Ergodic and statistical properties of smooth systems

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based on joint work with
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Ergodic properties

$T : (X, \kappa) \to (X, \kappa)$ – a (probability) measure preserving automorphism.

Definitions

T is:

1. ergodic – every measurable A for which $T(A) = A$ satisfies $\kappa(A) \in \{0, 1\}$.
2. weakly mixing – if $\frac{1}{N} \sum_{n \leq N} |\kappa(T^n A \cap B) - \kappa(A)\kappa(B)| \to 0$ for every measurable A, B.
3. mixing – $\kappa(T^N A \cap B) \to \kappa(A)\kappa(B)$ for every measurable A, B.
4. has positive entropy– if there is a finite partition with a linear growth of information.
5. K-system – if every (non-trivial) factor of T has positive entropy.
6. Bernoulli – if T is isomorphic to a Bernoulli shift.

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Bernoulli shifts

- $\Sigma = \{1, \ldots, d\}^{\mathbb{Z}}$;
- $\mathbf{p} = (p_1, \ldots, p_d)$, $\sum_{i=1}^{d} p_i = 1$ – probability vector;
- $\sigma : (\Sigma, \mathbf{p}^{\mathbb{Z}}) \to (\Sigma, \mathbf{p}^{\mathbb{Z}})$ – Bernoulli shift,

$$\sigma((x_i)_{i\in\mathbb{Z}}) = (x_{i+1})_{i\in\mathbb{Z}}.$$

Bernoulli systems

T is Bernoulli if T is isomorphic to a Bernoulli shift.

Relations

ergodic $\subsetneq$ *weak mixing* $\subsetneq$ *mixing* $\subsetneq$ *K* $\subsetneq$ *Bernoulli*.

The above inclusion also hold in smooth category.

K non Bernoulli

First Example – Ornstein
(T, T^{-1}) - transformation – Kalikow

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Quantitative properties that use smooth structure

The above properties are qualitative and do not require smooth structure.

Let $f \in C^k(M,\mu)$, be μ preserving, where μ is a smooth measure on M.

Definition: Central Limit Theorem

We say that f satisfies the classical CLT if for every $\phi \in C^k$ with $\mu(\phi) = 0$,

$$\frac{1}{\sqrt{N}} S_N(\phi) := \frac{1}{\sqrt{N}} \sum_{n \leq N} \phi \circ f^n \to \mathcal{N}(0, \sigma_\phi^2),$$

for some $\sigma_\phi^2 \geq 0$. We say that the classical CLT is non-trivial for f if $\sigma_\phi^2 > 0$ for some $\phi \in C^k$.

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Quantitative properties

Trivial CLT

$R_\alpha x = x + \alpha$, for a.e. α,

$$S_N(\phi) = o(N^\epsilon), \text{ for every } \epsilon > 0.$$

Definition: Exponential mixing

$f \in C^k(M, \mu)$ is exponentially mixing if there exists $C, \eta > 0$ and $\ell \in \mathbb{N}$ such that for every $\phi, \psi \in C^\ell$,

$$\left|\mu(\phi \circ f^n \cdot \psi) - \mu(\phi)\mu(\psi)\right| < C\|\phi\|_\ell \|\psi\|_\ell e^{-\eta n}.$$

There are other statistical properties, eg. large deviations...

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Some results

Known results: rigidity

1. K-property implies mixing (of all orders) (Kolmogorov).
2. K- property implies Bernoulli in dimension 2 (Pesin).
3. exponential mixing of all orders implies CLT (Chernov, Bjorklund-Gorodnik).

Known results: flexibility

1. every manifold of dim ≥ 2 supports a Bernoulli diffeomorphism (Katok, Brin-Katok-Rudolph);
2. K not Bernoulli examples (Kalikow, Katok, Rudolph, K-Rodriguez-Hertz-Vinhage), generalized (T, T^{-1})-maps;
3. non-weakly mixing systems satisfying CLT (Kifer-Conze), $Anosov \times R_\alpha$.

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Open questions

exponential mixing obviously implies mixing.
CLT implies ergodicity but not weak mixing.

Questions:

1. Does exponential mixing imply positive entropy, higher order mixing, K,Bernoulli?
2. Does exponential mixing imply CLT?
3. Does CLT imply positive entropy? (J.-P. Thouvenot)

We will discuss questions 1. and 3.

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Main Result: rigidity

Theorem 1 (Dolgopyat, K, Rodriguez-Hertz)

Exponential mixing implies Bernoulli.

Consequences

1. Exponential mixing implies positive entropy (and K).
2. Exponential mixing implies mixing of all orders.
3. From ergodic point of view exponentially mixing systems are classified by entropy.

Example:

The system:

$$\begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} \times \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

on $SL(2,\mathbb{R})^2/\Gamma$ (Γ irreducible) is Bernoulli. This already follows from an earlier result (K.).

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Exponential mixing implies Bernoulli.

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Does (non-trivial) CLT +K imply Bernoulli?(J.-P. Thouvenot)

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Flexibility of CLT: Generalized (T, T^{-1}) transformations

Setting

1. $T : (X, \mu) \to (X, \mu)$ map (or flow),
2. $\alpha : (Y, \nu) \to (Y, \nu)$ an $\mathbb{R}^d$ (or $\mathbb{Z}^d$) action, $d \geq 1$,
3. $\tau : X \to \mathbb{R}^d$ a L^1 cocycle.

$$F(x, y) = (Tx, \alpha_{\tau(x)}(y))$$

acting on $(X \times Y, \mu \times \nu)$.

Classical example: random walk in random scenery

$(X, \mu) = (Y, \nu) = (\{0, 1\}^{\mathbb{Z}}, (1/2, 1/2)^{\mathbb{Z}}),\ T = \alpha = \sigma_2,$
$\tau(x) = (-1)^{x_0}.$
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More results

Smooth category, $d = 1$

Source of K non-Bernoulli examples:

1. Smooth K non-Bernoulli examples (Katok).
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Higher rank actions in the fiber

Higher rank abelian actions in the fiber

$T = \sigma$, α= full $\mathbb{Z}^d$ shift, then (T, T^{-1}) is:

- NOT Bernoulli if $d = 2$ (Hollander,Steif), $d = 1$ (Kalikow);
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For every r there exists a C^r diffeomorphism F_r which satisfies the classical CLT and is of zero entropy.

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1. $Tx = R_\theta x = x + \theta$ on $\mathbb{T}^m$, $\theta \in D(\kappa)$, where

$$D(\kappa) = \{v \in \mathbb{R}^m \ : \ \langle v, k\rangle \geq C_v \|k\|^{-\kappa} \text{ for } k \in \mathbb{Z}^m\}.$$

$Leb(D(\kappa)) = 1$ for $\kappa > m$. Let $\kappa/2 < r < m$ and $d > 20 \cdot \frac{1}{1-\frac{r}{m}}$.

2. α any smooth $\mathbb{R}^d$-action which is exponentially mixing of all orders.

3. $\tau \in C^r(\mathbb{T}^m, \mathbb{R}^d)$ with $Leb(\tau) = 0$ and such that

$$Leb\Big(\{x \in \mathbb{T}^m \ : \ \|S_n(\tau)\| < \log^2 n\}\Big) = o(n^{-5}).$$

Then

$$F(x, y) := (R_\theta x, S_{\tau(x)}(y))$$

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   satisfies the classical CLT.

Theorem 2, continued

Main example of α

α is the Weyl chamber flow on $SL(d, \mathbb{R})$, i.e. α is the group of diagonal matrices (isomorphic to $\mathbb{R}^{d-1}$).

Steps for Thm 2

- Show existence of τ as in 3.
- Show that if T, α, τ are as in 1,2 and 3, then F satisfies CLT.

The second step uses exponential mixing of all orders of α together with property 3. Main technical work is to show existence of τ.

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Pesin theory, explanation of (II)

Let $dim\, M = d$, $df_x : T_x M \to T_{f(x)} M$ denote the differential of f at x and let $df_x^{(n)} := df_{f^{n-1}x} \circ \ldots \circ df_x$.

Lyapunov exponents, Oseledets splitting

There exists numbers $(\lambda_i)_{i=1}^d$ called Lyapunov exponents and (for a.e. x) a decomposition $T_x M = \bigoplus_{i=1}^d E_i(x)$ such that for $v \in E_i(x)$, $\|v\| = 1$, we have

$$\lim_{N\to\infty} \frac{1}{N} \log(\|df_x^{(N)}(v)\|) = \lambda_i.$$

Let $E^u(x) := \bigoplus_{\lambda_i > 0} E_i(x)$ and $E^s(x) := \bigoplus_{\lambda_i < 0} E_i(x)$ be respectively the unstable and stable space. Moreover let $E^c(x) = \bigoplus_{\lambda_i = 0} E_i(x)$ be the center space.

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Entropy formula

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Equidistribution of unstable leaves

For every $\epsilon > 0$ there exists a $\delta > 0$ and a set $P_\epsilon \subset M$, $\mu(P_\epsilon) > 1 - \epsilon$ such that for every $x \in P_\epsilon$ we have the unstable local foliation of size δ, i.e. $W^u(x, \delta) \subset M$ such that $E^u_{f^{-n}x}$ is tangent to $W^u(f^{-n}x, \delta)$ and this local foliation is exponentially contracted when iterating backwards. The same with the stable foliation. We call the normalized unstable measure on $W^u(x, \delta)$ by m^u.

Theorem

Let f be exponentially mixing. There exists $\eta'' > 0$ such that for every n and every ball $B \in M$ of radius $\geq e^{-\eta'' n}$ we have

$$m^u(W^u(x, \delta) \cap f^{-n}(B)) \in (1 - \epsilon, 1 + \epsilon)\mu(B).$$

Ornstein-Weiss reduction

Reduction

Let f be K. Let $W^u(x,\delta)$ and $W^u(y,\delta)$ be nearby unstable leaves of size δ. If for every N there exists an almost measure preserving map $\theta_{x,y,\delta,N} : (W^u(x,\delta), m^u_x) \to (W^u(y,\delta), m^u_y)$ such that

$f^n z$ and $f^n\theta z$ are close for most $0 \le n \le N$. then f is Bernoulli.

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