Open sets of partially hyperbolic systems having a unique SRB measure

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Introduction

Let $f : M \to M$ be a diffeomorphism on a manifold M. For μ an ergodic measure its **basin** is defined by

$$\mathcal{B}(\mu) := \left\{ x \in M : \lim_{n \to +\infty} \sum_{j=0}^{n-1} \delta_{f^j(x)} = \mu \right\}.$$

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Physical measure

We say that μ is **physical** if $\mathrm{Leb}(B(\mu)) > 0$.

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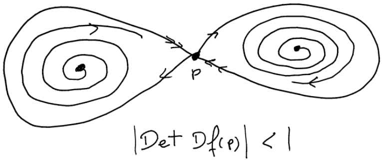

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Remark

The measures they constructed are nowadays called hyperbolic **SRB** (Sinai-Ruelle-Bowen) measures, and they form an important class of physical measures.

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For (f, μ), a real number λ is a **Lyapunov exponent** at the point $p \in M$, if $\exists v \in T_pM - \{0\}$ such that

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Hyperbolic measure

μ is a **hyperbolic measure** if for μ-almost every point, all the Lyapunov exponents are non-zero.

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Hyperbolic SRB measure

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Ledrappier, Ledrappier-Young

$$\Downarrow$$

μ is SRB $\Leftrightarrow$ μ verifies Pesin entropy formula.

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Two ingredients are important in Sinai, Ruelle and Bowen's construction of SRB measures:

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2. "Good angle" between expanding/contracting direction (Dominated splitting).

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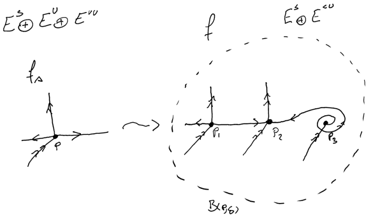
Question

How much can we "break" conditions 1,2 and 3 and still get SRB measures?

Some Derived from Anosov (DA) systems(Bonatti-Viana (2000), Alves-Bonatti-Viana (2000), Tahzibi (2004)):



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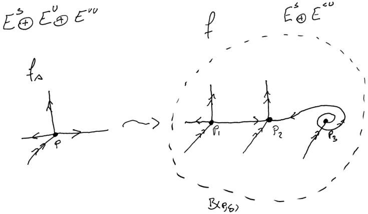

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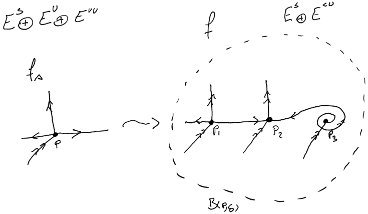

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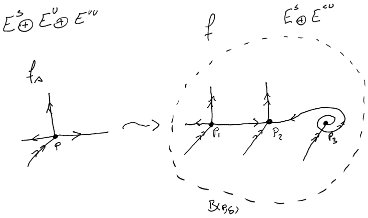

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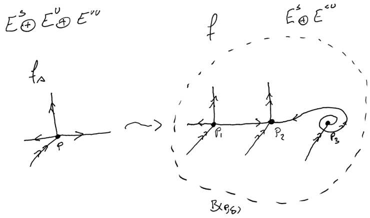

1. **Non-uniform** exp./cont.
2. “Good angles”.
3. It can preserve volume or not.

Conclusion

C^1-Open sets among $C^{1+\alpha}$-diffeomorphisms having a unique (or finitely many) hyperbolic SRB measures.

Remark

Pesin theory $\Rightarrow$ in the volume preserving setting: existence of hyperbolic SRB measure $\Leftrightarrow$ $\exists$ a non-uniformly hyperbolic region.

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Conclusion

C^2-open sets of volume preserving diffeomorphisms having an SRB measure.

Hénon maps (Benedicks-Young (1993)): $\exists \Delta \subset \mathbb{R}^2$ with positive Lebesgue measure such that for any $(a, b) \in \Delta$, the map

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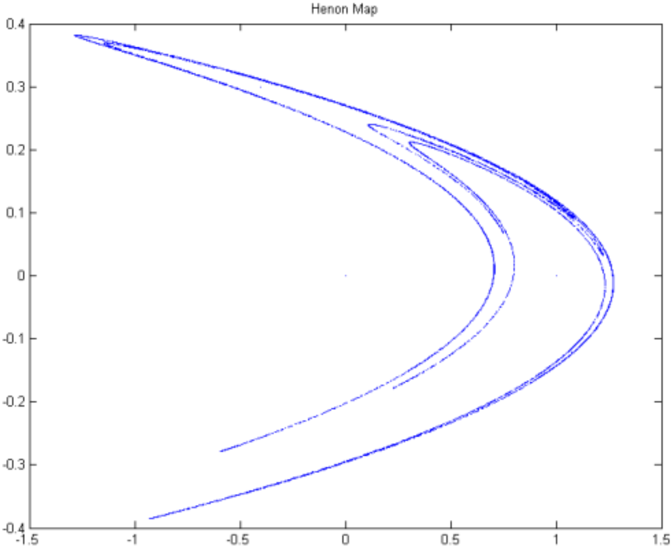
Conclusion

A set of positive Lebesgue measure of parameters Δ having a unique SRB measure. However the set Δ has empty interior.

Introduction



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3. Hénon: Non-uniform exp./cont. + Not "good angles" + Not vol. preserving.
   Conclusion: Not robust existence of hyperbolic SRB measures.

Problem

To obtain open sets (in some space) of systems with

1. non-uniform exp./contr.,
2. not "good angles" between exp./cont. directions,
3. non volume preserving,

and having a unique (or finitely many) hyperbolic SRB measures.

The Berger-Carrasco's example

Let $\mathbb{T}^2 = \mathbb{R}^2/2\pi\mathbb{Z}^2$, with coordinates (x, y). For $N \in \mathbb{N}$ consider the **standard map**

$$\begin{array}{rcl} s_N : \mathbb{T}^2 & \to & \mathbb{T}^2 \\ (x, y) & \mapsto & (N \sin x + 2x - y, x) \end{array}$$

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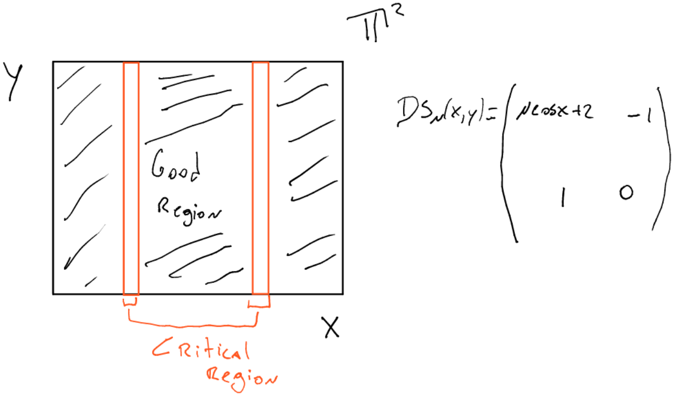

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Conjecture (Sinai, 1994)

If N is large then s_N has positive metric entropy.

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For $N \in \mathbb{N}$ define the skew-product

$$\begin{array}{rcl} f_N : \mathbb{T}^2 \times \mathbb{T}^2 & \to & \mathbb{T}^2 \times \mathbb{T}^2 \\ (x,y,z,w) & \mapsto & (s_N(x,y) + P_1 \circ A^N(z,w), A^{2N}(z,w)). \end{array}$$

The Berger-Carrasco's example

For N large enough, f_N has the following properties:

1. it is partially hyperbolic with splitting

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Theorem (O., 2018)

For N large enough, f_N is C^2-stably ergodic.

Statement of the main theorem

Let $\mathrm{Sk}^r(\mathbb{T}^2 \times \mathbb{T}^2)$ to be the set of diffeomorphism $g \in \mathrm{Diff}^r(\mathbb{T}^4)$ such that

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Main Theorem

Fix $\alpha \in (0, 1)$. For N large enough, $\exists \mathcal{U} \subset \mathrm{Sk}^2(\mathbb{T}^2 \times \mathbb{T}^2)$, C^2-nbd of f_N, and $\mathcal{V}$ an open and dense subset of $\mathcal{U}$ s.t. if $g \in \mathcal{V} \cap \mathrm{Sk}^{2+\alpha}(\mathbb{T}^2 \times \mathbb{T}^2)$, then $\exists!$ SRB measure μ_g for g.

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- μ_g is Bernoulli;
- $Leb(\mathcal{B}(\mu_g)) = 1$;
- $\mathrm{supp}(\mu_g) = \mathbb{T}^4$.

Statement of the main theorem

Problems to prove the main theorem

There are two different problems involved in the main theorem.

Existence and **Uniqueness**

Uniqueness

Theorem (Uniqueness)

For N large enough, $\exists \mathcal{U} \subset \mathrm{Diff}^2(\mathbb{T}^4)$ a C^2-nbd of f_N s.t. if $g \in \mathcal{U}$ then g has at most one SRB measure.

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For N large enough, $\exists \mathcal{U} \subset \mathrm{Diff}^2(\mathbb{T}^4)$ a C^2-nbd of f_N s.t. if $g \in \mathcal{U}$ then g has at most one SRB measure. Furthermore, if μ_g is an SRB measure for g then

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- This is a problem of finding transverse intersections between invariant manifolds.
- The same type of techniques allow me to prove the uniqueness of the measure of maximal entropy for the standard map.

Existence

Theorem (Rigidity)

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1. either g has an SRB measure, or;
2. $\exists$ T^{su} a 2-torus tangent to $E^s \oplus E^u$.

Existence

Theorem (Rigidity)

Fix $\alpha \in (0,1)$. For N large enough, $\exists \mathcal{U} \subset \mathrm{Sk}^2(\mathbb{T}^2 \times \mathbb{T}^2)$, C^2-nbd of f_N, s.t. if $g \in \mathcal{U}$ is $C^{2+\alpha}$ then:

1. either g has an SRB measure, or;
2. $\exists$ T^{su} a 2-torus tangent to $E^s \oplus E^u$.

Remark 1

This is an adaptation to the skew product p.h. setting of a rigidity theorem for stationary measures of random products of surface diffeomorphisms by Aaron Brown and Federico Rodriguez-Hertz.

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Remark 2

In this adaptation, I use many times the fact that the center foliations is smooth.

Questions

- In the Main Theorem, can we extend the result to the entire neighborhood?

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- How "typical" is this example?
- Can we remove the "skew product" hypothesis? (smooth center foliation)
- Is f_N, for N large, accessible?

Thank you!