of workers outside of C who will observe a wage in C, which is given by

$$\bar{P}(C) = \sum_{j \in W \setminus C} P^j(C)$$

The following lemma says that there are diminishing returns to observability in the size of the set $C.^6$

⁵This distinction will matter later on when I introduce dynamics into the stage game.

⁶Submodularity of P^j is the key property of information sharing. In particular, we can relax the assumption that workers' observations are independent.

Lemma 1. $P^{j}(\cdot)$ is submodular, i.e. $P^{j}(A) + P^{j}(B) \geq P^{j}(A \cap B) + P^{j}(A \cup B)$. Moreover, $P^{j}(A) + P^{j}(B) > P^{j}(A \cap B) + P^{j}(A \cup B)$ if and only if the following conditions jointly hold

- 1. $\exists k \in A \cap B \text{ with } \rho_{jk} < 1$
- 2. $\exists i \in A \setminus B \text{ with } \rho_{i,i} > 0$
- 3. $\exists \ell \in B \setminus A \text{ with } \rho_{i,\ell} > 0$

Proof. Using the definition of $P^{j}(\cdot)$, we have $P^{j}(A) + P^{j}(B) \geq P^{j}(A \cap B) + P^{j}(A \cup B)$ iff

$$\prod_{k \in A \cap B} (1 - \rho_{jk}) + \prod_{k \in A \cup B} (1 - \rho_{jk}) \ge \prod_{k \in A} (1 - \rho_{jk}) + \prod_{k \in B} (1 - \rho_{jk})$$

which can be written as

$$\prod_{k \in A \cap B} (1 - \rho_{jk}) \cdot \left(\prod_{k \in A \setminus B} (1 - \rho_{jk}) \cdot \prod_{k \in B \setminus A} (1 - \rho_{jk}) + 1 \right)$$

$$\geq \prod_{k \in A \cap B} (1 - \rho_{jk}) \cdot \left(\prod_{k \in A \setminus B} (1 - \rho_{jk}) + \prod_{k \in B \setminus A} (1 - \rho_{jk}) \right)$$

The result follows.

A sufficient condition for strict submodularity if P^j for all j is that $\rho_{i,j} \in (0,1)$ for all j. One important implication of submodular observability is that the payoffs of the high-type firm will be supermodular in the set of screening offers it accepts. This corresponds to the intuition discussed earlier; when there are many screening workers the probability that the wage of each screening worker reveals pivotal information to other workers is lower. Additionally, when the high type firm reveals its type to many workers there are fewer workers for whom others' wages convey new information. Lemma 1 formalizes this intuition.

I make two assumptions on the environment, which are both crucial for the characterization of equilibrium. I maintain both assumptions throughout.

Assumption (Monotonicity). $q \mapsto U(q|s)$ is weakly decreasing.

At this stage, monotonicity is really a normalization, since no additional meaning has been attached to the type of the firm. Later on the firm's type will take on additional significance.

Say that the menu $\hat{\kappa}$ ε -worker-dominates the menu κ if the following three conditions hold:

i.
$$v(\hat{\kappa}(h)|h) > v(\kappa(h)|h)$$
 and $v(\hat{\kappa}(l)|l) > v(\kappa(l)|l)$

ii.
$$u(\hat{\kappa}(l)|l) \ge u(\hat{\kappa}(h)|l)$$
 and $u(\hat{\kappa}(l)|l) \ge 0$.

iii.
$$u(\hat{\kappa}(h)|h) \ge u(\kappa(h)|h) - \varepsilon$$
.

Assumption (Continuous Quasi-Transferability). For any menu κ and any $\varepsilon > 0$ there exists a menu $\hat{\kappa}$ that ε -worker-dominates κ .

The monotonicity assumption is straightforward. It captures the reputational risk associated with being the high-type firm. Continuous Quasi-Transferability, as the name suggest, relates to transferability of utility. The essential idea is that it is always possible for the worker to make themself better off (condition i) without making the high-type worker too much worse off (condition ii). Condition ii says that this can be done in a way that respects the incentive constraint of the low type. This assumption is weaker than assuming transferable utility.

I will focus now on the problem of the high-type firm deciding how much screening to allow. Suppose that C is the set of workers who are expected to screen. Once the contract proposals have been made, the high-type firm decides which of the screening contracts to accept (by accept I mean reveal itself by choosing $\kappa(h)$). The payoff to the high-type firm of accepting a set $A \subseteq C$ and rejecting $C \setminus A$ is given by

$$\begin{split} \pi(A|C,\omega) &= \sum_{j \in A} \left(u(\kappa^{j}(h)|h) + U(1,h) \right) \\ &+ \sum_{j \in W \setminus C} \left(u(\kappa_{0}^{j}|h) + P^{j}(A)U(1,h) + \left(1 - P^{j}(A) \right) \left(P^{j}(C \setminus A)U(0,h) + (1 - P^{j}(C \setminus A))U(p,h) \right) \right) \\ &+ \sum_{j \in C \setminus A} \left(u(\kappa^{j}(l)|h) + P^{j}(A)U(1,h) + \left(1 - P^{j}(A) \right) U(0,h) \right) \end{split}$$

The set function $A \mapsto \pi(A|C,\omega)$ is supermodular if $\pi(A \cup B|C,\omega) + \pi(A \cap B|C,\omega) \ge \pi(A|C,\omega) + \pi(B|C,\omega)$ for all $A,B \subseteq C$.

Proposition 1. $A \mapsto \pi(A|C, w)$ is supermodular, strictly so when $\rho_{ij} \in (0, 1) \ \forall \ i, j.^7$

Proof. I want to show that $\pi(A \cup B) - \pi(B) \ge \pi(A) - \pi(A \cap B)$. In words, the firm has a greater incentive to accept the contracts from $A \setminus B$ when also accepting those in $B \setminus A$, as opposed to when rejecting those in $B \setminus A$. Consider separately the payoff for the firm generated by four different groups of workers: 1) those in $A \setminus B$, 2) those in $B \setminus A$ who screen only in the first scenario, 3) those in $A \cap B$ who screen in both scenarios, and 4) those in $W \setminus (A \cup B)$ who do not successfully screen in either scenario.

⁷It is easy to see from Lemma 1 that weaker conditions for supermodularity of π can be stated.

First consider the payoffs from $A \setminus B$. The payoff from accepting $A \setminus B$ of course does not depend on what the firm does with other workers. However if the firm accepts $B \setminus A$ then the payoff from rejecting $A \setminus B$ is lower than if the firm rejects $B \setminus A$, since $X \mapsto P^j(X)$ is increasing (in the set inclusion order). Thus, based on the payoff from $A \setminus B$, the firm has a higher incentive to accept in $A \setminus B$ when also accepting $B \setminus A$.

If the firm accepts $B \setminus A$ then there is no effect on the payoff from these workers of also accepting $A \setminus B$. On the other hand, if the firm rejects $B \setminus A$ then the payoff from these workers is lower if the firm accepts $A \setminus B$, since $X \mapsto P^j(X)$ is increasing. Thus, based on the payoff from $B \setminus A$, the firm has a higher incentive to accept in $A \setminus B$ when also accepting $B \setminus A$.

Workers in $A \cap B$ screen in either scenario, so accepting $A \setminus B$ does not affect the payoffs from them in either case.

The trickier bit is the payoffs from $W \setminus (A \cup B)$. This group can be divided into $C \setminus (A \cup B)$ and $W \setminus C$. The payoff from $j \in C \setminus (A \cup B)$ when the firm accepts set X is given by $u(\kappa^j(l)|h) + U(0,h) + P^j(X) (U(1,h) - (U(0,h))$, which is supermodular since $X \mapsto P^j(X)$ is submodular and under Monotonicity U(1,h) - (U(0,h) < 0. The continuation payoff from $j \in W \setminus C$ when a set X is accepted is given by

$$P^{j}(X)U(1,h) + \left(1 - P^{j}(X)\right)\left(P^{j}(C \setminus C)U(0,h) + (1 - P^{j}(C \setminus C))U(p,h)\right)$$

or equivalently

$$U(p,h) + P^{j}(X)(U(1,h) - U(p,h)) + (1 - P^{j}(X))P^{j}(C \setminus X)(U(0,h) - U(p,h))$$

Under Monotonicity, supermodularity of this function follows form submodularity of $X \mapsto P^{j}(X)$ and supermodularity of $X \mapsto (1 - P^{j}(X))P^{j}(C \setminus X)$.

The function π describes the payoff to the firm of deviating, given that all worker's have made their equilibrium initial offers in the first round. In order to characterize equilibrium however, we need also to understand the firm's payoff when a worker deviates. The distinction is important because a worker's beliefs may depend on whether or not they them-self have deviated. Fix an equilibrium, and suppose $i \in C$ deviates to a menu with a new high-type contract $\hat{\kappa}^i(h)$, and which ε -worker-dominates κ^i for some $\varepsilon > 0$. It must be that the high type firm rejects this offer, i.e chooses either $\hat{\kappa}^i(l)$ or their outside option, since otherwise this would be a profitable deviation for i. The low-type firm will also reject such an offer: their incentives in the first period are satisfied by construction, and they have no reputational incentive to deviate. Thus in the sub-game in which i has deviated and the offer has been

rejected, i does not update their beliefs about the firm's type. Let $X^i \subseteq C \setminus i$ be the set that workers expect the firm to accept if i deviates.

Let $\pi^i(A|C,\omega,X)$ be the payoff from the firm of accepting the initial screening offers of $i\in A$ when the screening set and offers are C,ω and $i\in C$ believes that a) both types will reject it's initial offer, and b) the high-type firm will accept the initial offers of $j\in X\subseteq C\setminus i$ and reject all others. In a subgame-perfect equilibrium, worker i believes that the firm will behave optimally should i deviate, and i's beliefs about which screening offers will be accepted must be consistent with this. Thus, if i conjectures that screening offers in $X^i\subseteq C\setminus i$ will be accepted then the set of possible subsets which the firm would actually like to accept is given by

$$\chi^{i}(X^{i}) = \underset{X \subseteq C \setminus i}{\operatorname{argmax}} \, \pi_{i}(X|C, \omega, X^{i}).$$

In equilibrium it must be that $X^i \subseteq \chi^i(X^i)$. The set $\chi^i(X^i)$ need not be single valued, but when it will not cause confusion I will discuss as if it is.

In order to understand equilibrium play, it is important to know what inference a worker j will draw if they observe the outcome from a worker k who has deviated. This will depend both on the structure of the game and the nature of inter-worker observations. If j observe not just the contract that the firm agrees to with i, but also the menu that i offered, then the answer is simple: j will draw the same inferences from i's outcome as i (although i and j may end with different beliefs depending on what other information they observe). If information sharing is more limited then j's inferences may differ from i's. In what follows I assume that if $i \in C$ deviates from a screening menu κ^i to another screening menu $(\hat{\kappa}^i,$ and j observes i then j can identify whether i's contract was the one intended for the high-type or the low-type firm. This structure of inference would of course be implied if j observes the entire menu, but it also arises in more limited information-sharing environments, such as that discussed in Section 2.2.

Given this assumption, the payoff to a high-type firm from accepting a set $A \subseteq C \setminus i$ following i's deviation is given by

$$\begin{split} \pi_i(A|C,\omega,X^i) &= \sum_{j \in A} \left(u(\kappa^j(h)|h) + U(1,h) \right) \\ &+ \sum_{j \in W \setminus C} \left(u(\kappa_0^j|h) + P^j(A)U(1,h) + \left(1 - P^j(A) \right) \left(P^j(C \setminus A)U(0,h) + (1 - P^j(C \setminus A))U(p,h) \right) \right) \\ &+ \sum_{j \in C \setminus (A \cup i)} \left(u(\kappa^j(l)|h) + P^j(A)U(1,h) + \left(1 - P^j(A) \right) \left(U(0,h) \right) \\ &+ u(\hat{\kappa}^i(l)|h) + P^i(A)U(1,h) + (1 - P^i(A)) \left((P^i(X^i \setminus A)U(0,h) + (1 - P^i(X^i \setminus A))U(p,h) \right) \end{split}$$

The same argument as Proposition 1 implies that $A \mapsto \pi_i(A|C,\omega_1,X^i)$ is supermodular.

Lemma 2. $A \mapsto \pi^i(A|C,\omega,X^i)$ is supermodular for all i,C,ω,X_i , strictly so when $\rho_{ij} \in (0,1) \ \forall \ i,j$.

Proof. The only difference from Proposition 1 is the payoff from worker i. Supermodularity of this term follows from the same argument as that given in the proof of Proposition 1 for workers in $W \setminus C$.

The assumptions of Continuous Quasi-Supermodularity and Monotonicity also have the following implication.

Lemma 3. In equilibrium, $\pi_i(X^i|C,\omega,X^i) = \pi_i(C|C,\omega,X^i) = \pi(C|C,\omega) = \pi(X^i|C,\omega)$ for all $i \in C$.

Proof. The second equality is immediate. Given the assumption of Continuous Quasi-Transferability, optimality of i's initial offer implies $\pi_i(X^i|C,\omega,X^i) \geq \pi_i(C|C,\omega,X^i)$. If this did not hold then there would be a profitable deviation of i to a contract that ε -worker-dominates their equilbrium contract. Moreover, $\pi(C|C,\omega) \geq \pi(X^i|C,\omega) \geq \pi_i(X^i|C,\omega,X^i)$, where the first inequality follows from firm optimality and the second from the fact that i has beliefs that are more favorable to the firm in the π case than in the π_i case (under Monotonicity). \square

The important piece of Lemma 3 is $\pi_i(X^i|C,\omega,X^i) = \pi(X|C,\omega)$. Under Monotonicity, it will always be the case that $\pi_i(X^i|C,\omega,X^i) \leq \pi(X^i|C,\omega)$; the only potential difference in the two cases is the belief of i. In order for equality to hold it must be that U(0,h) = U(p,h), $P^i(A) = 1$, or $P^i(X^i \setminus A) = 1$, the latter two of which cannot hold if $\rho_{ij} \in (0,1)$ for all i,j.

Lemma 4. If U(0,h) > U(p,h) then in any equilibrium $C = \emptyset$.

If U(0,h) = U(p,h) then $\pi_i(A|C,\omega,X^i) = \pi(A|C,\omega)$ for all $A \subseteq C$. Thus under this assumption I abuse notation supress the dependence of $\chi^i(X^i)$ on X^i , denoting this set simply as χ^i .

The main result of this section is that supermodularity of the high-type firm's payoffs implies that their binding incentive constraint will be to mimic the low-type in their negotiation with all workers, i.e. choose $\kappa_L(i)$ for all $i \in C$.

Theorem 1. Assume $\rho_{ij} \in (0,1)$ for all i,j and $U(0,h) = U(p,h).^8$ If an equilibrium is characterized by C, ω , it must be that $\pi(C|C, \omega) = \pi(\varnothing|C, \omega)$, and $\varnothing \in \chi^i$ for all $i \in C$.

⁸It is possible to prove a version of this result without the assumption U(0,h) = U(p,h), although additional assumptions on the structure of information sharing are needed. Since in any event the case of U(0,h) = U(p,h) is the interesting one, as it allows for $C \neq \emptyset$ in equilibrium, I focus on the result under this assumption.

Proof. Recall that if U(0,h) = U(p,h) then $\pi_i(A|C,\omega,X^i) = \pi(A|C,\omega)$ for all $A \subseteq C$. I will therefore proceed only with the function π .

If $\rho_{ij} \in (0,1)$ for all i,j then the conditions for strict supermodularity of $\pi(\cdot|C,\omega)$ are satisfied for all non-nested A, B. I will first show that $\emptyset \in \chi^i$ for all $i \in C$.

Claim 1. For any $i \in C$, any $X^i \in \chi^i$, any $k \in X^i$ and any $X^k \in \chi^k$, we have $X^k \subsetneq X^i$.

Let $X^i \neq \emptyset$, and let $k \in X^i$. By definition of X^i , $\pi(X^i|C,\omega) \geq \pi(X^i \cap X^k|C,\omega)$. But then Proposition 1 implies $\pi(X^i \cup X^k|C,\omega) \geq \pi(X^k|C,\omega)$. Suppose X^i and X^k satisfy the conditions for strict supermodularity of π . Then we have $\pi(X^i \cup X^k|C,\omega) > \pi(X^k|C,\omega)$. Under Continuous Quasi-Transferability worker k can deviate by offering a menu $\hat{\kappa}^k$ that ε -worker-dominates κ^k , so that the payoff to the high-type firm from accepting the offers of workers in $X^i \cup X^k$ changes only slightly. Since $\pi(X^i \cup X^k|C,\omega) > \pi(X^k|C,\omega)$, the firm will still accept k's offer. But then ω and C cannot characterize an equilibrium, since k has a profitable deviation. The conclusion is that X^i and X^k cannot satisfy the conditions for strict supermodularity of π . However strict supermodularity holds for any non-nested X^i, X^j when $\rho_{ij} \in (0,1)$ for all i,j. This proves Claim 1.

Claim 2. There exists $k \in C$ such that $\chi^k = \emptyset$.

Let i be an arbitrary worker with $X^i \in \chi^i$ such that $X^i \neq \emptyset$, and let $j \in X^i$. Then by Claim 1, $X^j \subsetneq X^i$ for all $X^j \in \chi^j$ (the inclusion must be strict since $j \in X^i$, and $j \notin X^j$ by definition). Then since there are finitely many workers there must be $k \in X^i$ such that $\chi^k = \emptyset$.

Claim 3. Even without conditions on ρ , for any equilibrium C, ω we will have $\pi(X^i|C,\omega) = \pi(C|C,\omega)$ for all i.

To see this, first note that under Continuous Quasi-Transferability, optimality of κ^i for worker i implies $\pi(C|C,\omega) \leq \pi(X^i|C,\omega)$. Moreover, firm optimality implies $\pi(C|C,\omega) \geq \pi(A|C,\omega)$ for all $A \subseteq C$. Thus $\pi(C|C,\omega) = \pi(X^i|C,\omega)$. This proves Claim 3.

By Claims 2 and 3, for any i we have $\pi(X^i|C,\omega) = \pi(C|C,\omega) = \pi(X^k|C,\omega) = \pi(\varnothing|C,\omega)$, so $\varnothing \in \chi^i$, as desired.

Pooling workers can also deviate to screening. Suppose that some contract profile C, ω satisfies the IC constraints of the high-type firm and of all workers in C. Suppose $i \in \mathcal{W} \setminus C$ offers a screening contract κ^i such that the IC constraint of the low-type firm is satisfied. Let ω' the new contract profile following i's deviation. Under the assumption that U(p|h) = U(|h), Theorem 1 implies that $\pi(C \cup i|C \cup i, \omega') = \pi(\varnothing|C \cup i, \omega')$. Therefore i chooses their deviating contract to maximize their own payoff, subject to $\pi(C \cup i|C \cup i, \omega') = \pi(\varnothing|C \cup i, \omega')$. This constraint depends on the set $\{\kappa^j(h)\}_{j\in C}$ of high-type contracts offered by workers in C

only through $\sum_{j\in C} u(\kappa^j(h)|h)$. Thus we have the following lemma, which will be useful for proving Proposition 2.

Lemma 5. If C, ω characterize an equilibrium and ω' differs from ω only in terms of the contracts $\kappa^i(h)$ offered to the high type firm for $i \in C$, then the IC constraint of all pooling workers $j \in W \setminus C$ is satisfied under C, ω' .

The fact that $\varnothing \in \chi^i$ for all $i \in C$ does not preclude the existence of equilibria in which a worker i expects others to be able to successfully screen even if i deviates and asks for a higher wage. Such equilibria can be ruled out if $\chi^i = \varnothing$ for all $i \in C$. I say that C, ω characterize an *interior* equilibrium if they characterize an equilibrium and $\chi^i = \{\varnothing\}$ for all $i \in C$. Proposition 2 shows that equilibria for which $\chi^i \neq \varnothing$ are in a sense boundary cases.

Quasi-linearity. The space of available contracts can be written as $\mathcal{K} = \mathcal{Y} \times \mathbb{R}$. Moreover $t \mapsto u(y,t|s)$ is strictly decreasing and v(u,t|s) is strictly increasing for any $y \in \mathcal{Y}$ and $s \in \{l,h\}$; and u(y,t|h) = f(y|h) + g(h)t.

Under Quasi-linearity there are no equilibria that are isolated from interior equilibria (with the metric being on the real component of contracts alone). Under Quasi-linearity, I write contracts as $(y,t):\{l,h\}\to\mathcal{Y}\times\mathbb{R}$.

Proposition 2. Assume Quasi-linearity.⁹ For any equilibrium C, ω and any $\varepsilon > 0$, there exists $\hat{\omega} = \{(\hat{y}^i, \hat{t}^i)\}_{i \in W}$ such that for all $i \in W$, $y^i = \hat{y}^i$, $t^i(l) = \hat{t}^i(l)$ and $|t^i(h) - \hat{t}^i(h)|$; and such C, ω' characterize an interior equilibrium.

Proof. Start with C, ω such that $\chi^i \neq \{\varnothing\}$ for some $i \in C$. Let A be the set of such workers. I define an algorithm to shrink χ^i by 1 for all i in A. The result follows by iteratively applying this algorithm. For simplicity of notation, I will therefore assume here that $|\chi^i \setminus \{\varnothing\}| = 1$ for all $i \in A$, and denote the non-empty element of χ^i by X^i . Claim 1. There exists a set $K \subset C$ such that i) $\chi^k = \{\varnothing\}$ for all $k \in K$, ii) $|K \cap X^i| = 1$ for all $i \in A$, and iii) if $j \in X^i \cap A$ then $K \cap X^i = K \cap X^j$.

I describe an algorithm to construct such a set. Start with $i \in A$. Let $X_1 = X^i$ and $B_1 = A \cap X_1$. By Claims 1 and 2 in the proof of Theorem 1, there exists $k_1 \in X_1$ such that $\chi^{k_1} = \emptyset$. Moreover, by the same claims, k_1 can be chosen such that for any $j \in X_1 \cap A$ and any X^j , we have $k_1 \in X^j$. This defines a the first step in the algorithm. Now let $B_2 = B_1 \cup i$ and choose j from $A \setminus B_2$. Repeat the above procedure to define k_2 .

⁹In fact, Quasi-linearity is stronger than needed. In particular, we can allow for u(y,t|h) = f(y|h) + g(y|h)t if $g(y^i(h)|h) = g(y^j(h)|h)$ for all $i, j \in C$. The example studied in Section 2 satisfies this condition, but not Quasi-linearity.

Since A is finite, this algorithm yields a set K such that $K \cap X^i \neq \emptyset$ for all $i \in A$ and $X^i \in \chi^i$. Moreover, by Claim 1 in the proof of Theorem 1, $j \in X^i \cap X^l$ then there is $k \in K$ such that $k \in X^i \cap X^j \cap X^l$. Thus the set K can be constructed such that $|K \cap X^i| = 1$ for all $i \in A$ and $X^i \in \chi^i$. Thus condition ii) is satisfied. Condition iii) is satisfied by construction. This proves the claim.

Using this claim, I show how to modify ω to obtain the desired $\hat{\omega}$. For each $i \in A$, let $\hat{t}^i(h) = t^i(h) - \delta/|A|$. For each $k \in K$, let $B_k = \{i \in A : K \cap X^i = k\}$, and let $\hat{t}^k(h) = t^k(h) + \delta |B_k|$. Then $\sum_{i \in C} \hat{t}^i(h) = \sum_{i \in C} t^i(h)$, and so firm's payoff from accepting all high-type contracts in C is unchanged. However for each $i \in A$, $\sum_{j \in X^i} \hat{t}^j(h) < \sum_{j \in X^i} t^j(h)$. Since $\pi(X^i|C,\omega) = \pi(\varnothing|C,\omega) = \pi(\varnothing|C,\omega')$, this implies $\pi(X^i|C,\omega') < \pi(\varnothing|C,\omega')$. By taking $\delta > 0$ small enough we can guarantee that $\chi^i = \varnothing$ for all $i \in C$, as desired. With this new set of offers, $i \in A$ will not want to deviate since their payoff from screening decreases only slightly, while their payoff from pooling decreases by a discrete amount since i can no longer expect to receive information from other workers.

Lemma 5 implies that pooling worker IC is not violated after this change. \Box

Proposition 2 is useful for welfare analysis. It is convenient to conduct comparative statics on interior equilibria, since they have the special property that worker's in C who deviate expect to receive no information.

2 A simple bargaining model

I now consider a simple bargaining model. I show that this model can be analyzed as a special case of the general model considered in Section 1, despite the fact that there are within-period dynamics.

The firm has private information about the per-period output generated by a worker, which is $s \in \{s', s''\}$, with s'' > s' > 0. The firm's total output is additively separable across workers. The worker's outside option is -d < 0, so there are always gains from cooperation. The worker's prior belief is that the firm is of type s'' with probability p. Workers and firms discount at rate β between periods.

I present here a simple bargaining game which illustrates the important forces involved in bargaining with multiple workers.¹⁰ In the first period there are two rounds of bargaining. First, the worker makes an initial wage offer. If this is rejected then the firm makes a counter

¹⁰The general qualitative results regarding the complementarity of screening do not depend on the specific details of the stage game between the workers and the firm. For clarity I will focus on a simple bargaining game.

offer. The minimum allowable wage is $\underline{w} \leq s'$ (as we will see, in equilibrium a firm will always propose \underline{w} if its turn comes). If the counter offer is rejected then both players get their outside option in the first period. If an offer is accepted then the firm enjoys its surplus and the worker their wage. Both workers and the firm discount at rate δ between bargaining rounds within the first period, so there is a cost to delaying agreement until the second round of offers.

In the second period, worker's simply make a take-it-or-leave-it offer to the firm. This asymmetry in bargaining games between the first and second periods allows us to focus on the implications of information sharing in the first period. Moreover, it seems reasonable to expect that workers' bargaining power increases after the first year, since they will have acquired firm-specific skills which makes retaining them more attractive than seeking an outside hire.¹¹

The worker will make a second-period proposal of s'' if they are sufficiently certain that the firm is the high type; otherwise, they offer s'. Given Lemma 4, I assume that s' > ps'' - d(1-p). This means that a worker who has received no information between the first and second periods will make an offer of s' in the second period, so U(p,h) = U(0,h) = s'' - s'.

The solution concept is PBE as in Fudenberg and Tirole (1991) (i.e. including the "no signaling what you don't know" condition). I impose one additional refinement, to be discussed below.

2.1 Preliminary observations

I say that a worker screens in a given period if, in equilibrium, it learns the type of the firm. The worker pools if it does not update it's belief about the firm's type. Partial screening, which will occur when the firm follows a mixed strategy, will be discussed later on.

Since a worker will propose either s'' or s' in the second period, the low-type firm always receives a payoff of zero in the second period, regardless of the worker's beliefs. As a result, in the first period the low-type firm has no reputation concerns.

Lemma 6. The low-type firm will never accept a first-period first-round offer above $w' \equiv (1 - \delta)s' + \delta \underline{w}$.

Proof. Since the low type has no reputation concerns, they accept an offer of w iff $s' - w \ge \delta(s' - \underline{w})$.

¹¹In fact, if the second period bargaining game is identical to that of the first then we show in Appendix B that no screening can occur in equilibrium. However the specification of take-it-or-leave-it offers in the second period is not essential to avoid this negative result. The key properties are discussed in Appendix B

Given this observation, I impose the following refinement, which restricts off-path beliefs when workers observe mixed evidence about the firm's type. This refinement can be justified via forward-induction type arguments.

Assumption 1. A worker who observes a wage (their own or another worker's) strictly greater than $w' \equiv (1 - \delta)s' + \delta \underline{w}$ infers that the firm is the high type.

Next, observe that the high-type firm has more to lose from delaying agreement. Thus whenever a worker screens it must be that only the high-type firm accepts the initial offer.

Lemma 7. Any worker that screens makes an initial offer w_1 that only the type s'' firm accepts in equilibrium.

Proof. Suppose instead that some worker i makes an initial offer w_1 that only the low type firm accepts. The high-type firm loses more from delay, since $(s'' - w_1) - \delta(s'' - \underline{w}) > (s' - w_1) - \delta(s' - \underline{w})$. Thus if the low-type firm finds it optimal to accept the initial offer, absent reputation concerns, then so does the high-type. Moreover, by mimicking the low type and accepting i's initial offer, the firm ensures that the beliefs of any worker who observes i's wage are more favorable (this is true even if the information set of such a worker is off path, by Assumption 1).

The above observations imply that in each worker in the first period chooses between a pooling and a screening strategy. Pooling entails making an initial offer of w' that both types of the firm accept. In order to screen, the worker makes an initial offer of $w_1 > w'$, which the low-type rejects and the high-type accepts.

It will never be worthwhile for the firm to reject an initial offer of w'.

Lemma 8. Any firm will accept an initial offer of w'.

Proof. The low-type firm is indifferent between accepting and rejecting such an offer from worker i, and as usual we assume that it is accepted. If the strategy of the high-type firm specifies rejection of such an offer, then any worker who observes acceptance will infer that the firm is the low-type. Then the argument of Lemma 7 applies.

2.2 Equilibrium

The bargaining game described above can be interpreted in the framework of the general model in Section 1. Each worker can either pool by making an initial offer of w', or screen by making an initial offer w > s'. Any worker who sees that another received a wage above w' infers that the firm must be the high type. Monotonicity of $p \mapsto U(p,h)$ is satisfied, as

is Continuous Quasi-Transferability (a screening worker can create an ε -worker-dominating contract by increasing their screening offer by ε).

2.3 Screening workers

One implication of Theorem 1 is to pin down the sum of screening initial offers. Let $\bar{W}(C)$ be the sum of the equilibrium initial offers of workers in C, which is defined by the high-type firm's indifference condition

$$|C|\left(s'' - \frac{\bar{W}(C)}{|C|}\right) + (|W| - |C|)(s'' - w') + \beta \left(1 - \bar{P}(C)\right)U(p, s'')$$

$$= |C|\left(\delta(s'' - \underline{w}) + \beta(s'' - s')\right) + (|W| - |C|)(s'' - w') + \beta \bar{P}(C)(s'' - s') + \beta(1 - \bar{P}(C))U(p, s'').$$

Thus $\overline{W}(C)$ is given by

$$\bar{W}(C) = |C| \left(s'' - \left[(\delta(s'' - \underline{w}) + \beta(s'' - s') \right] \right) - \beta \bar{P}(C)(s'' - s'). \tag{1}$$

A necessary condition for screening to be possible with set C is that $\overline{W}(C)/|C| \geq w'$, otherwise the low type firm will want to accept initial screening offers as well. This holds iff

$$(1 - \delta - \beta) \ge \beta \frac{\bar{P}(C)}{|C|}.$$
 (2)

The right hand side of (2) non-negative, so it must be that $\beta + \delta \leq 1$. Under symmetric information sharing $C \mapsto \bar{P}(C)/|C|$ is decreasing (as shown in the proof of Lemma 12), so (2) holds for C above some cut-off.

A worker who is supposed to screen in equilibrium can always deviate to pooling (knowing that no other workers will be able to screen, as we will see below).

Lemma 9. Let C be the equilibrium set of screening workers. If $i \in C$ deviates and makes an initial offer of w' then both firms accept.

Proof. This is obvious for the type s' firm. Consider the type s'' firm. If the firm accepts w', any other workers for whom i's wage is pivotal information will infer that the firm is type s', which is the best outcome for the firm. Thus the only reason to reject s' is to convince i to pool rather than screen in the second period. But, as shown in the proof of Lemma 8, this is not sufficient to justify delay in the first period, so the high-type firm accepts the initial offer of w'.

Consider now the set of (C, ω) pairs such that incentive compatibility of the high-type firm and the screening workers is satisfied. Maintain the assumption that U(p, s'') = s'' - s'.

Then $\pi_i(A|C,\omega,X_i)=\pi(A|C,\omega)$ for all i,A. Moreover, let $C'\subset C$ and $\omega_i'=\omega_i$ for all $i\in C'$. Then for any $A\subseteq C'$, we have $\pi(A|C',\omega')=\pi(A|C,\omega)$. Thus for any X^i , it must be that $\pi(X^i|C,\omega)=\pi(X^i|X^i,\omega)=\pi(X^i|X^i,\omega)$, where the second equality follows from Theorem 1. In other words, if there is some non-empty X^i then it must itself constitute a screening set satisfying the firm's incentive constraint. This implies that for any X^i , the wage sum $\sum_{j\in X^i}\omega_j=\bar{W}(X^i)$ (where $\bar{W}(\cdot)$ is given by (1)).

Lemma 10. If $i \in C$ and $X^i \in \chi^i$, then $\sum_{j \in X^i} \omega_j = \bar{W}(X^i)$.

all $i \in A$, and iii) if $j \in X^i \cap A$ then $K \cap X^i = K \cap X^j$.

Equilibria in which some i have $\chi^i \neq \{\varnothing\}$ are edge cases. For any such equilibrium, we can find equilibria arbitrarily nearby that have $\chi^i = \{\varnothing\}$ for all i. Call such equilibria interior.

Definition. An equilibrium with screening set C is interior if $\chi^i = \{\emptyset\}$ for all $i \in C$.

Proposition 3. For any equilibrium C, ω and any $\varepsilon > 0$, there exists ω' such that $|\omega - \omega'| < \varepsilon$ and C, ω' characterize an interior equilibrium.

Proof. Start with C, ω such that $\chi^i \neq \{\varnothing\}$ for some $i \in C$. Let A be the set of such workers. I define an algorithm to shrink χ^i by 1 for all i in A. The result follows by iteratively applying this algorithm. For simplicity of notation, I will therefore assume here that $|\chi^i \setminus \{\varnothing\}| = 1$ for all $i \in A$, and denote the non-empty element of χ^i by X^i .

Claim 1. There exists a set $K \subset C$ such that i) $\chi^k = \{\varnothing\}$ for all $k \in K$, ii) $|K \cap X^i| = 1$ for

I describe an algorithm to construct such a set. Start with $i \in A$. Let $X_1 = X^i$ and $B_1 = A \cap X_1$. By Claims 1 and 2 in the proof of Theorem 1, there exists $k_1 \in X_1$ such that $\chi^{k_1} = \emptyset$. Moreover, by the same claims, k_1 can be chosen such that for any $j \in X_1 \cap A$ and any X^j , we have $k_1 \in X^j$. This defines a the first step in the algorithm. Now let $B_2 = B_1 \cup i$ and choose j from $A \setminus B_2$. Repeat the above procedure to define k_2 .

Since A is finite, this algorithm yields a set K such that $K \cap X^i \neq \emptyset$ for all $i \in A$ and $X^i \in \chi^i$. Moreover, by Claim 1 in the proof of Theorem 1, $j \in X^i \cap X^l$ then there is $k \in K$ such that $k \in X^i \cap X^j \cap X^l$. Thus the set K can be constructed such that $|K \cap X^i| = 1$ for all $i \in A$ and $X^i \in \chi^i$. Thus condition ii) is satisfied. Condition iii) is satisfied by construction. This proves the claim.

Using this claim, I show how to modify ω to obtain the desired ω' . For each $i \in A$, let $\omega'_i = \omega_i - \delta/|A|$. For each $k \in K$, let $B_k = \{i \in A : K \cap X^i = k\}$. For each $k \in K$, let $\omega' = \omega + \delta |B_k|$. Thus the total sum of screening wages remains the same. However for each $i \in A$, $\sum_{j \in X^i} \omega'_j < \sum_{j \in X^i} \omega_j$. Since $\pi(X^i|C,\omega) = \pi(\varnothing|C,\omega) = \pi(\varnothing|C,\omega')$, this implies

 $\pi(X^i|C,\omega') < \pi(\varnothing|C,\omega')$. By taking $\delta > 0$ small enough we can guarantee that $\chi^i = \varnothing$ for all $i \in C$, as desired. With this new set of wage offers, $i \in A$ will not want to deviate since their payoff from screeening decreases only slightly, while their payoff from pooling decreases by a discrete ammount since i can no longer expect to recieve information from other workers. \square

Proposition 3 is useful for welfare analysis. It is convenient to conduct comparative statics on interior equilibria, since they have the special property that worker's in C who deviate expect to receive no information.

Lemma 11. In any interior equilibrium with screening set C, if $i \in C$ deviates by making a higher initial offer or deviates to pooling then the high-type firm rejects all initial offers in $C \setminus i$.

Proof. If $i \in C$ deviates by making a higher offer then the result follows immediately from Theorem 1 and the definition of an interior equilibrium. The same argument applies if i deviates to pooling, since U(p, s'') = s'' - s'. \Box

When information sharing is symmetric $(\rho_{ij} = \rho \in (0,1))$ for all i,j, the average screening worker initial offer is increasing in |C|. This implies in particular that equilibria in which all screening workers make the same offer are interior.

Lemma 12. Assume $\rho_{ij} = \rho \in (0,1)$ for all i,j. Then $C \mapsto \overline{W}(C)/|C|$ is increasing in |C|.

Proof. Recall that

$$\frac{\bar{W}(C)}{|C|} = \left(s'' - \left[\left(\delta(s'' - \underline{w}) + \beta(s'' - s')\right)\right) - \beta \frac{\bar{P}(C)}{|C|}(s'' - s').$$

The claim follows if $C \mapsto \bar{P}(C)/|C|$ is decreasing in |C|. Under symmetry, $\bar{P}(C) = (|W| - |C|) (1 - (1 - \rho)^{|C|})$. Then the claim is that the function $|C| \mapsto \frac{|W| - |C|}{|C|} (1 - (1 - \rho)^{|C|})$ is decreasing. The derivative of this function is negative iff

$$\frac{|W|}{|C|(|W|-|C|)} > -\ln(1-\rho)\frac{(1-\rho)^{|C|}}{1-(1-\rho)^{|C|}}.$$

The right hand side of this expression is bounded above by 1/|C| (the limit as $\rho \to 0$). The inequality follows.

In summary, the equilibrium incentive constraints in a symmetric equilibrium with symmetric information sharing are as follows:

¹²In fact, the result continues to hold even if U(p, s'') > s'' - s'.

- The high type firm must be indifferent between accepting all initial offers from workers in C and rejecting all offers from workers in C. Thus the sum of these offers must be $\bar{W}(C)$.
- The low type firm will accept an offer if and only if it is no greater than w'. This implies that we must have $\bar{W}(C)/|C| > s'$, the condition for which is (2).
- Workers in C must prefer screening to deviating to pooling. If they deviate they know that the firm will reject all other screening initial offers, so they will receive no information from other workers.

2.4 Pooling workers

I turn now characterizing the relevant deviation for pooling workers. In particular, I identify the highest period 1 initial wage offer that the high-type firm will accept from a pooling worker. Assume that there is an equilibrium with screening set C, and suppose $i \in W \setminus C$ deviates by making a wage offer greater than w'. Let $\tilde{w}(C,i)$ be the highest wage offer that i can make such that the high-type firm is indifferent between accepting all initial offers in $C \cup i$ and rejecting all such offers. $\bar{W}(C) + \tilde{w}(C,i)$ may differ from $\bar{W}(C \cup i)$ because of the inferences made by workers when the firm rejects i's initial offer; when i is expected to screen in equilibrium, other workers who observe a wage of w' for i will infer that the firm is type s', while when i deviates from pooling to screening and is rejected, other workers make no inference upon observing a wage of w' for i. As a result $\tilde{w}(C,i)$ is given by

$$\tilde{w}(C, i) = s'' - [(\delta(s'' - \underline{w}) + \beta(s'' - s')] - \beta(\bar{P}(C \cup i) - \bar{P}(C))U(p, s'')$$

Lemma 13. Assume $\rho_{i,j} \in (0,1)$ for all i,j. In an equilibrium with screening set C, if a pooling worker i makes an initial offer of $\hat{w} > w'$, the type s'' firm will accept iff $\hat{w} \leq \tilde{w}(C,i)$.

Proof. Suppose that the type s'' firm rejects i's offer. Then i will believe that the firm is type s' as long as it does not observe evidence to the contrary. Since the second period payoff to the firm from a worker who believes it is type s' is weakly higher when workers maintain their prior, the incentive of the firm to screen with the workers in C is lower than in equilibrium. By Theorem 1, for all $A \subseteq C$, on path the firm weakly prefers rejecting all initial offers in A to accepting all initial offers in A, and only these initial offers. Thus following its rejection of i's initial offer, the firm will also weakly prefer rejecting all initial offers in A to accepting all initial offers in A, for all $A \subseteq C$.

By the definition of $\tilde{w}(C, i)$, the firm is indifferent between accepting all initial offers in $C \cup i$ and rejecting all such offers. Combined with the preceding paragraph, this implies that

if i makes an initial offer of $\tilde{w}(C, i)$, the type s'' firm will accept. In particular, the firm will prefer accepting all initial offers in $C \cup i$ to rejecting all initial offers in $C \cup i$. (Moreover, given acceptance of i's offer, the high-type firm is more willing to be screened by other workers than it was with no deviation by i.)

It remains to show that no higher offer by worker i will be accepted. This would only be the case if and only if there was some set $A \subset C$ such that the firm strictly preferred accepting only the initial offers in $A \cup i$ to accepting all initial offers in $C \cup i$. The existence of such a set A is ruled out however by submodularity of P^j (Lemma 1), using a similar argument to that given for Proposition 1. (It does not follow directly from Proposition 1 because we are comparing a case in which i pools to one in which it screens. However the difference in the argument is minor.)

A full characterization of (pure-strategy) equilibrium requires characterizing the set of screening sets C for which the pooling worker does not want to deviate as identified in Lemma 13. Unfortunately this set does not have a convenient characterization in general. Rather than focus on this question here, I will return to the question of pooling worker incentives in Section 3.

We can easily see that there will always be an equilibrium in which all workers pool, provided the degree of transparency is high enough. If $C = \emptyset$ then the highest wage a worker can obtain by deviating to screening is $\tilde{w}(\emptyset, i) = s'' - [(\delta(s'' - \underline{w}) + \beta(s'' - s'))] - \beta \bar{P}(\{i\})U(p, s'')$. Since U(p, s'') = (s'' - s') we have the following condition.

Lemma 14. Assume $\rho_{i,j} \in (0,1)$ for all i,j. There exists a threshold $\bar{p} < 1$ such that there is an equilibrium in which all workers pool if and only if $\min_i \bar{P}(\{i\}) \leq \bar{p}$.

Proof. The existence of a threshold follows from the fact that $\tilde{w}(\emptyset, i)$ is decreasing in P(i). That \bar{p} is less than 1 follows from the worker's incentive constraint and the assumption that $\delta + \beta < 1$.

 $\bar{P}(\{i\})$ is decreasing in |W| and the degree of transparency. Thus the larger is |W|, the lower is the degree of transparency needed to guarantee the existence of an all-pooling equilibrium. This is consistent with the standard reputational intuition regarding the effect of transparency. However, it is not the case that increased transparency necessarily leads to less screening.

Even without fully characterizing equilibrium, we can examine one of the comparative statics of primary interest: the effect of transparency. Let \mathcal{E} be the set of (C, ω) that can be supported in equilibrium. Say that \mathcal{E}'' has higher screening that \mathcal{E}' if the following two

conditions hold: a) $\mathcal{E}'' \subset \mathcal{E}'$, and b) if $(C', \omega') \in \mathcal{E}' \setminus \mathcal{E}''$ and $(C'', \omega'') \in \mathcal{E}''$ then |C''| > |C'|. When $\rho_{ij} = \rho \in (0, 1)$ for all i, j, let \mathcal{E}_{ρ} be the set of supported screening sets and screening offers supported under ρ .

Proposition 4. Assume $\rho_{ij} = \rho \in (0,1)$ for all i,j. There is a threshold $\bar{\rho}$ such that $\rho'' > \rho' \geq \bar{\rho}$ implies $\mathcal{E}_{\rho''}$ has higher screening than $\mathcal{E}_{\rho'}$.

Proof. I first show that $\tilde{w}(C, i)$ is decreasing in ρ , which implies that increasing from ρ' to ρ'' creates no new profitable deviations for pooling workers. I therefore show that

3 Worker heterogeneity

Thus far I have assumed that workers are identical. This poses a problem for characterizing equilibrium. As Section 2 explains, the set C of screening workers fully characterizes the incentive constraints of high and low-type firms, as well as those of the screening workers (assuming symmetric screening offers). It is important for each worker to know the identity of every worker in C, as this determines their inferences both on and off path. However it is not obvious how asymmetries between identical workers, with some screening and others pooling, as well as the knowledge of which workers are doing which activity, would arise in a decentralized environment.

In many settings workers may not in fact be identical. They may differ in their patience, degree of risk aversion, disutility of labor, outside option, etc. Heterogeneity among workers, particularly in observable characteristics, will facilitate coordination on the set of screening workers. Moreover, many sources of heterogeneity will make some workers more willing to engage in screening than others. In this section I characterize equilibrium in such settings, and study welfare comparative statics and implications for policy interventions. As might be expected, increasing information sharing between workers (i.e. increasing ρ) will adversely affect the welfare of screening workers by increasing in the information rents that must be paid to screen high-type firms. Moreover, when information sharing is too high no screening will be possible in equilibrium, reducing the welfare of pooling workers as well. The main insight however is that the positive externality imposed by one screening worker on another, embodied by the fact that $C \mapsto \bar{W}(C)/|C|$ is increasing in |C|, can be leveraged to offset the negative effects of increasing ρ . Thus policies aimed at increasing worker welfare should not only promote transparency, but also provide additional incentives for screening.

Assume $W = P \cup S$, where P is the set of workers who do not screen when their belief is p, their prior, even if all other workers are screening. These workers make an initial wage offer of w' in the first period. If they believe the firm to be type s'' for sure then they demand s'' in the

second period. These workers need not be "behavioral", in the sense of mechanically playing a given strategy, only that the threshold belief above which they find it optimal to screen is higher than that of workers in S. There are a number of reasons that this might be the case: higher discounting between rounds, worse outside option (if within-round discounting is reinterpreted as a separation probability), greater risk aversion, cultural norms. I will not explicitly model any of these reasons here. Rather, I will explore the implications of such preferences on equilibrium outcomes, and discuss welfare comparative statics that apply to a broad class of preferences for workers in P.

Workers in S have payoffs as described in the previous section. I will focus on symmetric equilibria in which all workers in S follow the same strategy. Note that if there is no equilibrium in which all workers in S screen then there is no equilibrium in which any workers screen. Thus restricting attention to symmetric equilibria does not artificially disadvantage the workers.

Proposition 5. Assume symmetric information sharing. There exists an integer $n(\rho, P)$ such that there exists a unique unique symmetric pure-strategy equilibrium in which

- 1. if $|S| \ge n(\rho, P)$ all workers in S screen in first period,
- 2. if $|S| < n(\rho, P)$ no workers screen in the first period,

where $n(\rho, P)$ is increasing in both arguments.

Proof. The existence of n, and its monotonicity properties, are immediate consequences of by Lemmas 12, 9, and 11. Existence follows from the results of Section 2. If workers in S are not willing to screen when the screening set is |S| then they will be unwilling to do so for any smaller screening set, by Lemmas 12, 9, and 11. Then the only equilibrium is for no workers to screen.

This characterization has important welfare and policy implications. The following lemma follows immediately from Proposition 5.

Lemma 15. Pooling-worker welfare is non-monotone in ρ : it increases in ρ as long as $|S| \geq n(\rho, P)$ and is minimized when $n(\rho, P) > |S|$. Screening-worker welfare is decreasing in ρ .

Worker welfare is increasing in |S|, fixing |P|.

Lemma 16. Fix |P|. The welfare of all workers is increasing in |S|. In particular, each screening worker receives higher wages from the high-type firm, and pooling workers receive more information, the higher is |S|.

Proof. This follows from showing that $S \mapsto \frac{|P|}{|S|} (1 - (1 - \rho)^{|S|})$ is decreasing. The derivative with respect to |S| is negative iff

$$-\frac{1}{|S|^2}(1-(1-\rho)^{|S|})-|S|\ln(1-\rho)(1-\rho)^{|S|}<0.$$

The limit of the LHS as $\rho \to 1$ is $-1/|S|^2 < 0$ and the limit as $\rho \to 0$ is 0. The derivative of the LHS with respect to ρ is

$$-\frac{1}{|S|}(1-\rho)^{(|S|-1)} + |S|(1-\rho)^{(|S|-1)} + |S|^2 \ln(1-\rho)(1-\rho)^{(|S|-1)}.$$

The sign of this is the same as that of

$$-\frac{1}{|S|} + |S| + |S|^2 \ln(1 - \rho)$$

which is (weakly) negative for any $|S| \ge 1$. Thus the LHS term is decreasing in ρ . Since the limit of the LHS term as $\rho \to 0$ is 0, this implies that the derivative of $\frac{|P|}{|S|}(1-(1-\rho)^{|S|})$ with respect to S is negative.

Worker welfare is decreasing in |P|, fixing |S|.

Lemma 17. Fix |S|. The welfare of all workers is decreasing in |P|. In particular, each pooling worker receives lower wages from the high-type firm. Pooling worker's information is unchanged as long as $n(p) \ge |S|$, and they receive no information otherwise.

The main policy take-away is that increasing ρ alone may have adverse effects. These can be mitigated by measures which encourage workers in P to engage in screening. The specific ways in which this can be done, and the potential for welfare gains, will depend on the reasons for which workers in P are reluctant to screen. Nonetheless, the positive externalities of screening, both for pooling and screening workers, justify interventions along these lines.

4 Application: discrimination

One of the primary motives for increasing pay transparency is to reduce discrimination. The logic generally given is that greater transparency makes it easier for workers to identify discriminatory compensation patterns. This should in turn prevent firms from discriminating. In order to more fully address the benefits of pay-secrecy policies, it is important therefore to incorporate discrimination into the model. I will be particularly interested in how various anti-discrimination policies affect equilibrium outcomes, and how these policies interact with transparency.

The legal background of pay discrimination policies will form the basis for incorporating discrimination into the model. Pay discrimination occurs along many lines, including race, gender, and sexual orientation. In this discussion I will focus on laws regarding sex-based pay discrimination, although the model will apply equally well to other forms of discrimination. The primary legal basis for the prohibition of sex-based discrimination is the Equal Pay Act of 1963. This law "prohibits employers from discriminating among employees on the basis of sex by paying higher wages to employees of the opposite sex for 'equal work on jobs the performance of which requires equal skill, effort, and responsibility, and which are performed under similar working conditions.' "Belfi v. Prendergast, 191 F.3d 129, 135 (2d Cir.1999) (quoting 29 U.S.C. §206(d)(1)). The law allows for four defenses for observed pay discrepancies: the employer must show that the discrepancies are the result of a seniority system, a merit system, a system that measures quality or quantity of production, or "any factor other than sex." The interpretation of this latter catch-all defense has naturally been the subject of great controversy. One such factor which has at times been cited in defense of pay differences is the so-called "salary negotiation defense". This is the argument that the difference in pay arises from differences in negotiation tactics. Essentially, the employer may assert that a man is paid more than a woman for the same work because he bargained hard, while she did not. The legal precedent for this type of defense is mixed. The argument was rejected in Dreves v. Hudson Group Retail LLC, 2013 WL 2634429, **8-9 (D. Vt. June 12, 2013). However versions of the salary negotiation defense were accepted in Muriel v. SCI Arizona Funeral Services, Inc., 2015 WL 6591778, *3 (D. Ariz. Oct. 30, 2015) and Horner v. Mary Institute, 613 F.2d 706, 714 (8th Cir. 1980). The use of the salary negotiation defense raises the question of how well an anti-discrimination law which admits such a defense does in achieving it's goals of eliminating pay discrimination.

4.1 Verifiable discrimination

I begin by assuming that the anti-discrimination law admits a salary negotiation defense. This in turn defines the *prima facie* burden of proof for an employee who wishes to bring a suit against their employer. Let $Y \subset W$ be the set of workers which is protected by the anti-discrimination law. I define verifiable discrimination as that which can not be justified via the salary negotiation defense.

Definition. There is **verifiable discrimination** against $i \in Y$ if $i \in Y$ makes an offer of $w \in (w', s'']$ which is rejected, and i observes $j \notin Y$ with $w^j > w'$.¹³

¹³As will be shown below, the results will establish the irrelevance of penalties for verifiable discrimination.

I will assume that the firm pays a penalty ℓ for cases of verifiable discrimination, a fraction $\alpha \in [0,1]$ of which goes directly to the injured worker. A worker will bring a discrimination case against the firm if and only if they have evidence of verifiable discrimination. This rules out cases in which the worker brings the suit without making a *prima facie* case. Such cases are unlikely to survive an initial motion to dismiss, and even if they do, the worker runs the risk that in fact no discrimination has occurred, in which case they will be responsible for the legal costs associated with brining the suit.

The general intuition for why the penalty for verifiable discrimination may help Y workers obtain higher wages is the following. Suppose $i \in Y$ is receiving a relatively low wage, either because they are pooling or because they are screening, but at a low wage. The larger is ℓ , the more willing the firm should be to accept should i deviate and make a higher wage offer, provided this offer is still below the wage obtained by some worker $j \in C \setminus Y$. This is because rejection of i's deviation would form the basis for a verifiable case of discrimination. Greater transparency should reinforce this effect, since it means that i will be more likely to observe j's wage. This intuition seems to be behind the arguments in favor of both large penalties for discrimination and greater pay transparency.

The intuition outlined above however is incomplete, as it implicitly takes as given the probability that i will observe a high wage from j. Both j's offer and the firm's decision whether to accept or reject are endogenous. Faced with i's deviation, the firm considers not only whether or not to accept i's offer, but also which offers in C to accept. It turns out that this endogeneity severely limits the efficacy of penalties and transparency for reducing pay discrimination. In many cases, the anti-discrimination penalty will in fact have no effect on equilibrium outcomes. The reason again has to do with supermodularity of the firm's payoffs (Proposition 1). In the language of this paper, the intuition for why transparency and discrimination penalties help prevent discrimination can be rephrased as follows: if a pooling Y worker tries to deviate to screening, the firm will have a greater incentive to accept the offer when ρ and ℓ are high. This makes it harder to sustain equilibria in which Y workers do not screen. The problem with this reasoning is Lemma 13. If the firm rejects the initial offer of a worker $i \in Y$ who deviates to screening then it will reject all initial screening offers when $\ell = 0$, and all the more so when $\ell > 0$. Thus there will be no verifiable discrimination, and the penalty ℓ is irrelevant. This conclusion is robust across a number of

These negative results continue to hold with the following narrower definition of verifiable discrimination: i makes an offer of w^i which is rejected, and i observes $j \notin Y$ with a wage $w^j \ge w^i$. This definition corresponds more directly to the intuitive idea of the salary negotiation defense. However I focus on the more permissive notion because to highlight the fact that even if the bar for a successful salary negotiation defense is high, the anti-discrimination policy will not achieve its goal.

different discrimination settings, a few of which I will discuss below.

4.2 Statistical discrimination

Statistical discrimination occurs when the firm draws inferences about worker quality from observable characteristics. I model this form of discrimination as follows. Workers can be either high ability (g) or low ability (b). Ability is irrelevant at the low type firm. At the high-type firm, a high ability worker produces output s'', while a low ability worker produces output s'' - c > s'. Neither workers nor the firm know each worker's ability at the start of the first period, but both learn it by the end of the first period (independent of the firm's type). The firm discriminates because it believes that Y workers are low ability with higher probability than non-Y workers. Without loss of generality, assume that it believes non-Y workers are always high ability. This belief may well be incorrect; there may be no differences in ability distributions across groups. What matters is the perception of the firm about the different groups, and the fact that workers know how they are perceived.

Because the worker and the firm learn the worker's type after the first period, there is no basis for statistical discrimination in the second period. I assume that in the second period a firm cannot be penalized for rejecting a wage which is greater than the value of the worker, meaning a high type firm will accept a worker's offer if and only if it is less than s'' for a high ability worker, or s'' - c for a low-ability worker. Workers who know the firm's type will therefore demand their value. Workers who have no information will demand a wage of s' (given the maintained assumption on p).

Consider now the first-period interaction. There are two differences, relative to the analysis of Section 2.2. First, worker's are heterogeneous (at least from the firm's perspective). The expected surplus generated by a Y worker is less than that of a non-Y worker. This has no impact on the analysis however; with $\ell = 0$ all results of Section 2.3 go through, the proofs unchanged. The second differences is of course the penalty for verifiable discrimination.

Throughout I will fix c and the probability with which the firm believes that Y workers are low-ability, and therefore suppress these variables in the notation. Let $\pi(A|C,\omega,\ell)$ be the payoff to the high-type firm of accepting initial offers from $A \subseteq C$ when the penalty for verifiable discrimination is ℓ . Note that

$$\pi(A|C,\omega,\ell) = \pi(A|C,\omega,0) - \ell \cdot \left(\sum_{j \in (Y \cap C) \setminus A} P^{j}(A \setminus Y)\right)$$

Lemma 18. $A \mapsto \pi(A|C,\omega,\ell)$ is supermodular.

Proof. $\pi(A|C,\omega,0)$ is supermodular, by the same proof as Proposition 1. Then $A \mapsto -\ell \cdot \left(\sum_{j\in (Y\cap C)\setminus A} P^j(A\setminus Y)\right)$ is supermodular since $X\mapsto P^i(X)$ is submodular and increasing in the set-inclusion order. To see this, write

$$\begin{split} \sum_{j \in (Y \cap C) \backslash A} P^j(A \setminus Y) + \sum_{j \in (Y \cap C) \backslash B} P^j(B \setminus Y) &= \sum_{j \in (Y \cap C) \backslash (A \cup B)} \left[P^j(A \setminus Y) + P^j(B \setminus Y) \right] \\ &+ \sum_{j \in (Y \cap C) \cap (A \backslash B)} P^j(B \setminus Y) + \sum_{j \in (Y \cap C) \cap (B \backslash A)} P^j(A \setminus Y) \end{split}$$

and

$$\sum_{j \in (Y \cap C) \setminus (A \cup B)} P^{j}((A \cup B) \setminus Y) + \sum_{j \in (Y \cap C) \setminus (A \cap B)} P^{j}((A \cap B) \setminus Y) =$$

$$\sum_{j \in (Y \cap C) \setminus (A \cup B)} \left[P^{j}((A \cup B) \setminus Y) + P^{j}((A \cap B) \setminus Y) \right]$$

$$+ \sum_{j \in (Y \cap C) \cap (A \setminus B)} P^{j}((A \cap B) \setminus Y) + \sum_{j \in (Y \cap C) \cap (B \setminus A)} P^{j}((A \cap B) \setminus Y)$$

Given Lemma 18, a version of Theorem 1 continues to hold.

Proposition 6. Assume $\rho_{ij} \in (0,1)$ for all i,j and U(0,h) = U(p,h). Under statistical discrimination, if an equilibrium is characterized by C, ω , it must be that $\pi(C|C, \omega, \ell) = \pi(\varnothing|C, \omega, \ell)$, and $\varnothing \in \chi^i$ for all $i \in C$.

Since $\pi(C|C,\omega,\ell)$ and $\pi(\varnothing|C,\omega,\ell)$ are independent of ℓ , this means that the sum of screening wages associated with any screening set C is independent of ℓ . In fact, the effect of ℓ on equilibrium outcomes is very limited. We first need to verify that there are no strategies available to workers, other than screening or pooling.

Lemma 19. For any $\ell \geq 0$ and any α , all workers either pool or screen.

Proof. I need to show that there are no strategies other than screening or pooling that could occur on path. The only other strategy that could arise is for a Y worker to make an offer in the first period that the high-type firm will reject, in the hope of then encountering verifiable discrimination and winning a lawsuit. Call this the entrapment strategy.

Suppose there is an equilibrium in which a set B of Y workers plays the entrapment strategy. Let $i \in Y$ be a worker pursuing the entrapment strategy, and let C be, as before, the set of screening workers. The entrapment strategy can only be profitable if $C \setminus Y \neq \emptyset$.

I will show that i would be better off making an offer that the high type firm would accept. For any $\ell'>0$, the firm's payoffs can be written as the sum of its operating surplus, i.e. output minus wages, and the costs arising from lawsuits. For the purposes of inference, a worker playing the entrapment strategy is the same as a pooling worker. In other words, the informational effect on operating surplus is the same whether a worker pools or entraps. For any screening set C it must still be the case that $\pi(C|C,\omega,\ell)=\pi(\varnothing|C,\omega,\ell)$; the presence of entrapment workers does not affect the supermodularity of π . However $\pi(C|C,\omega,\ell)$ will now depend on B, since $C\setminus Y\neq\varnothing$ implies that lawsuits occur with positive probability on-path. Let $\tilde{w}(C)$ be the highest initial wage offer from $i\in B$ that would be accepted if ℓ' . In other words, $\tilde{w}(C)$ is the highest wage that the firm would be willing to accept from i if it only cared about its operating surplus. By Lemma 13 and the fact that workers in C are willing to screen, it must be that $\tilde{w}(C)>w'$.

Let $P(C \setminus Y)$ be the probability that i observes a wage in $C \setminus Y$, which is exactly the probability that i brings a lawsuit. Suppose that instead of making an unacceptable initial offer, $i \in B$ instead proposes $\tilde{w}(C) + P(C \setminus Y)\ell$. If the firm accepts, i strictly prefers this deviation to the entrapment strategy, since $\tilde{w}(C) > w'$. I claim that the firm accepts i's offer, and continues to accept all initial offers in C. This is because i screening has no effect on expected lawsuit costs arising from workers in $B \setminus i$. The firm's expected lawsuit costs will decrease by exactly $P(C \setminus Y)\ell$ if it accepts i's offer. Its expected operating surplus is the same, by definition of $\tilde{w}(C)$. Thus the firm obtains it's equilibrium payoff whether it accepts or rejects i's deviation offer (to break the firm's indifference, i could offer a slightly lower wage and still be better off). Thus there can be no equilibrium in which any workers entrap.

Proposition 7. Assume there is statistical discrimination and $\rho_{ij} = \rho \in (0,1)$ for all i, j. Let $c \geq 0$ and $\alpha \in [0,1]$ be arbitrary, and let $\ell'' > \ell'$. Then any C, ω that characterize an equilibrium under ℓ' also do so under ℓ'' .

Proof. I show that interior equilibria are preserved under increases in ℓ . If C, ω characterize an equilibrium that is not interior, i.e. where $X^i \neq \emptyset$ for some $i \in C$, then they also characterize an interior equilibrium.

Assume for simplicity of notation that $\alpha = 1$; the proof is identical for any $\alpha \in [0, 1]$. Let C, ω characterize an equilibrium under ℓ' . I wish to show that they also characterize an equilibrium under ℓ'' .

To do this, I verify that there will be no profitable deviations for any workers or for the firm. Assume that $C \setminus Y \neq \emptyset$, otherwise it is obvious that the penalty has no effect.

Every worker will either screen or pool, so there are potentially four types of workers whose deviations we must consider when ℓ' .

Worker $i \in C \cap Y$. Suppose first that such a worker deviates to pooling. Under both ℓ' and ℓ'' , the firm will then reject all initial offers in C (by Lemma 11). Thus the payoff to i from this deviation is the same in both cases, and therefore not profitable under ℓ' . The same is true if i deviates by making a higher offer.

Worker $i \in C \setminus Y$. Again, if i deviates to pooling or makes a higher offer all other offers in C will be rejected, so the payoff to i from deviating is independent of ℓ .

Worker $i \in Y \setminus C$. This is the most interesting direction. Let \tilde{w} be the highest wage that i could offer under ℓ' which would be accepted by the high-type firm. By Lemma 11, under ℓ' the firm will then be indifferent between accepting all initial offers in $C \cup i$ and rejecting all such offers, should i offer \tilde{w} . Suppose that under ℓ'' , i deviates by making an offer $w \leq \tilde{w}$. The payoff to the firm of accepting all offers in $C \cup i$ is the same as under ℓ' . The payoff of rejecting i's offer is weakly worse, since this opens the possibility for a lawsuit. Thus the firm will accept any such offer, and so the payoff to i from deviating is the same as under ℓ' . Suppose instead that i offers $w > \tilde{w}$. If the firm accepts i's offer it will also accept all offers in C (by supermodularity of π). If the firm rejects i's offer then it will also reject all offers in C. This is because under ℓ' the firm at least weakly prefers rejecting all C in this case (Theorem 1), and this preference is strict for any ℓ'' , since otherwise there is the possibility of a lawsuit (recall the assumption that $C \setminus Y \neq \emptyset$). The payoffs to the firm of accepting all $C \cup i$ are independent of ℓ , and so it will reject under ℓ'' , given that this was weakly optimal under ℓ' . Thus the payoff to the worker form this deviation is weakly worse under ℓ'' than under ℓ' .

Worker $i \in (W \setminus C) \setminus Y$. The previous argument also applies to this type of worker.

Consider now the potential deviations by the firm. The low type firm of course has no deviations, so consider a high-type firm. The only relevant deviation is rejecting some initial offers in C. Doing so is weakly worse under ℓ'' , since it creates the possibility of a lawsuit. Thus the firm will accept all C.

Proposition 7 implies that increasing penalties for discrimination will not eliminate discriminatory equilibria. However penalties for verifiable discrimination can make possible new equilibria in which discriminated-against workers receive higher wages. To see how this could occur, suppose that $|C| \geq 2$ and $C \cap Y = \{i\}$. In other words, there are multiple screening workers, but only one is of group Y. As noted above, the sum of screening wages is $\bar{W}(C)$, independent of ℓ . Suppose that the share of this wage sum going to i is large, so that $\pi(C \setminus i|C,\omega,0) > \pi(C|C,\omega,0)$. Clearly this cannot be an equilibrium, as the hightype firm prefers accepting only $C \setminus i$ to either accepting all of C or rejecting all of C (the later follows by the definition of $\bar{W}(C)$). However $\ell \mapsto \pi(C \setminus i | C, \omega, \ell)$ is strictly decreasing, since the firm in this case runs the risk of a lawsuit. Thus for ℓ high enough, we will have $\pi(C|C,\omega,\ell) > \pi(C \setminus i | C,\omega,\ell)$.

More generally, larger penalties increase the share of $\bar{W}(C)$ that can go to workers in Y in equilibrium. In some situations, this may make it possible to sustain screening sets that cannot be sustained without penalties.

4.3 Other forms of discrimination

Aside from statistical discrimination, observed patterns of pay disparities have been attributed to "taste-based" discrimination and the attitudes towards negotiation held by various groups of workers. I show that both factors lead to conclusions that are similar to those discussed above in the context of statistical discrimination.

4.3.1 Taste-based discrimination

The formulation of statistical discrimination discussed above is very similar to "taste-based" discrimination. The only difference is that I have assumed that the uncertainty regarding the worker's type is resolved after the first period, so that there is no discrimination in the second period. Taste-based discrimination, on the other hand, would occur in both periods.

Penalties for verifiable discrimination which are paid to the injured worker could in theory create perverse incentives for a worker $i \in Y$ who pools in the first period. Suppose the firm is high-type, and i and observes the wage of $j \in C \setminus Y$; it could be that i prefers in the second period to hide the fact that they know the firm's type, and instead make an offer that they knows the firm will reject. This would benefit i if the payoff they receive from winning the subsequent lawsuit is greater then the wage that they could demand if they know the firm's type.

I will assume that cross-period evidence cannot be used in discrimination cases. This means that a worker i who observes $j \notin Y$ with a period 1 wage greater than w' will simply demand a wage of s'' in the second period. Ruling out the behavior described in the previous paragraph is the only effect of this cross-period evidence assumption.

Suppose that there is tasted based discrimination, which takes the form of a utility cost C payed by the firm when it employs a type Y worker. This can be interpreted with as the psychological cost associated which captures the firm's bias. For simplicity, assume that this cost is not discounted; the firm pays the cost if it employs a Y worker in a given period, whether they reach agreement in the first or second round of negotiation. As long as s''-c>s'

the conclusion of Proposition 7 continues to hold; penalties for discrimination will not change the set of equilibrium outcomes. The proof is unchanged.

Corollary 1. Assume there is taste-based discrimination and $\rho_{ij} = \rho \in (0,1)$ for all i, j. Let $c \geq 0$ and $\alpha \in [0,1]$ be arbitrary, and let $\ell'' > \ell'$. Then any C, ω that characterize an equilibrium under ℓ' also do so under ℓ'' .

4.3.2 Perception of negotiators

There is significant empirical evidence that racial and gender pay gaps arise in part from the perceptions of workers who engage in negotiations. Bowles et al. (2007) find that women who attempt to negotiate for higher salaries are perceived as demanding "less nice", particularly be male evaluators. Hernandez et al. (2019) show that Black workers face similar perception effects when dealing with White evaluators.

Formally, we can model this perception effect as a utility cost for Y workers who attempt to negotiate. I will assume that this cost takes the form of a fixed cost c paid by a worker who proposes a wage above s' (the same results will hold if instead the cost is proportional to the wage proposal). This cost is also paid if the worker brings a lawsuit against the company. I assume that this cost is not so large that it completely outweighs the benefit of learning the firm's type. To be precise, s'' - c > s', so a Y worker is willing to demand a high wage in the second period if they know that the firm is the high type.

Aside from this negotiation cost and the penalty for discrimination, the model is unchanged. I will consider fixed negotiation costs c throughout, and so will suppress dependence on c in the notation. For any $\ell \geq 0$, let $\pi(A|C,\omega,\ell)$ be defined as before, except that the penalty ℓ for verifiable discrimination is incorporated.

The fact that Y workers pay a perception cost for negotiation will make them less willing to attempt to screen, in the absence of penalties for verifiable discrimination.

Corollary 2. Assume there is taste-based discrimination and $\rho_{ij} = \rho \in (0,1)$ for all i, j. Let $c \geq 0$ and $\alpha \in [0,1]$ be arbitrary, and let $\ell'' > \ell'$. Then any C, ω that characterize an equilibrium under ℓ' also do so under ℓ'' .

4.4 Discriminatory low-type firm

Consider either taste-based or statistical discrimination, and assume w' > s' - c. Then a discriminatory low-type firm will employ no Y workers in the second period (unless there is an additional round of information sharing after the second period), the low-type firm will either accept any offer of s' made by a Y worker, and thus avoid paying any penalty for

discrimination, or reject all such offers and risk paying penalties. The higher the penalty ℓ and the higher the information sharing parameter ρ , the more likely it is that the firm will avoid lawsuits by hiring type A workers.

4.5 Empirical observations

The results on discrimination can be summarized as follows

- 1. For low-type discriminatory firms, increasing ℓ and ρ reduces hiring discrimination.
- 2. The penalty ℓ has no impact on the actions of high-type discriminatory firms. Increasing ρ will not have a direct impact on discrimination.

The second point conforms with empirical evidence, such as that discussed by Babcock and Laschever (2009), that women are paid less than men for the same job, and are less likely to ask for higher pay. The gender pay gap has remained virtually the same over the past decade, despite regulations such as the 2009 Lilly Ledbetter Fair Pay Act, which increased the penalty for pay discrimination and made it easier for employees to file discrimination cases.

Pay gaps between men and women are highest in professions such as financial services, in which there is potentially large variability in productivity across firms.¹⁴. In industries in which firms are homogeneously low-type, such as food service, the gender pay gap is smaller.

These results are also consistent with patterns of racial discrimination. Black and Hispanic/Latino workers are over-represented in low paying jobs such as food servers and porters. ¹⁵ While there are a host of reasons for this pattern, it is consistent with the observation that measures meant to prevent discrimination, through increased wage transparency and higher penalties for discrimination, are more effective in preventing hiring discrimination in low-type firms than in closing the pay gap in high-type firms.

¹⁴Stebbins and Frohlich (2019)

¹⁵Solomon et al. (2019)

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A Existence of intermediate equilibria

I show here that there can be equilibria with $|C| \in [1, |W| - 1]$. This means showing that the incentive constraints of pooling and screening workers can be satisfied simultaneously.

The payoff to a pooling worker i is

$$(1+\beta)s' + p\beta P^{i}(C)(s'' - s').$$

If this worker deviates to screening then the highest wage they can obtain is $\tilde{w}(C, i)$. Their payoff from deviating is

$$(\delta + \beta)s' + p\left(\tilde{w}(C, i) + \beta s'' - (\delta + \beta)s'\right)$$

Thus pooling worker IC is satisfied if and only if

$$(1 - \delta)s' + p\left(\beta P^i(C)(s'' - s') - \tilde{w}(C, i) - \beta s'' + (\delta + \beta)s'\right) \equiv \Delta^{pool} \ge 0.$$

Interior equilibria, in which $\chi^i = \{\varnothing\}$ for all i, are most conducive to screening (and in a sense generic, as demonstrated by Proposition 3). Moreover, equilibria with asymmetric offers among the screening workers are possible only if there is an equilibrium with symmetric screening offers and the same screening set; asymmetric offers necessarily mean someone screening with a wage below $\bar{W}(C)/|C|$ (the converse holds if screening worker IC does not bind in the symmetric-offers equilibrium). I will therefore focus on interior equilibria with symmetric screening offers. The payoff to a screening worker i is

$$(\delta + \beta)s' + p\left(\frac{\overline{W}(C)}{|C|} + \beta s'' - (\delta + \beta)s'\right).$$

If this worker deviates to pooling then their payoff is $(1 + \beta)s'$. Thus screening worker IC is satisfied if and only if

$$-(1-\delta)s' + p\left(\frac{\bar{W}(C)}{|C|} + \beta s'' - (\delta + \beta)s'\right) \equiv \Delta^{screen} \ge 0.$$

Assume that $\rho_{ij} = \rho \in (0,1)$ for all i,j. Substituting for $\tilde{w}(C,i)$ and $\bar{W}(C)/|C|$ and simplifying, we obtain $\Delta^{pool} \geq 0$ iff

$$\rho(1-\rho)^{|C|}(|W|-|C|-1) + \frac{\beta'-\beta}{\beta'}(1-\rho)^{|C|} \ge \frac{(1-\delta)(ps''-s')}{p\beta'(s''-s')}$$
(3)

and $\Delta^{screen} > 0$ iff

$$\frac{(1-\delta)(ps''-s')}{p\beta'(s''-s')} \ge \frac{|W|-|C|}{|C|} \left(1-(1-\rho)^{|C|}\right) + \frac{\beta'-\beta}{\beta'}.\tag{4}$$

The right hand side of a (3) (same as the left hand side of (4)) can be made small by making ps'' - s' small, and can be made arbitrarily large by taking β' to zero. Both IC constraints are satisfied for $|W| = 10, |C| = 3, \beta = .5, \beta' = .1, r \ge .7$ and ps'' - s' sufficiently small.

B An impossibility result

Suppose that the second-period bargaining game is the same as the first; that is, workers make offers in two rounds. It turns out that in this case there can be no equilibrium with screening. This result hinges on the off-path beliefs of workers.

Consider the second period equilibrium. From this point on there is no information sharing, so the problem separates completely across workers. In the second round of the second period negotiation, a worker with belief q will either offer a wage of s'' (if $qs'' \geq s'$) or s' (if qs'' < s'). In either case the low type firm would receive a payoff of 0. Thus the low type firm strictly prefers to accept any first round offer $w_1 < s'$ than to reject such an offer. This has the following implication.

Lemma 20. In the second period negotiation, the unique pooling equilibrium wage is s'.

Proof. Suppose that a there is a pooling equilibrium in which the worker makes an initial offer of $w_1 < s'$ and both types accept. If the worker deviates and makes an initial offer of $w' \in (w_1, s')$ the low type firm strictly prefers acceptance to rejection. Thus if the firm rejects w' the worker will believe that it is type s'', and make a wage offer of s'' in the second period. Therefore both firm types accept w', so w_1 cannot be an equilibrium offer. Thus the unique pooling equilibrium wage is s'

Suppose the worker screens the firm. Lemma 7 says that the high type firm will be the one to accept first. The type s'' firm will accept an initial offer of w_1 iff $s'' - w_1 \ge \delta(s'' - s')$. So the equilibrium screening wage is $w^s = \delta s' + (1 - \delta)s''$. The worker with belief q prefers screening to pooling in the second period iff $qw^s + (1 - q)\delta s' \ge s'$, or equivalently, $qs'' \ge s'$. Notice that the low type firm always gets a payoff of zero in the second period, regardless of whether the worker screens or pools. Let $U_2(s'')$ be the equilibrium second period payoff of a type s'' firm when the worker has belief p, so $U_2(s'') = \delta(s'' - s')$ if the worker screens and s'' - s' if the worker pools.

Consider now the first-period negotiations. Under the minimum-wage restriction, it will never be worthwhile for the firm to reject the offer of s' of workers who are supposed to pool on-path.

Lemma 21. Under the minimum-wage restriction, if a worker pools on-path then the firm never benefits from rejection of the initial offer s'.

Proof. This is obvious for the low-type firm, so consider a high-type firm. By rejecting the initial offer of s' from some pooling worker i, the high type firm can only hope to gain by

convincing some set of workers not to screen in the second period. If the firm accepts a second-round offer w > s' then all workers who observe this will infer that the firm is type s' for sure, so this cannot be beneficial. If instead the firm accepts a second-round offer of s' then all workers other than i who observe this will make the equilibrium inference and retain their prior. The gains from convincing i not to screen in the second period are sufficient to justify the delay in the first period from the first to the second round iff

$$\delta(s'' - s') + \beta(s'' - s') \ge (s'' - s') + \beta\delta(s'' - s'),$$

or equivalently $\delta + \beta \ge 1 + \beta \delta$. This holds with equality if $\beta = 1$, but is violated for any $\beta < 1$ and $\delta < 1$. Thus the firm will never find it beneficial to reject the initial offer.

Fix an equilibrium, and suppose $i \in C$ deviates by making a higher initial wage proposal. It must be that the high type firm rejects this offer, since otherwise this would be a profitable deviation for i. Thus in the sub-game in which i has deviated and the offer has been rejected, i does no update their beliefs about the firm's type. Therefore the payoff to a type s'' firm from accepting a set $A \subseteq C \setminus i$ following i's deviation is given by

$$\pi_{i}(A|C,\omega_{1},X^{i}) = \sum_{i \in A} \left(s'' - w_{1}^{i}\right)$$

$$+ \sum_{j \in W \setminus C} \left(s'' - s' + \beta \left(1 - P^{j}(A)\right) \left(P^{j}(C \setminus A)(s'' - s') + (1 - P^{j}(C \setminus A))U_{2}(s'')\right)\right)$$

$$+ \sum_{j \in C \setminus (A \cup i)} \left(\delta(s'' - s') + \beta \left(1 - P^{j}(A)\right) \left(s'' - s'\right)\right)$$

$$+ \delta(s'' - s') + \beta(1 - P^{i}(A)) \left((P^{i}(X^{i} \setminus A)(s'' - s') + (1 - P^{i}(X^{i} \setminus A))U_{2}(s'')\right)$$

The same argument as Proposition 1 implies that $A \mapsto \pi_i(A|C,\omega_1,X^i)$ is supermodular.

Lemma 22. $A \mapsto \pi^i(A|C, \omega, X^i)$ is supermodular for all i, C, ω, X_i , strictly so when $\rho_{ij} \in (0,1) \ \forall \ i,j$.

Proof. The only difference from Proposition 1 is the payoff from worker i. Supermodularity of this term follows from that of $X \mapsto P^i(X)$ and $X \mapsto (1 - P^i(X))P^i(B \setminus X)$.

Lemma 23. In equilibrium,
$$\pi_i(X^i|C,\omega,X^i) = \pi_i(C|C,\omega,X^i) = \pi(C|C,\omega)$$
.

Proof. The second equality is immediate. Optimality of i's initial offer implies $\pi_i(X^i|C,\omega,X^i) \ge \pi_i(C|C,\omega,X^i)$. Moreover, $\pi(C|C,\omega) \ge \pi(X^i|C,\omega) \ge \pi_i(X^i|C,\omega,X^i)$, where the first inequality follows from firm optimality and the second from the fact that i has more favorable beliefs in the π case then the π_i case.

Corollary 3. If C is non-empty then $U_2(s'') = s'' - s'$ (equivalently, $ps'' \le s'$).

Proof. Otherwise $\pi(X^i|C,\omega) > \pi_i(X^i|C,\omega,X^i)$ and so Lemma 23 cannot hold.

Proposition 8. If $ps'' \neq s'$ then there is no equilibrium with screening in the first period. If ps'' = s' then there are two equilibria: all workers screen, or no workers screen.

Proof. Suppose $C \neq \emptyset$. The sum of screening wage offers is given by $\bar{W}(C)$ in (1). Note that $\bar{W}(W)/|W| \geq \bar{W}(C)/|C|$ for any $C \subseteq W$, where $\bar{W}(W)/|W| = s'' - (\delta + \beta)(s'' - s')$.

Since $U_2(s'') = s'' - s'$, a worker is prefers screening in the first period at wage w to pooling and receiving no information from other workers if and only if

$$p(w + \beta s'') + (1 - p)(\delta s' + \beta s') \ge (1 + \beta)s',$$

or equivalently, $w \ge \tilde{w} \equiv \frac{1}{p}(1-\delta)s' + \delta s' - \beta(s''-s')$.

In any equilibrium, at least one screening worker receives a wage less than or equal to $\bar{W}(C)/|C|$. If $\tilde{w} > \bar{W}(W)/|W|$ then there is no equilibrium with all worker's screening: workers would prefer to deviate to pooling even if they expect no other workers to successfully screen. Since $\bar{W}(W)/|W| \geq \bar{W}(C)/|C|$ for any $C \subseteq W$, this means that there is no screening in equilibrium. $\tilde{w} \leq \bar{W}(W)/|W|$ iff $ps'' \geq s'$. By Corollary 3 this holds iff ps'' = s'. In this case the unique equilibrium with screening involves all workers screening in the first period (it must also be that the firm expects worker's to pool in the second period if they deviate in the first).

C Alternating offers bargaining

There is a set W of workers, one firm, and 2 periods. The firm has private information about the per-period surplus generated by a worker, which is $s \in \{s', s''\}$, with s'' > s' > 0. The worker's outside option is 0, so there are always gains from cooperation. The worker's prior belief is that the firm is of type s'' with probability p. The firm discounts between periods at rate β , and workers do not discount between periods.

In each period the firm and worker engage in alternating-offers bargaining over a wage to be paid to the worker in the first period, à la Rubinstein (1982) (so there is an infinite time horizon within each period).¹⁶ Workers and firms have a common discount factor δ in the

¹⁶I assume that the worker and firm cannot commit to a second period wage in the first period. If they could then this model would be equivalent to a single worker-single firm model. I will discuss a different but related model in which workers arrive sequentially. In this case we can just assume that each worker works for only one period.

bargaining game. After the first period, but before the second round of wage negotiations, worker i observes worker j's first period wage with probability ρ_{ij} . The solution concept is SPBNE, with some restrictions which I will introduce later on.

C.1 Period 2

Consider first the negotiation in the second period, which I refer to as the P2 game. In period 2 the problem separates completely across workers, so we only need to consider a single-worker negotiation. An important preliminary observation is that discounting generates single crossing in the firm's propensity to accept a given wage. In other words, high type firms are more impatient, since delay is more costly.

Lemma 24. In any P2 equilibrium if firm s' accepts a wage proposal then so does s''.

Proof. Suppose that the worker has proposed w in round t. Let w_{t+j} be the lowest wage the firm can obtain at time t+j for $j \in \mathbb{N}$ by rejecting and making counter offers. Type s accepts w if and only if

$$s - w_{t+j} \ge \delta^{t+j} (s - w_{t+j}) \quad \forall \ j \in \mathbb{N}.$$

Clearly if this condition holds for s' then it holds for s''.

Recall that in the complete information stage game in which the worker knows that the firm is of type s, Rubinstein (1982) shows that there is a unique subgame perfect equilibrium in which the worker proposes a wage of $w(s) := s(1 - \delta)/(1 - \delta^2)$ and the firm accepts. Another general property of equilibrium strategies is that a worker will never propose a wage below w(s').

Lemma 25. No P2 equilibrium strategy involves the worker proposing a wage below w(s').

Proof. Let \tilde{w} be the infimum of the set of wages proposed by the worker following any history, in any equilibrium. A necessary condition for type s to reject a proposal of w' is

$$s - w' \le \delta(s - \delta \tilde{w})$$

or equivalently,

$$w' \ge (1 - \delta)s + \delta^2 \tilde{w}.$$

Notice that $\tilde{w} < (1 - \delta)s + \delta^2 \tilde{w} \Leftrightarrow \tilde{w} < w(s)$. So if $\tilde{w} < w(s') < w(s'')$ there exists a wage $w' > \tilde{w}$ such that both types of firms accept any $w \leq w'$ following any history. This means that proposing any w < w' at any point is dominated by proposing w', which contradicts the definition of \tilde{w} .

Lemma 24 implies that, when the worker does not know the firm's type, equilibrium in the P2 game take one of two forms, pooling or separation. In the pooling equilibrium the worker makes a wage offer in round 1 which is accepted by both firms. In the separating equilibrium the worker first makes an offer w_h which is accepted by the type s'' firm, and then the type s' firm offers $\delta w(s')$, which is accepted by the worker. The high type firm will accept w_h if and only if

$$s'' - w_h \ge \delta(s'' - \delta w(s'))$$

and so the highest acceptable wage that the worker can offer is $w_h = (1 - \delta)s'' + \delta^2 w(s')$. This can be written as $w_h = w(s'') - \delta^2(w(s'') - w(s'))$. The payoff to the worker of engaging in screening, when the worker attaches probability \hat{p} to the firm being type s'', is given by

$$V_S(\hat{p}) = \hat{p}w_h + (1 - \hat{p})\delta^2 w(s').$$

Alternatively the worker can make an offer in the first period that is accepted by both firms. Without restriction off-path beliefs there are many equilibria that can be supported, including a wage offer of s'. However these equilibria are ruled out by a mild monotonicity condition.¹⁷ It turns out that under this condition the worker can do no better than if they were facing the low type firm alone.

Inner Weak Monotonicity. Suppose the worker in the P2 game assigns positive probability to the firm being type s'. If the firm rejects an offer and makes a lower one then the worker continues to assign positive probability to type s'.

Lemma 26. Assume Inner Weak Monotonicity. In any P2 equilibrium, the type s' firm will never accept a wage greater than w(s').

Proof. Let \hat{w} be the supremum of the set of wages accepted by type s' and offered by a worker who assigns positive probability to type s'. Suppose the worker makes a proposal of w. The low type firm accepts w if this is better than rejecting w and proposing a lower wage. Under Weak Monotonicity, following such a rejection the worker continues to assign positive probability to type s'. Thus a necessary condition for w to be accepted by s' is.

$$s' - w > \delta(s' - \delta \hat{w})$$

or equivalently,

$$w \le (1 - \delta)s' + \delta^2 \hat{w}.$$

¹⁷This condition is satisfied under various refinements of SPBNE, such as the perfect sequential equilibrium solution concept of Grossman and Perry (1986)

Notice that $(1-\delta)s'+\delta^2\hat{w}<\hat{w}\Leftrightarrow\hat{w}>w(s')$. So if $\hat{w}>w(s')$ then there exists $\varepsilon>0$ such that no wage greater than $\hat{w} - \varepsilon$ is accepted by type s'. But this contradicts the definition of \hat{w} .

Corollary 4. Under Inner Weak Monotonicity, the unique NIP2 pooling equilibrium wage is w(s').

By Corollary 4, the worker screens the firms if and only if

$$pw_1 + (1-p)\delta^2 w(s') \ge w(s') \Leftrightarrow ps'' \ge s'.$$

It will be important in what follows that the low type is indifferent between the outcomes of the complete information equilibrium, the pooling equilibrium, and the screening equilibrium. In the former two cases the firm immediately accepts the wage offer of w(s'). In the latter case the firm delays one round and then proposes the wage $\delta w(s')$, which is accepted. Indifference follows since by construction $s' - w(s') = \delta(s' - \delta w(s'))$.

C.2Period 1 - no information

Consider now the game beginning in period 1, but assume that there is no information sharing between workers; $\rho_{i,j} = 0$ for all i, j. Call this the NIP1 (no information period 1) game. In this case the conclusion of Lemma 26 continues to hold.

Lemma 27. Under Inner Weak Monotonicity, in the NIP1 game the type s' firm never accepts a wage offer above w(s').

Proof. Recall that the period 2 payoff of the type s' firm is the same if in the second period it's type is fully revealed, screened, or pooled. Thus the firm's second period payoff is independent of worker beliefs, so long as these assign positive probability to s'. The proof is then the same as that of Lemma 26.

The separating equilibrium in the NIP1 game takes a similar form. Define w_h^1 by

$$s'' - w_b^1 + \beta(s'' - w(s'')) = \delta(s'' - \delta w(s')) + \beta(s'' - w(s')).$$

In words, w_h^1 is the highest wage that type s'' is willing to accept in period 1 and be revealed, rather then delay by one round of negotiation to pass as type s'. ¹⁸ Rearranging,

$$w_h^1 = (1 - \delta)s'' + \delta^2 w(s') - \beta(w(s'') - w(s')).$$

$$w_h^1 \ge w(s') \text{ iff } 1 - \delta^2 > \beta$$

Lemma 28. Under Inner Weak Monotonicity, in the NIP1 game, if the equilibrium in the negotiation with a worker is separating then the worker first offers wage w_h^1 , and the high type firm accepts. If the first offer is not accepted then the firm offers $\delta w(s')$, which is accepted.

Proof. Since the equilibrium is not pooling, the firm's type will be revealed. Then it must be that on-path the negotiation with type s'' ends before that with s'. This is because the highest continuation payoff for s'' comes from convincing workers it is type s', combined with the fact that waiting is more costly for type s'' (see Lemma 24). Once the low type is revealed we are back to the complete information game, so the worker will offer w(s').

Without further restrictions on off-path beliefs we cannot rule out pooling equilibria with wages below w(s'). Such an equilibrium could be supported if the worker's believes that the firm is type s'' for sure flowing acceptance of a wage above the equilibrium level. Such equilibriua are ruled out under natural refinements, for example, the Perfect Sequential Equilibrium (PSE) of Grossman and Perry (1986). Roughly speaking, off path beliefs in PSE are guided by the following question: for a given deviation, is there a set S of firm types that would benefit from deviating if the worker's belief upon observing the deviation is the restriction of the prior to S? If so then this should be the belief of the worker. In the case of a NIP1 pooling equilibrium with first period wage $\bar{w} < w(s')$, the deviation would be accepting a wage in $(\bar{w}, w(s')]$. If the worker believes that both types will accept such a wage then it is optimal for both types to do so. Thus no such PSE can be supported.

Lemma 29. Under Inner Weak Monotonicity, the unique pooling PSE in the NIP1 game is at a wage of w(s').

I will say that an equilibrium is a symmetric PSE if it is a PSE and all workers $j \neq i$ have the same beliefs about the firm's type after observing i's wage.

C.3 Period 1 - information sharing

The central question I want to answer is how the equilibria with and without information sharing compare. Consider now the game with information sharing beginning in the first period. A type s' firm receives the same period 2 payoff regardless of worker beliefs, so long as workers continue to assign positive probability to the firm being type s'. The first step is to show that, as in the P2 game, the type s' firm will never accept a wage above w(s'). This depends on a mild monotonicity condition on beliefs of workers after observing the wages of others.¹⁹

¹⁹If workers shared their beliefs, rather than just their wages, then this assumption would not be needed.

Outer Weak Monotonicity. If worker i assigns positive probability to the firm being type s' after observing a wage of w for worker j, then i continues to do so after observing a wage for j less than w.

Lemma 30. Under Inner and Outer Weak Monotonicity, the type s' firm never accepts a wage offer above w(s').

Proof. Recall that the period 2 payoff of the type s' firm is the same if in the second period it's type is fully revealed, screened, or pooled. Thus the firm's second period payoff is independent of worker beliefs, so long as these assign positive probability to s'. The proof is then the same as that of Lemma 30, where Outer Weak Monotonicity guarantees that the beliefs of other workers do not become degenerate on s''.

Without further refinements the conclusion of Lemma 25 does not hold in the first period, and so we cannot conclude that the unique pooling equilibrium wage in period 1 is w(s') as in 4. This is because the type s'' firm wants to influence period 2 beliefs. The s'' is indifferent between passing as a type s' and pooling with s' if the P2 equilibrium is pooling. It strictly prefers either of these outcomes to the separating P2 equilibrium, which in turn it strictly prefers to being fully revealed as type s''. Depending on off-path beliefs, it may be possible to sustain pooling wages below w(s') in equilibrium. Fortunately, we can do comparative statics and draw welfare conclusions without further refinements. In particular, Lemma 30 allows us to identify what the separating equilibrium in period 1 will look like.

Lemma 31. Under Inner and Outer Weak Monotonicity, there exist w_1^i, w_2^i with $w_2^i \leq w(s')$ such that if the equilibrium in the negotiation with worker i is separating then the worker first offers w_1^i , and the high type firm accepts. If the first offer is not accepted then the firm offers δw_2^i , which is accepted. In the unique separating symmetric PSE $w_2^i = w(s')$, and in the unique pooling symmetric PSE the wage is w(s').

Proof. If the equilibrium is separating, the firm's type will be revealed. Then it must be that on-path the negotiation with type s'' ends before that with s'. This is because the highest continuation payoff for s'' comes from convincing workers it is type s', combined with the fact that waiting is more costly for type s'' (see Lemma 24). Lemma 30 implies $w_2^i \leq w(s')$.

Suppose the equilibrium is pooling at wage w. If $w > w_2^i$ then firm s' would also be willing to accept w when its type has been revealed to worker i in the separating equilibrium. If $w < w_2^i$ then worker i could instead propose w_2^i in the first period. Since the s' firm was willing to accept w_2^i in the separating equilibrium, it will be willing to do so in the first

period. If all workers believe that both firms accept w_2^i then both will, and so in a symmetric PSE this should be the off path belief.

If $w_2^i < w(s')$ consider the deviation by a firm in the pooling equilibrium of accepting a wage $w \in (w_2^i, w(s')]$. If all workers believe that both types accept such a wage then it is optimal for both to do so, so in the symmetric PSE all workers believe that both types accept w. But then the worker would offer w rather than w_2^i , so this cannot be an equilibrium. \square

It is not necessary to assume symmetric PSE to rule out $w_2^i = w(s')$. We can rule out equilibria in which $w_2^i < w(s')$ as long as workers who observe a wage $w \in (w_2^i, w(s')]$ do not believe that the firm is type s'' with probability 1.

It need not be that $w_1^i > w_2^i$, so we cannot ignore the incentives of type s' firms to accept w_1^i . In what follows I will use the incentive constraint of type s'' to pin down w_1^i . If it turns out that the incentive constraint of type s' is violated at this wage then it will not be possible for i to screen in equilibrium.

In the separating equilibrium, w_1^i depends on the information sharing parameters. On the other hand, w_2^i depends only on off-path beliefs. The willingness of a type s'' firm to reveal itself to worker i depends on two characteristics of equilibrium information sharing; the number workers other than i that are not screening the firm in period 1, and the probability that these workers observe i's wage, conditional on not receiving information on the firm's type from another worker. The first effect I refer to as 'information free-riding''. Workers are willing to pay a cost in the first period to screen, since the information will be useful in the second period. If they are likely to learn the firm's type from other workers then they will be less willing to do so. The second effect is the firms "reputation effect". The high type firm will demand a greater premium for being revealed to worker i in the first period if other workers are likely to observe i's wage and learn the firms type. The magnitude of this effect depends on the information sharing parameters, as well as the number of workers that are not screening in the first period.

If worker i does not screen in the first period, and receives no information about the firm's type from other workers' wages, then i will screen in the second period if and only if

$$pw_h + (1-p)\delta^2 w(s') \ge w(s').$$

Substituting for w_h , this condition becomes

D Equilibrium in the screening game

I will focus on equilibria in which the worker never offers a wage below w(s'). As Lemma 31 and the subsequent discussion show, any other equilbrium can be ruled out by assuming PSE, or simply ruling out extreme belief revisions that would support $w_2^i < w(s')$. It is also necessary to identify how workers who pool will react if they see wage w_1^i from worker i and δw_2^j from worker j. I assume that in this case the worker infers that the firm is type s''.

Let V_2 be the value that workers would obtain in the second period if they did not receive any additional information about the firm's type following the first period (so either $V_2 = V_S(p)$ or $V_2 = w(s')$). Let $U_2(s)$ be the corresponding second period value the type s firm derives from a single worker who has not received information the first period.

Fix the first period separating wage offers $\omega_1 = \{w_1^i\}_{i \in C}$. Let $\pi(A|C,\omega)$ be the payoff of a type s'' firm that accepts w_1^i iff $i \in A$, and for $j \in C \setminus A$ offers $\delta w(s')$ (and accepts w(s') for all $i \in W \setminus C$). Then the firms on path payoff is $\pi(C|C,\omega)$, given by

$$\pi(C|C,\omega) = \sum_{i \in C} \left(s'' - w_1^i + \beta(s'' - w(s'')) \right)$$

$$+ \sum_{j \in W \setminus C} \left(s'' - w(s') + \beta \left(P^j(C)(s'' - w(s'')) + (1 - P^j(C))U_2(s'') \right) \right).$$

For any $A \subseteq C$ we have

$$\begin{split} \pi(A|C,\omega) &= \sum_{i \in A} \left(s'' - w_1^i + \beta(s'' - w(s'')) \right) \\ &+ \sum_{j \in W \setminus C} \left(s'' - w(s') + \beta \left[P^j(A)(s'' - w(s'')) + (1 - P^j(A)) \left(P^j(C \setminus A)(s'' - w(s')) + (1 - P^j(C \setminus A)) U_2(s'') \right) \right] \right) \\ &+ \sum_{j \in C \setminus A} \left(\delta(s'' - \delta w(s')) + \beta \left[P^j(A)(s'' - w(s'')) + (1 - P^j(A))(s'' - w(s')) \right] \right) \end{split}$$

Lemma 32. For any C and ω , $\pi(\cdot|C,\omega)$ is supermodular, and strictly supermodular if $\rho_{i,j} \in (0,1)$ for all $i,j.^{20}$

Proof. For simplicity I will write $\pi(X)$ rather than $\pi(X|C,\omega)$. I wish to show that $\pi(A \cup B) - \pi(B) \ge \pi(A) - \pi(A \cap B)$. In all cases, the payoff the firm derives from workers in $A \cap B$ is unchanged. I will consider separately the payoffs of workers in $W \setminus C$, $C \setminus (A \cup B)$, $A \setminus B$ and $B \setminus A$.

²⁰The condition $\rho_{i,j} \in (0,1)$ for all i,j is clearly not necessary for strict supermodularity to hold for a given C. Lemma 1) gives the relevant conditions for strict supermodularity.

Consider the payoff derived from workers in $W \setminus C$. Let $\Pi^j(X) = \prod_{k \in X} (1 - \rho_{jk})$. Then the, using the fact that $\Pi^j(X)\Pi^j(C \setminus X) = \Pi^j(C)$, the $W \setminus C$ component of $\pi(X)$ can be written as

$$\sum_{j \in W \setminus C} \left(s'' - w(s') + \beta \left[(1 - \Pi^{j}(X))(s'' - w(s'')) + \Pi^{j}(X)(1 - \Pi^{j}(C \setminus X))(s'' - w(s')) + \Pi^{j}(C)U_{2}(s'') \right] \right)$$

$$= \sum_{j \in W \setminus C} \left(s'' - w(s') + \beta \left[s'' - w(s'') + \Pi^{j}(X)(w(s'') - w(s')) + \Pi^{j}(C)(U_{2}(s'') - s'' + w(s')) \right] \right)$$

Then the $W \setminus C$ component of $\pi(X) - \pi(Y)$ is given by

$$\beta \sum_{j \in W \setminus C} \left(\Pi^{j}(X) - \Pi^{j}(Y) \right) \left(w(s'') - w(s') \right)$$

For all j, $\Pi^{j}(A \cup B) - \Pi^{j}(B) \geq \Pi^{j}(A) - \Pi^{j}(A \cap B)$ (with strict inequality whenever the conditions in Lemma 1 for strict inequality are satisfied). This shows supermodularity on the $W \setminus C$ component of firm payoffs.

Now for the $C \setminus (A \cup B)$ component of $\pi(X)$. These are given by

$$\sum_{j \in C \setminus (A \cup B)} \left(\delta(s'' - \delta w(s')) + \beta \left[s'' - w(s') + P^j(X) \left(w(s') - w(s'') \right) \right] \right).$$

Then the $C \setminus (A \cup B)$ component of $\pi(X) - \pi(Y)$ is given by

$$\beta \sum_{j \in C \setminus (A \cup B)} \left(P^j(X) - P^j(Y) \right) \left(w(s') - w(s'') \right)$$

Then submodularity of $P^{j}(\cdot)$ (Lemma 1) implies that supermodularity holds for this part of payoffs (strictly when the conditions given in Lemma 1 are satisfied).

Now consider the $A \setminus B$ component of payoffs. In both $\Pi(A \cup B)$ and $\Pi(A)$, these workers are successfully screening, so they generate the same payoff for the firm. Supermodularity here will follow by showing that the $A \setminus B$ component of $\pi(B)$ is less than that of $\pi(A \cap B)$. In $\pi(B)$, this is given by

$$\sum_{j \in A \setminus B} \left(\delta(s'' - \delta w(s')) + \beta \left[s'' - w(s') + P^j(B) \left(w(s') - w(s'') \right) \right] \right).$$

The expression in $\pi(A \cap B)$ is the same, except that $P^j(A \cap B)$ replaces $P^j(B)$. Since $P^j(B) \geq P^j(A \cap B)$ (with strict inequality as long as $P^j(A \cap B) < 1$ and there exists $k \in B \setminus A$ with $\rho_{jk} > 0$) we conclude that supermodularity holds on $A \setminus B$.

Finally, consider the $B \setminus A$ component of payoffs. Since these workers successfully screen in both $\pi(A \cup B)$ and $\pi(B)$, we need only show that the $B \setminus A$ component of payoffs is greater in $\pi(A \cap B)$ than in $\pi(A)$. This follows from the same argument given for the $A \setminus B$ component above.

I will first characterize the wage that each screening worker offers to the high type firm. The subtlety here is that the binding incentive constraint under which type s'' accepts a wage is generally not the "single deviation" of rejecting i and accepting all other $k \in C$. To see this, consider an equilibrium in which the set of screening workers is C, and these workers make initial offers of $\omega = \{w_1^i\}_{i \in C}$. C and ω fully characterize on-path play in equilibrium. For each $i \in C$, let

$$\chi^i = \operatorname*{argmax}_{X \subseteq C \setminus i} \pi(X|C, \omega).$$

 χ^i need not be single valued, but when it will not cause confusion I will discuss as if it is, and refer to this set as X^i . X^i is the set of screening offers that type s'' will accept if it rejects i's initial offer. If $X^i \neq \emptyset$, let $k \in X^i$. By definition of X^i , $\pi(X^i|C,\omega) \geq \pi(X^i \cap X^k|C,\omega)$. But then Lemma 32 implies $\pi(X^i \cup X^k|C,\omega) \geq \pi(X^k|C,\omega)$.

Suppose X^i and X^k satisfy the conditions for strict supermodularity of π . Then we have $\pi(X^i \cup X^k | C, \omega) > \pi(X^k | C, \omega)$. Consider what happens if k offers a slightly higher wage. This can only make the payoff from rejecting k's offer worse, since by rejecting w_1^k the firm would have signaled that it was type s' (it could be strictly worse, since k may reject the firms proposal of $\delta w(s')$ in the next round). If the type s'' firm accepts k's offer then at worst worker k, and any other worker who observes k's wage, will believe that the firm is type s'' for sure. But this is the same belief they would hold if the firm accepted w_1^k . Thus by raising it's wage offer slightly, worker k makes only a small change to the firm's payoff from accepting the offers of workers in $X^i \cup X^k$. Since $\pi(X^i \cup X^k | C, \omega) > \pi(X^k | C, \omega)$, the firm will still accept k's offer. But then ω and C cannot characterize an equilibrium, since k has a profitable deviation. The conclusion is that X^i and X^k cannot satisfy the conditions for strict supermodularity.

Lemma 33. Assume $\rho_{ij} \in (0,1)$ for all i,j. In any equilibrium characterized by C,ω , it must be that $\pi(C|C,\omega) = \pi(\varnothing|C,\omega)$, and $\varnothing \in \chi^i$ for all $i \in C$.

Proof. I will show that $\emptyset \in \chi^i$ for all $i \in C$; the remainder of the result then follows from optimality of the worker's wage offers. If $\rho_{ij} \in (0,1)$ for all i,j then the conditions for strict supermodularity of $\pi(\cdot|C,\omega)$ are satisfied for all non-empty A,B. This, and the discussion preceding Lemma 33, imply that there is no pair of workers i,k with $k \in X^i$ and $X^k \neq \emptyset$.

Even without conditions on ρ , for any equilibrium C, ω we will have $\pi(X^i|C,\omega) = \pi(X^k|C,\omega)$ for all $k \in X^i$. To see this, first note that optimality of w_1^i for worker i implies $\pi(C|C,\omega) \leq \pi(X^i|C,\omega)$. Moreover, firm optimality implies $\pi(C|C,\omega) \geq \pi(A|C,\omega)$ for all $A \subseteq C$. Thus $\pi(C|C,\omega) = \pi(X^i|C,\omega) \geq \pi(X^k|C,\omega)$. If $\pi(X^i|C,\omega) > \pi(X^k|C,\omega)$ then

firm k can make a slightly higher wage offer that will still be accepted, so we must have $\pi(X^i|C,\omega) = \pi(X^k|C,\omega)$.

Suppose $X^i \neq \emptyset$ and $X^k = \emptyset$ for all $k \in X^i$. Then by the previous claim, $\pi(X^i|C,\omega) = \pi(\emptyset|C,\omega)$, so $\emptyset \in \chi^i$.

The condition $\pi(C|C,\omega)=\pi(\varnothing|C,\omega)$ pins down $\sum_{i\in C}w_1^i$. This condition is given by

$$\begin{split} & \sum_{i \in C} \left(s'' - w_1^i + \beta(s'' - w(s'')) \right) + \sum_{j \in W \setminus C} \left(s'' - w(s') + \beta \left(P^j(C)(s'' - w(s'')) + (1 - P^j(C))U_2(s'') \right) \right) \\ & = \sum_{i \in C} \left(\delta(s'' - \delta w(s')) + \beta(s'' - w(s')) \right) + \sum_{j \in W \setminus C} \left(s'' - w(s') + \beta \left(P^j(C)(s'' - w(s')) + (1 - P^j(C))U_2(s'') \right) \right) \end{split}$$

This simplifies to

$$|C| \left(\delta(s'' - \delta w(s')) - \left(s'' - \frac{1}{|C|} \sum_{i \in C} w_1^i \right) \right) + \beta \left(|C| + P^j(C) \right) \left(w(s'') - w(s') \right) = 0$$

so

$$\begin{split} \sum_{i \in C} w_1^i &= \bar{W}(C) := |C| \left((1 - \delta)s'' + \delta^2 w(s')) \right) - \beta \left(w(s'') - w(s') \right) \left(\bar{P}(C) + |C| \right) \\ &= |C| \left((1 - \delta^2)w(s'') + \delta^2 w(s')) \right) - \beta \left(w(s'') - w(s') \right) \left(\bar{P}(C) + |C| \right) \end{split}$$

where $\bar{P}(C) = \sum_{j \in W \setminus C} P^j(C)$ is the expected number of pooling workers who will observe a wage from a worker in C. As expected, the sum of screening wages is decreasing in $\bar{P}(C)$.

The individual wage offers in ω are constrained by the conditions that $\emptyset \in X^i$ for all i. This condition implies that there cannot be too much dispersion in ω .²¹ As we will see, it is satisfied automatically, conditional on the sum being $\bar{W}(C)$, in the symmetric equilibria of interest.

I now turn to the incentives of the workers to engage in screening. If worker i decides not to screen, they must offer a wage that both firm types will accept. By Lemma 30, the highest such wage is w(s'). I will show now that the high type firm will accept an offer of w(x') from any worker in C. This is easiest to show under the assumption that workers' prior beliefs are intermediate.

Intermediate Beliefs. I say that workers have intermediate beliefs if they screen in the NIP1 game, but not in the NIP2 game. This holds iff

$$\frac{s'}{s''} \ge p \ge \frac{(1 - \delta^2)}{(2 - \delta^2 - \beta)\frac{s''}{s''} + \beta - 1}$$

²¹Bounds on wages satisfying this condition can be determined by looking recursively at smaller screening sets.

When workers have intermediate beliefs the high type firm is indifferent in the second period between pooling and passing as the low type. This simplifies the analysis. If p > s'/s'' the worker screens in the NIP2 game. In this case the high type firm strictly prefers passing as the low type to pooling.

Lemma 34. Assume Intermediate Beliefs. In any equilibrium in which the set of screening workers is C, the type s'' firm will accept an initial offer of w(s') from any worker in C.

Proof. Let i be the deviating worker. Assume that if the firm rejects the offer of w(s') and proposes $\delta w(s')$ then i believes that the firm is type s'. I will show that if this is the case the type s'' firm prefers to accept w(s''), so these should indeed be the beliefs under PSE. Similarly if the firm accepts w(s'), all workers who observe this will retain their prior belief about the firm's type.

By Lemma 33, if the firm rejects i's initial offer then it should also reject all offers in $C \setminus i$. Therefore a sufficient condition for the type s'' firm to accept w(s') from i is that it prefers accepting w(s') from i to rejecting, given that it is rejecting all other offers in C. Given the assumption of intermediate beliefs, from the firm's perspective, convincing workers that it is type s' is the same as getting them to maintain their prior belief. Thus there is no informational difference between accepting w(s') from i or rejecting and proposing $\delta w(s')$. The type s'' firm then prefers to accept w(s'), since $s'' - w(s') > \delta(s'' - \delta w(s'))$.

Lemma 34 does not imply that if $i \in C$ deviates by proposing w(s'), the firm will reject all initial offers in $C \setminus i$. Asymmetry in the initial offers of workers in C may help provide incentives for workers to engage in screening. This is because the distribution of these offers affects the outside option of workers. To see this, suppose for illustrative purposes that $w_1^j = 0$ for all $j \in C \setminus i$ (ignoring the fact that these workers would then prefer to deviate and offer w(s')). If w_1^j were to deviate and offer w(s') then the firm would continue to accept the initial offers of $j \in C \setminus i$. On the other hand, if $w_1^i = 0$ and i offers w(s'), the firm will in general reject the initial offers of $j \in C \setminus i$.

This suggest that when information-sharing parameters differ across workers, screening will be best supported by the following pattern of initial wage offers in C (holding fixed $\sum_{i \in C} w_1^i$): workers with high $P^i(C \setminus i)$, i.e. workers who are likely to observe another wage from C, should make low initial wage offers w_1^i , while workers with low $P^j(C \setminus j)$ should make high initial wage offers. This is because workers with high are those with the highest ability to free-ride off the information generated by workers in $C \setminus i$. When their offer w_1^i is low, which means $\sum_{C \setminus i} w_1^j$ is relatively high, a deviation to pooling will lead the firm to reject all initial offers in $C \setminus i$, rendering i's informational advantage useless. Thus, somewhat

counter-intuitively, i's informational advantage if it deviates means precisely that it should receive a lower wage on path.

Given this discussion, it is natural to wonder if asymmetric offers can help support screening even when workers are symmetric. It turns out that this is not the case; if screening can be supported with asymmetric offers than it can be supported with symmetric offers. This will follow from the fact that, with symmetric information-sharing parameters, the *average* initial wage offer of screening workers must be increasing in the size of the set of screening workers. The intuition is that as the set of pooling workers shrinks, it becomes less costly for the high type firm to reveal itself to the screening workers.

Lemma 35. Assume $\rho_{ij} = \rho \in (0,1)$ for all i,j. Let C,ω and C',ω' be equilibrium screening sets and wage offers, with |C'| > |C|. Then $\frac{1}{|C'|} \sum_{i \in C'} w_1^{ii} > \frac{1}{|C|} \sum_{i \in C} w_1^{ii}$

Proof. Recall that

$$\frac{1}{|C|} \sum_{i \in C} w_1^i = \left((1 - \delta^2) w(s'') + \delta^2 w(s') \right) - \left(\frac{\bar{P}(C)}{|C|} + 1 \right) \beta \left(w(s'') - w(s') \right).$$

The claim follows in $\bar{P}(C)/|C| > \bar{P}(C')/|C'|$. Under symmetry, $\bar{P}(C) = (|W|-|C|) (1-(1-\rho)^{|C|})$. Then the claim is that the function $|C| \mapsto \frac{|W|-|C|}{|C|} (1-(1-\rho)^{|C|})$ is decreasing. The derivative of this function is negative iff

$$\frac{|W|}{|C|(|W|-|C|)} > -\ln(1-\rho)\frac{(1-\rho)^{|C|}}{1-(1-\rho)^{|C|}}.$$

The right hand side of this expression is bounded above by 1/|C| (the limit as $\rho \to 0$). The inequality follows.

As a result of Lemma 35, we can see that creating dispersion in the wages of screening workers cannot help their incentives to screen. To see this, suppose that there is a uniform screening wage $w = w_1^i$ for all $i \in C$. If a worker deviates and offers w(s'), Lemma 35 implies that the firm will be unwilling to accept any wages in C. This means that the deviating worker anticipates receiving no information. If workers want to deviate in this situation, then adding dispersion only increases the incentives to deviate of the workers who are to make lower initial offers.

Lemma 36. Assume Intermediate Beliefs and $\rho_{ij} = \rho \in (0,1)$ for all i, j. If screening cannot be supported with a uniform screening wage then in cannot be supported with non-uniform screening wages.

Similarly, if the incentive constraint of the screening workers is violated, i.e. they prefer to deviate and offer w(s'), when there is a large set of screening firms, then they will also prefer to deviate when there is a smaller set of screening firms.

Lemma 37. Assume Intermediate Beliefs and $\rho_{ij} = \rho \in (0,1)$ for all i,j. If the incentive constraint of screening workers is violated when N' workers screen, then it will be violated if N < N' workers screen.

Proof. Since screening workers that deviate to pooling anticipate receiving no information from other workers, their payoff from deviation is the same regardless of |C|. The claim follows from Lemma 35.

I now turn to the incentives of the low type firm. In equilibrium, the type s' firm receives a payoff of $(1+\beta)(s'-w(s'))|W|$. This is because the low type firm is indifferent between screening, pooling, and being revealed. In an equilibrium with screening set C, the firm could deviate by accepting the initial offers from workers in $A \subseteq C$. This is the so-called "take the money and run" deviation. The payoff from doing so, under the assumption that workers who receive mixed signals assume the firm is the high type, is given by²²

$$(|W| - |A|)(s' - w(s')) + \sum_{i \in A} (s' - w_1^i) + \beta \sum_{i \in W \setminus A} (1 - P^i(A)) (s' - w(s')).$$

The firm prefers not to deviate in this way iff the equilibrium payoff is higher, the condition for which is given by

$$\beta\left(\bar{P}(A) + |A|\right)\left(s' - w(s')\right) \ge \sum_{i \in A} \left(w(s') - w_1^i\right) \tag{5}$$

Here $\bar{P}(A) + |A|$ is the expected number of workers with whom the type s' firm will be unable to reach any agreement in period 2, and $\sum_{i \in A} \left(w(s') - w_1^i \right)$ is the potential gain in the first period of accepting wages that may be below w(s'). Note that the condition in (5) does not depend on C directly, only on A and ω . I say that the low type incentive constraint is satisfied for C, ω if (5) is satisfied for all $A \subseteq C$.

Lemma 38. Let C be the set of screening workers, and suppose $w_1^i = \overline{W}(C)/|C|$ for all $i \in C$. If $\rho_{i,j} = \rho \in (0,1)$ for all i,j then the low type incentive constraint is satisfied iff (5) holds for A = C.

 $^{^{22}}$ If workers become convinced that the type s' firm is actually type s'', the firm gets a payoff of zero in the second period.

Proof. Rewriting (5), we have

$$\beta\left(\frac{\bar{P}(|A|)}{|A|} + 1\right)\left(s' - w(s')\right) \ge w(s') - \frac{\bar{W}(C)}{|C|}$$

As shown in the proof of Lemma 35, $\bar{P}(A)/|A|$ is decreasing under symmetric information sharing. The claim follows.

I now want to consider the incentives of pooling workers. A pooling worker may deviate by trying to screen the firm. Assume that the deviating pooling worker first proposes a wage \hat{w} , which it expects to be accepted if and only if the firm is type s''. I will show when these beliefs are consistent with the incentives of the firm.

Suppose there is an equilibrium with |W| > |C| > 0. If a pooling worker $i \in W \setminus C$ deviates and tries to screen, the firm can either accept i's initial offer or reject and propose $\delta w(s')$ in the second round.

Lemma 39. Assume $\rho_{i,j} \in (0,1)$ for all i,j. In an equilibrium with screening set C, if a pooling worker i makes an initial offer of \hat{w} , the type s'' firm will accept if and only if $\bar{W}(C \cup i) - \bar{W}(C) \ge \hat{w}$.

Proof. Suppose that the type s'' firm rejects i's offer. Then i will believe that the firm is type s' as long as it does not observe evidence to the contrary. Since the second period payoff to the firm from a worker who believes it is type s' is weakly higher when workers maintain their prior, the incentive of the firm to screen with the workers in C is lower than in equilibrium. By Lemma 33, on path the firm weakly prefers rejecting all initial offers in A to accepting all initial offers in A, for all $A \subseteq C$. Thus following it's rejection of i's initial offer, the firm will also weakly prefer rejecting all initial offers in A to accepting all initial offers in A, for all $A \subseteq C$. (Under Intermediate Beliefs the firm is indifferent between rejecting all initial offers in C and accepting all initial offers in C, both on path and following the rejection of i's initial offer.)

By the definition of $W(C \cup i)$, the firm is indifferent between accepting all initial offers in $C \cup i$ and rejecting all such offers. Combined with the preceding paragraph, this implies that if i makes an initial offer of $\bar{W}(C \cup i) - \bar{W}(C)$, the type s'' firm will accept. In particular, the firm will prefer accepting all initial offers in $C \cup i$ to rejecting all initial offers in $C \cup i$. Moreover, given acceptance of i's offer, the high-type firm is more willing to be screened by other workers than it was with no deviation by i.

It remains to show that no higher offer by worker i will be accepted. This would only be the case if there was some set $A \subset C$ such that the firm strictly preferred accepting only

the initial offers in $A \cup i$ to accepting all initial offers in $C \cup i$. This is ruled out however by submodularity of P^j (Lemma 1), using a similar argument to that given for Lemma 32. (It does not follow directly from Lemma 32 because we are comparing a case in which i pools to one in which it screens. However the difference in the argument is minor.)

Lemma 39 does not imply that the deviating pooling worker will be able to successfully screen by offering $\bar{W}(C \cup i) - \bar{W}(C)$: it may be that the low type firm will also want to accept. Mild conditions on primitives guarantee that this is not the case. The following Lemma is an intermediate step. It narrows down the set of possible responses by the low type firm to the pooling workers i's deviation; either the low type firm rejects i's initial offer, or it accepts all initial offers.

Lemma 40. Assume $\rho_{i,j} = \rho \in (0,1)$ for all i,j and the low type incentive constraint is satisfied with screening set C and $w_1^i = \bar{W}(C)/|C|$ for all $i \in C$. If i deviates by offering $\hat{w} = \bar{W}(C \cup i) - \bar{W}(C)$ and the type s' firm accepts, then it must be that the firm also accepts all initial wage offers in C.²³

Proof. As shown in the proof of Lemma 35, with symmetric information sharing $\bar{W}(C)/|C|$ is decreasing in |C|. This implies that $\bar{W}(C \cup i) - \bar{W}(C) > \bar{W}(C)$. Suppose that in response to i's deviation, the type s' firm accepts all initial offers in $i \cup S$ for $S \subset C$, (or $S = \emptyset$). This means that condition (5) is violated for $A = i \cup S$. Since $\bar{W}(C \cup i) - \bar{W}(C) > \bar{W}(C)$, this meas there also exists a set $X \subset S$ with $|X| = |S \cup i|$ such that $\sum_{k \in X} w_1^k < \bar{W}(C \cup i) - \bar{W}(C) + \sum_{k \in S} w_1^k$. But then (5) does not hold for A = X, so this cannot be an equilibrium.

Using Lemma 40, we can identify conditions under which the low type firm will not accept the initial wage offer of a pooling worker who deviates to offer $\bar{W}(C \cup i) - \bar{W}(C)$. In other words, the pooling worker will be able to screen with this deviation.

Lemma 41. Assume $\rho_{i,j} = \rho \in (0,1)$ for all i,j and the low type incentive constraint is satisfied with screening set C and $w_1^i = \overline{W}(C)/|C|$ for all $i \in C$. If $j \in W \setminus C$ deviates by offering $\hat{w} = \overline{W}(C \cup i) - \overline{W}(C)$ then the low type will reject this initial offer iff

$$\beta\left(\frac{\bar{P}(C\cup j)}{|C|+1}+1\right) \ge (1-\delta)\frac{s'-s''}{s'-w(s'')}.$$

Proof. By construction, the type s'' firm is indifferent between accepting all initial offers in $C \cup j$, given j's proposal of $\bar{W}(C \cup j) - \bar{W}(C)$, and rejecting all these offers. This indifference

²³It is easy to see from condition (5) that weaker conditions can be given for the conclusion, but they are not as convenient to state. I present this version since it is what will be used later on.

condition is given by

$$\beta \left(\bar{P}(C \cup j) + |C| + 1 \right) \left(w(s'') - w(s') \right) = (|C| + 1) \left(s'' - \frac{1}{|C| + 1} W(\bar{C} \cup j) - \delta(s'' - \delta w(s')) \right). \tag{6}$$

The left hand side of this expression is the reputational benefit of delay, and the right hand side is the period 1 cost.

By Lemma 40, if the low type firm accepts j's initial offers then it accepts all initial offers in $C \cup j$. On the other hand, the fact that condition (5) was satisfied for all $A \subseteq C$ implies that if the low type rejects j's initial offer than it also rejects all initial offers in C. Thus the low type rejects j's initial offer if and only if it prefers rejecting all initial offers in $C \cup j$ to rejecting all such offers. The condition for this is

$$\beta \left(\bar{P}(C \cup j) + |C| + 1 \right) (s' - w(s')) \ge (|C| + 1) \left(s' - \frac{1}{|C| + 1} \bar{W}(C \cup j) - \delta(s' - \delta w(s')) \right). \tag{7}$$

Combining conditions (6) and (7) yields the desired condition.

Since $\bar{P}(C)/|C|$ is decreasing, it is sufficient to check the condition of Lemma 41 for |C| = |W| - 1, in which case it reduces to

$$\beta \ge (1 - \delta) \frac{s' - s''}{s' - w(s'')}.$$

A worker in $W \setminus C$ can deviate and offer at most $\bar{W}(C \cup i) - \bar{W}(C)$ if they hope the high type firm to accept. Under the conditions of Lemma 41, this leads to successful screening; the firm will accept this initial offer if and only if it is type s''. Therefore the worker's belief that offering $\bar{W}(C \cup i) - \bar{W}(C)$ will lead to screening is consistent, and this constitutes the best deviation from pooling.

Under the conditions of Lemma 41, $\overline{W}(C)/|C|$ is decreasing. Therefore a pooling worker who deviates when |C| = N does better than a screening worker in the symmetric-offers equilibrium when |C| = N + 1. Since the payoff of screening workers is increasing in |C|, this means that in equilibrium pooling workers always do better than screeners.

D.1 Properties of equilibrium

The properties of equilibrium can be summarized as follows, beginning with those that hold under the least restrictive assumptions.

Assume $\rho_{i,j} \in (0,1)$ for all i,j.

• Lemma 33. Interpretation: Fix the set C of workers who will screen in equilibrium, and consider a) the choice of initial wage offers by workers in C, and b) the incentives of the high type firm to accept the initial offer, rather than delay to pass as the low type. Supermodularity in information transmission implies that each worker knows that if they increase their offer the firm will reject not just theirs, but all other initial offers in C. Thus the workers know they will get no information about the firm's type. This pins down the sum of the initial offers for workers in C.

Assume $\rho_{i,j} \in (0,1)$ for all i,j and Intermediate Beliefs.

• Lemma 34. Interpretation: Any screening worker can deviate to pooling by making an initial offer of w(s').

Assume $\rho_{ij} = \rho \in (0,1)$ for all $i, j.^{24}$

- Lemma 35. Interpretation: given the supermodularity of information sharing, there are two positive externalities of screening. First, pooling workers may get some information. Second, other screening workers need to pay lower information rents.
- Screening is fragile; if any screening worker deviates no screening occurs.

Assume $\rho_{ij} = \rho \in (0,1)$ for all i,j and symmetry of screening wage offers $(w_1^i = \bar{W}(C)/|C|$ for all $i \in C$).

- In this case Lemma 35 implies that when all workers in C make the same initial wage offer $\bar{W}(C)/|C|$, we have $X^i = \emptyset$ for all $i \in C$.
- Lemma 38. Interpretation: The binding constraint for the low type firm is to not accept all initial offers in C.
- Lemma 41. Interpretation: Under the condition given in the lemma, workers who are supposed to pool in equilibrium can deviate to screening by making an offer of $\bar{W}(C \cup i) \bar{W}(C)$. By Lemma 39, this is the relevant incentive constraint for workers who pool in equilibrium.

Assume $\rho_{ij} = \rho \in (0,1)$ for all i, j and Intermediate Beliefs.

• Lemma 37. Interpretation: Incentives of workers to screen are stronger when more workers screen.

²⁴Definitely not necessary for Lemma 35

In the intersection of the restrictions, we have the following (partial) characterization of equilibrium.

Assume $\rho_{ij} = \rho \in (0,1)$ for all i, j, Intermediate Beliefs, and symmetric screening wage offers. The incentive constraints in an equilibrium in which the screening set is C are as follows.

- Screening workers: either screen by offering $\bar{W}(C)/|C|$ or deviate to pooling by offering w(s'), in which case the high type firm will reject all other initial offers from screening workers, and so there will be no information revealed by wages.
- Pooling workers: pool and get wage w(s') or deviate to screening by making initial wage offer $\bar{W}(C \cup i) \bar{W}(C)$. Following a deviation screening will continue with all workers in $C \cup i$.
- High type firm. Accept all initial offers of workers in C or deviate and reject all initial offers of workers in C. The firm will be indifferent as long as all workers offer $\bar{W}(C)/|C|$.
- Low type firm. Reject all initial offers in C or deviate and accept all initial offers in C.

Moreover, equilibrium has the following properties.

- Pooling workers are always better off than screening workers.
- The payoffs of both pooling and screening workers are increasing in |C|.
- Screening is fragile; if any screening worker deviates then no screening occurs.

D.2 Key takeaways

The key constraint is that screening workers must prefer screening to offering w(s') and receiving no information. When information sharing increases this constraint eliminates equilibria with smaller screening sets. As long as the incentive constraints of the pooling worker and the low type firm do not bind, this is the comparative static.