# Why Are U.S. Parties So Polarized? A "Satisficing" Dynamical Model* 

Vicky Chuqiao Yang ${ }^{\dagger}$ Daniel M. Abrams ${ }^{\ddagger}$<br>Georgia Kernell ${ }^{\S}$<br>Adilson E. Motter ${ }^{\top}$


#### Abstract

Since the 1960s, Democrats and Republicans in the U.S. Congress have taken increasingly polarized positions, while the public's policy positions have remained centrist and moderate. We explain this apparent contradiction by developing a dynamical model that predicts ideological positions of political parties. Our approach tackles the challenge of incorporating bounded rationality into mathematical models and integrates the empirical finding of satisficing decision making-voters settle for candidates who are "good enough" when deciding for whom to vote. We test the model using data from the U.S. Congress over the past 150 years and find that our predictions are consistent with the two major political parties' historical trajectories. In particular, the model explains how polarization between the Democrats and Republicans since the 1960s could be a consequence of increasing ideological homogeneity within the parties.


Key words. dynamical systems, social systems, voting, bounded rationality, satisficing
AMS subject classifications. 00A69, 62P25
DOI. 10.1137/19M1254246
I. Introduction. The U.S. Democratic and Republican parties have polarized drastically since the 1960s [2,22,23]. Legislators' ideological positions are distributed bimodally, with an increasingly vast distance separating the two parties' modes [4,17]. Although what it means to be moderate has changed over the past half century, the general public's dispersion in ideology remains mostly the same: unimodal, centrist, and stationary $[12,16,17]$. Figure 1 illustrates contrasting ideological dispersion between the U.S. public and the U.S. Congress. In a system designed to create democratic responsiveness, why have members of Congress of the two major parties shifted while the general public appears to have remained unchanged?

[^0]Political party polarization is a complex social phenomenon. Recent research demonstrates that many key aspects of social processes can be explained by simple mathematical models. Examples include opinion dynamics [9], terrorist events [10], the spread of rumors [6], and shifts in religious affiliation [1]. A number of studies address aspects of voting and elections, such as consensus formation [27, 28], how opinion clusters form in the compromise process [5], and how democratic voting can lead to totalitarianism [14]. Party polarization has now started to gain attention in the dynamical systems community. A recent study, for example, used a group competition model and found that the two major parties in the U.S. are increasingly benefiting from polarization in Congress [20].

In political science, many theoretical models examine how individuals' voting behavior shapes political parties' positions. The classic Downsian model [11] assumes two-party competition in a one-dimensional ideology space, where citizens cast their vote for the ideologically closest party, and parties adjust their positions to maximize votes. The Downsian model predicts that both parties should converge to the median voter's position. In reality, however, the two major U.S. parties have dramatically polarized over the past half century.


Fig. I Polarization in the U.S. public and Congress (House and Senate combined), as measured by the standard deviation of ideological positions. The ideological positions are drawn from [17] for the public and [7] for Congress. Congress's standard deviation is scaled so that its mean for the first three data points matches that of the public.

One reason we believe the Downsian model fails empirical validation is the assumption that voters maximize their utility. Empirical research suggests that voters often behave in ways that are only boundedly rational [13]. Instead of maximizing, people tend to satisfice; that is, they accept what is "good enough" and do not obsess over other options [3, 24, 25, 26]. The decisions of a maximizer may approach those of a satisficer in the presence of noise, misinformation, missing information, decision fatigue, and other factors that limit one's ability to choose optimally. Moreover, experimental evidence demonstrates that for complex decisions, maximizers often make suboptimal choices [21]. Thus, even if some voters follow a maximizing decisionmaking strategy, in practice they will often fail to find the optimum and instead behave similarly to satisficers.

In this article, we (1) develop a simple mathematical model for political party positions, taking the more realistic satisficing decision making of voters into account,
and (2) explain why political party polarization may develop in the absence of changes in voters' ideological positions. This work helps address the outstanding challenge in political polarization, and in social systems more generally [15], of establishing unified mathematical models that are grounded in empirical findings. In addition to explaining key aspects of political polarization, our model's framework provides a foundation for future studies on other aspects of party dynamics, such as minor party influence and success.
2. Overview. Our modeling framework considers the voter population distributed in a one-dimensional ideology space, with two parties adjusting their positions along this continuum in order to win the most votes. Voters decide which party to support based on satisficing - choosing one party randomly out of those that are satisfactory. The probability that a party is satisfactory to any given voter decays with the distance between that party and the voter, with the speed of decay determined by a parameter describing the party's inclusiveness, or tolerance of ideological diversity. We present a dynamical model for the parties' positions in ideology space. We then validate our model's predictions using empirical data on the distribution of U.S. legislators' ideological positions from 1861 to 2015, from which we estimate the position and inclusiveness of the Democratic and Republican parties. The model employs as an input party inclusiveness, estimated from the data in each Congress and predicts the corresponding position of each party over time. We then compare the predicted positions against historical positions of the two major U.S. parties to establish a relationship between party polarization and party inclusiveness. Our model proposes a possible mechanism for the polarization of political parties without any change in the distribution of the public's ideological positions.
3. Deriving the Mathematical Model. Using congressional roll call voting records, empirical research has found that the U.S. political division can be well represented by a single dimension [23]. ${ }^{1}$ We refer to this dimension as the ideology space, with left being liberal and right being conservative. We also describe how liberal or conservative a party or voter is by their position in the same ideology space, henceforth referred to as ideological position, or simply as position. Survey data reveals that the U.S. public's ideology is distributed in this space in a unimodal manner, peaked at a moderate position and well approximated by a Gaussian (see the supplementary material, section 1). Thus, we consider the voters to be distributed in the ideology space according to a Gaussian function $\rho(x)$. Without loss of generality, we set the mean of the Gaussian to 0 and the standard deviation to $\sigma_{0}$. Each party $i$ adopts a position, denoted as $\mu_{i}$, along this ideology space. See Figure 2 for a sketch of the model setup.

Consider an election among $n$ parties. We assume the probability that a voter positioned at $x$ is satisfied with party $i$ decays with the voter's distance from $\mu_{i}$ and that this decay is symmetrical. We refer to this decaying probability as the satisficing function, $s_{i}\left(d_{i}\right)$, where $d_{i}=\left|x-\mu_{i}\right|$. We also assume $s_{i}(0)=1$ (meaning that a voter perfectly aligned with a party will be satisfied with that party), and $s_{i}\left(d_{i}\right) \rightarrow 0$ as $d_{i} \rightarrow \infty$. One function that satisfies these properties is, again, a Gaussian: $s_{i}\left(d_{i}\right)=\exp \left[-d_{i}^{2} /\left(2 \sigma_{i}^{2}\right)\right]$ (here scaled to have unit peak amplitude). The parameter $\sigma_{i}$ represents how tolerant voters are of parties with ideologies different

[^1]

Fig. 2 Sketch of the model setup showing the interpretation of parameters in the ideology space. The function $\rho$ (solid curve) represents the opinion distribution in the population of voters. The parameter $\mu_{i}$ (triangle) marks the position of one out of possibly $n$ parties competing for voters, and function $s_{i}$ (dashed curve) represents that party's satisficing function.
from their own (Figure 2). It can also be interpreted as the inclusiveness of party $i$; that is, how much a party appeals to voters at distant ideological positions. Parties with lower inclusiveness are more ideologically homogeneous.

We break down the satisficing decision-making process into explicit statements:

1. Voters who are satisfied with none of the $n$ parties abstain from voting.
2. Voters who are satisfied with a subset $M$ of the $n$ parties vote for each party in $M$ with probability $p_{i}$, where $\sum_{M} p_{i}=1$. (Below, we assume for simplicity that $p_{i}$ is the same for all parties in this set, but this assumption is not essential.)
We first examine the case in which two parties, party 1 and party 2 , compete. Since the governing equations for the two parties are similar, in the following equations we use index $i$ to denote either party 1 or 2 , and $j$ the other party. The expected share of potential voters at position $x$ who vote for party $i$ can be expressed as

$$
\begin{equation*}
p_{i}\left(x \mid \mu_{i}, \mu_{j}\right)=\underbrace{s_{i}\left(d_{i}\right)\left(1-s_{j}\left(d_{j}\right)\right)}_{\mathrm{A}}+\underbrace{\frac{1}{2} s_{i}\left(d_{i}\right) s_{j}\left(d_{j}\right)}_{\mathrm{B}}, \tag{3.1}
\end{equation*}
$$

where term A represents the expected share of voters at position $x$ who are satisfied with party $i$ only, and term B represents half of those who are satisfied with both parties. The expected total number of voters for party $i$, which we denote $V_{i}$, is obtained by integrating $p_{i}\left(x \mid \mu_{i}, \mu_{j}\right)$ over the ideology space weighted by the population density $\rho$ :

$$
\begin{equation*}
V_{i}\left(\mu_{i}, \mu_{j}\right)=\int_{-\infty}^{\infty} \rho(x) p_{i}\left(x \mid \mu_{i}, \mu_{j}\right) d x \tag{3.2}
\end{equation*}
$$

To model dynamical changes in positions over time, we assume parties move in the direction that increases the number of votes received at a speed proportional to the potential gain:

$$
\begin{equation*}
\frac{d \mu_{i}}{d t}=k \frac{\partial V_{i}}{\partial \mu_{i}} \tag{3.3}
\end{equation*}
$$

where $k$ is a positive constant determined from data that sets the time scale. Equation (3.3) generates predictions for party $i$ 's position over time, where the left side represents the speed of position change and the right side represents the change in


Fig. 3 (A) A vector field that demonstrates the movement of the two parties at various ideological positions. Arrows show the direction of the party positions' movement, with the length of each arrow proportional to the speed of movement. The parameters used in this figure are the inclusiveness parameters $\sigma_{1}=\sigma_{2}=0.5$ and time scale constant $k=1$. (B) Fixed points for party positions ( $\mu$ ) as a function of the inclusiveness parameter of the two major parties, assuming symmetrical inclusiveness $\sigma_{1}=\sigma_{2}=\sigma$. The stable fixed points are shown as solid curves, and the unstable fixed point is indicated by the dashed line. The shaded area indicates the inclusiveness parameter region corresponding to that estimated from the $D W$-NOMINATE data discussed in section 4.3 using methods discussed in section 4.5. (C) Polarization, as measured by the distance between parties at steady state, as a function of both $\sigma_{1}$ and $\sigma_{2}$. In all three panels, the parameter $\sigma_{0}$ is set to 0.93 , which is the same value used in Figure 4.
the party's votes. While we derived this equation for the two-party case, it admits an immediate extension to the $n$-party case with the exact same form, except that the vote, $V_{i}$, is then defined relative to all $n$ parties. In the supplementary material, section 4, we explore an alternative model where parties maximize vote share. This model produces similar outcomes (although it requires additional complexity).

## 4. Results and Comparison with Empirical Data.

4.I. Fixed Points of the System. We solve for the fixed points of the system both analytically and numerically, with analytical results presented in section 4.2 and numerical results presented in Figures 3 and 4. The results reveal that for sufficiently large inclusiveness parameters, $\sigma_{1,2}$, the two parties are attracted to a single stable equilibrium: $\mu_{1}=\mu_{2}=0$, at the center of the ideology space. However, for smaller $\sigma_{1,2}$, the two parties stabilize at a finite separation. Figure 3A visualizes the dynamics of a two-party system by plotting the derivatives $d \mu_{1} / d t$ and $d \mu_{2} / d t$ in (3.3) as a vector field. In this example, the two parties stabilize at a finite separation. The fixed points and their stability are shown in Figure 3B for the symmetric case, $\sigma_{1}=\sigma_{2}$. We note that Figure 3B plots the positions of both parties on the same vertical axis. In the asymmetric case where $\sigma_{1} \neq \sigma_{2}$, the polarization, as measured by the distance between the two parties at steady state, is a function of both $\sigma_{1}$ and $\sigma_{2}$. The numerical results for this case are shown in Figure 3C. The system equilibrates at a finite separation in most of the parameter space.

An intuitive understanding of this result is that when the inclusiveness parameters are large, few voters abstain. Thus, parties are attracted to the center of the ideology space through the same mechanism as in the classic Downsian model that predicts the convergence of both parties to the median voter [11]. However, as the inclusiveness parameters decrease, convergence to the center increases the number of voters at the tails of the ideology distribution abstaining from voting. In that case, the parties benefit from moving away from the center, with an equilibrium distance determined
by the parameters. Although the model was presented in a one-dimensional ideology space, the model's behavior is similar in two dimensions (see the supplementary material, section 1).
4.2. Analytical Results for Two Identical Parties. The right-hand side of the system specified by (3.3) can be computed analytically for the symmetrical case $\sigma_{1}=$ $\sigma_{2}=\sigma$ :

$$
\begin{equation*}
\frac{d \mu_{i}}{d t}=\frac{\sigma_{0}\left[\mu_{i}\left(\sigma^{2}+\sigma_{0}^{2}\right)-\mu_{j} \sigma_{0}^{2}\right]}{2 \sigma\left(\sigma^{2}+2 \sigma_{0}^{2}\right)^{3 / 2}} e^{-\frac{\left(\mu_{i}^{2}+\mu_{j}^{2}\right) \sigma^{2}+\left(\mu_{i}-\mu_{j}\right)^{2} \sigma_{0}^{2}}{2 \sigma^{2}\left(\sigma^{2}+2 \sigma_{0}^{2}\right)}}-\frac{\mu_{i} \sigma_{0} \sigma e^{-\frac{\mu_{i}^{2}}{2\left(\sigma^{2}+\sigma_{0}^{2}\right)}}}{\left(\sigma^{2}+\sigma_{0}^{2}\right)^{3 / 2}} . \tag{4.1}
\end{equation*}
$$

Solving for the equilibria $d \mu_{i} / d t=0$ and imposing symmetry in party positions $\mu_{2}=$ $-\mu_{1}=\mu$ (this follows from the symmetry in $\sigma_{1}=\sigma_{2}$ ), we find

$$
\begin{equation*}
\mu\left[-2 \sigma^{2}\left(\sigma^{2}+2 \sigma_{0}^{2}\right)^{1 / 2} e^{-\frac{\mu^{2}}{2\left(\sigma^{2}+\sigma_{0}^{2}\right)}}+\left(\sigma^{2}+\sigma_{0}^{2}\right)^{3 / 2} e^{-\frac{\mu^{2}}{\sigma^{2}}}\right]=0 \tag{4.2}
\end{equation*}
$$

Clearly, $\mu=0$ is a solution, and thus $\mu^{\star}=0$ is a fixed point of the system for all values of $\sigma$ and $\sigma_{0}$. Other possible fixed points are given by the solution of the equation $\left(\sigma^{2}+\sigma_{0}^{2}\right)^{3 / 2} e^{-\frac{\mu^{2}}{\sigma^{2}}}-2 \sigma^{2}\left(\sigma^{2}+2 \sigma_{0}^{2}\right)^{1 / 2} e^{-\frac{\mu^{2}}{2\left(\sigma^{2}+\sigma_{0}^{2}\right)}}=0$, which can be solved for $\mu^{2}$ explicitly as

$$
\begin{equation*}
\left(\mu^{\star}\right)^{2}=\sigma^{2}\left(\frac{\sigma^{2}+\sigma_{0}^{2}}{\sigma^{2}+2 \sigma_{0}^{2}}\right) \ln \left[\frac{\left(\sigma^{2}+\sigma_{0}^{2}\right)^{3}}{4 \sigma^{4}\left(\sigma^{2}+2 \sigma_{0}^{2}\right)}\right] \tag{4.3}
\end{equation*}
$$

the solid line in Figure 3B is this curve. Note that this expression can be rewritten in terms of nondimensional parameters $\left\{\hat{\sigma} \equiv \sigma / \sigma_{0}, \hat{\mu} \equiv \mu / \sigma_{0}\right\}$ simply by setting $\sigma \mapsto \hat{\sigma}$ and $\sigma_{0} \mapsto 1$.

The trivial solution $\mu^{\star}=0$ is the only fixed point for $\sigma>\sigma_{c}$ for some $\sigma_{c}$, and it is of interest to understand how $\sigma_{c}$ depends on system parameters. ${ }^{2}$ This critical value can be understood by noting that the nontrivial fixed points for $\mu$ given in (4.3) cease to exist when the logarithm becomes negative, i.e., when the argument of the logarithm becomes less than $1 ; \sigma_{c}$ is the critical value at which the argument is exactly 1. Expressing this condition in nondimensionalized form gives

$$
\begin{equation*}
\frac{\left(\hat{\sigma}_{c}^{2}+1\right)^{3}}{4 \hat{\sigma}_{c}^{4}\left(\hat{\sigma}_{c}^{2}+2\right)}=1 \tag{4.4}
\end{equation*}
$$

and rearranging the terms leads to a cubic polynomial in $\hat{\sigma}_{c}^{2}$ :

$$
\begin{equation*}
3 \hat{\sigma}_{c}^{6}+5 \hat{\sigma}_{c}^{4}-3 \hat{\sigma}_{c}^{2}-1=0 \tag{4.5}
\end{equation*}
$$

This equation has three real roots for $\hat{\sigma}_{c}^{2}$, at $\hat{\sigma}_{c}^{2} \approx\{-2.07,-0.247,0.652\}$, and only the last root allows for a real-valued solution $\hat{\sigma}_{c} \approx 0.807$. The system undergoes a subcritical pitchfork bifurcation at this point.

[^2]The exact shape of the fixed point curve is defined by (4.3), which cannot be solved explicitly for $\sigma$ (or $\hat{\sigma}$ in nondimensional form). A convenient approximation can be written by expanding the right-hand side to the leading order in $\hat{\sigma}$, yielding $\hat{\mu}^{2} \approx-\hat{\sigma}^{2} \ln \left(\sqrt{8} \hat{\sigma}^{2}\right)$. Define $W \equiv-\hat{\mu}^{2} / \hat{\sigma}^{2}$, and rearrange the previous equation to obtain $-\sqrt{8} \hat{\mu}^{2} \approx W \exp (W)$. This equation is solved by $W=W\left(-\sqrt{8} \hat{\mu}^{2}\right)$, where $W$ is now identified as the special function known as the Lambert $W$ function. Thus, $\hat{\sigma}^{2} \approx-\hat{\mu}^{2} / W\left(-\sqrt{8} \hat{\mu}^{2}\right)$.

The Lambert $W$ function is real valued for negative arguments only when the argument has magnitude less than $1 / e$. This sets a bound on the maximum $\hat{\mu}^{2}$ for which a solution exists, since $\sqrt{8} \hat{\mu}_{\text {max }}^{2} \approx 1 / e$ implies $\hat{\mu}_{\max } \approx 2^{-3 / 4} e^{-1 / 2} \approx 0.361$. The corresponding value of $\hat{\sigma}$ is then immediate since $W(-1 / e)=-1$, yielding $\hat{\sigma}_{\max } \approx$ $\hat{\mu}_{\max } \approx 2^{-3 / 4} e^{-1 / 2}$. These approximations are useful in understanding the general shape of the bifurcation curve, though the critical points of the exact expression in (4.3) can be easily computed numerically.
4.3. Empirical Data. To examine changes in real-world party positions over time, we employ ideological positions for Democratic and Republican legislators calculated from their congressional roll call voting records using the Dynamic, Weighted, Nominal Three-Step Estimation (DW-NOMINATE) method [7]. DW-NOMINATE is a multidimensional scaling method that first calculates a pairwise distance for every two members of the U.S. House of Representatives based on similarities in their roll call voting records. It then projects the resulting high-dimensional network of representatives to a low dimension while preserving the pairwise distance relation as much as possible. The representatives' relative positions in this low-dimensional space are referred to as their ideology scores. ${ }^{3}$ The same method can be used for scaling the positions of U.S. Senators, and in what follows we use a combined dataset of both representatives and senators (see the supplementary material, section 2 , for the data source).

As an example, Figure 4A displays a histogram of Democratic and Republican legislators' ideological positions during the 2013-2015 term, the most recent in the data.
4.4. Model Validation. For the purpose of comparing our model with data, we take the mean of each party's distribution to represent its position, $\mu_{i}$, in the model and the standard deviation of each party's distribution to represent its inclusiveness parameter, $\sigma_{i}$. As members of Congress move farther from the party mean, we infer that the party is more inclusive - and may thus appeal more to voters farther away from its center. Repeating that procedure for all available Congresses leads to a series of party positions over time since 1861, shown as the solid curves in Figure 4B. We note that two distinct polarization metrics - the distance between party means (used in Figure 4) and the standard deviation of the legislators' ideology distribution (used in Figure 1) - consistently demonstrate the striking increase in party polarization since the 1960s.

Importantly, Figure 4B also compares the model's predictions for the two major parties' positions to historical data. We set the two parties' positions in 1861 as their initial conditions and numerically simulate the dynamical system (3.3) to predict their positions in subsequent years. In the simulations, we update the inclusiveness parameter according to the data every Congress (2 years), and the model outputs the

[^3]

Fig. 4 Predictions and validations of the satisficing model. (A) Histogram of ideological positions for the Democratic and Republican parties in the 2013-2015 Congress. (B) Comparison of the model prediction with time series data for party positions. The solid curves indicate the empirical party positions, estimated by the mean position of the parties' members of Congress, while the shaded areas mark the associated standard deviations. The dashed curves show our model predictions. (C) Comparison of party polarization predicted by our model and polarization observed in the data. The Pearson correlation between the prediction and the data is 0.75 , with $p$-value $6.1 \times 10^{-15}$. The dashed line corresponds to correlation 1 and is a reference. (D) Party polarization as a function of $\sigma$ (average of the Democratic and Republican parties' inclusiveness parameters). Each marker represents one Congress term. In both the model's prediction and the data, party inclusiveness is negatively associated with polarization. In panels $(\mathrm{C})$ and (D), party polarization is measured as the distance between party positions.
positions of both parties according to (3.3). Parameters $\sigma_{0}, k$, and a proportionality constant associating the standard deviation in the DW-NOMINATE score with the inclusiveness parameter are determined from the fit to the data. An animated visualization of Figures 4A and 4B is included as a supplementary video.

Our theory shows good agreement with the data (Pearson correlation 0.75; see Figures 4C and 4D). Notable deviations are observed around the times of the First and Second World Wars and following the recent rightward move by the Republican Party, as shown in Figure 4B.

A central prediction of our model is that lower inclusiveness (more homogeneity within parties) will lead to higher party polarization. We find support for this prediction in empirical data, as shown in Figure 4D. This finding suggests that it is not
necessary for the voting population to become polarized in order for increased political party polarization to occur. Indeed, the electorate need not change at all; it is held constant in our simulations. We also perform robustness checks using an alternative measure of party polarization as well as an independent data source for the inclusiveness parameter and reach the same qualitative conclusions (see the supplementary material, section 3).
4.5. Parameter Fitting. When determining the parameters from the data, we relate the units of the DW-NOMINATE ideology scores to units for variables in our model by assuming a linear scaling. For example, the party inclusiveness parameter, $\sigma_{i}$, is assumed to be linearly related to the DW-NOMINATE data through the scaling $\sigma_{i}=b \sigma_{\text {data }, i}$. We fitted the parameters in the model by minimizing the 1-norm for differences between the time series predicted by the model and the time series from data. Three parameters are fitted: (1) $\sigma_{0}$, the standard deviation of the population ideology distribution; (2) $b$, the proportionality constant relating the standard deviation in the DW-NOMINATE data with the party inclusiveness parameter in the model, $\sigma_{i}$; and (3) $k$, the time scale constant. We set the initial conditions for the two parties as given by the data. The best fitting parameters are $\sigma_{0}=0.93, b=3.73$, and $k=2.54$.

## 5. Discussion.

5.I. Comparison between Satisficing and Maximizing Models. The satisficing assumption is essential to our model's behavior. Here, we compare the results of our satisficing model with a maximizing model in the same framework (assuming two parties) and show that the predictions from the two models are fundamentally different. We consider the maximizing voters to be defined in the Downsian sense [11]-they maximize their utility function by voting for the ideologically closest party. With maximizing voters, the number of votes each party receives is

$$
\begin{equation*}
V_{l}^{(m)}=\int_{-\infty}^{\left(\mu_{l}+\mu_{r}\right) / 2} \rho(x) d x \quad \text { and } \quad V_{r}^{(m)}=\int_{\left(\mu_{l}+\mu_{r}\right) / 2}^{\infty} \rho(x) d x \tag{5.1}
\end{equation*}
$$

where $(l, r)=(1,2)$ if $\mu_{1}<\mu_{2}$ and $(l, r)=(2,1)$ if $\mu_{1}>\mu_{2}$. If $\mu_{1}=\mu_{2}$, both parties receive the same number of votes,

$$
\begin{equation*}
V_{1}^{(m)}=V_{2}^{(m)}=\frac{1}{2} \int_{-\infty}^{\infty} \rho(x) d x \tag{5.2}
\end{equation*}
$$

We then solve for the fixed points of the system described by (3.3) with the two types of vote calculations and find their stabilities.

Figure 5 shows the stable fixed points for party positions as a function of $\sigma_{1}=$ $\sigma_{2}=\sigma$, for normalized variables $\hat{\sigma}=\sigma / \sigma_{0}$ and $\hat{\mu}=\mu / \sigma_{0}$. With satisficing voters, the two parties stabilize at a finite separation for a range of parameters (as shown above). However, with maximizing voters, the position $(0,0)$ is the only stable fixed point, recovering Downs' result [11], a classic finding in political science. ${ }^{4}$ This difference indicates that the voters' satisficing decision making is indeed a key ingredient in producing the model's results.

[^4]

Fig. 5 Stable fixed points for party positions ( $\hat{\mu}$ ) as a function of the inclusiveness parameter of the two parties ( $\hat{\sigma}$ ) for (A) the satisficing model and (B) the maximizing model. For each model, the positions of both parties are shown on the same vertical axis.
5.2. Implications. Our model offers new contributions to the literature. First, we present empirical evidence of and an explanation for the relationship between party homogeneity and polarization, showing that simply changing the shape of parties' satisficing functions (via changes to inclusiveness) is sufficient to lead to divergence in their positions. In particular, even though the impact of intraparty heterogeneity on a party's competitiveness has been discussed before [18], the mechanism linking increasing ideological homogeneity with diverging party positions has remained unclear. Second, the model also shows why appealing to an extreme segment of the electorate can be a winning political strategy in times of greater intraparty ideological homogeneity (i.e., decreasing party inclusiveness), which may be especially relevant for interpreting current trends in U.S. politics. Ideological homogeneity itself may be driven by other factors, such as partisan redistricting [8] or media echo chambers [19], and explicitly modeling how these factors influence intra-party cohesion remains an open area of research. Third, our approach offers a new quantitative framework that incorporates satisficing behavior into a voting model.

In addressing the call for combining interdisciplinary methods to study human social behavior [15], this article offers insight into the complex process of political elections and democratic responsiveness through a parsimonious model and suggests several directions for future work. For example, do all individuals engage in satisficing, and if so, do they do so in the same way? When and how can minor parties gain traction? And how might outcomes change if parties could control both their position and their level of inclusiveness? For simplicity, we left out a number of electoral variables, such as party primaries, campaign financing, and Southern realignment, but it will be important for future research to understand how these factors interact with vote satisficing. We hope our work will spur further development of quantitative frameworks to incorporate human bounded rationality into mathematical models.

Acknowledgments. We thank Jeffrey Lewis and Keith Poole for in-depth discussions about DW-NOMINATE. We also gratefully acknowledge babies Miriam, Desiree, and Yara for their contributions.

Data and Code Availability. A copy of the DW-NOMINATE dataset used is included as a supplementary file. The computer code for numerically simulating the
model is available in the following GitHub repository: https://github.com/vc-yang/ satisficing_election_model.

Author Contributions. All authors contributed to the design of the research. V.C.Y. developed the model and performed computer simulations. All authors analyzed the results and contributed to the writing of the manuscript, which was led by V.C.Y. and D.M.A.

## REFERENCES

[1] D. M. Abrams, H. A. Yaple, and R. J. Wiener, Dynamics of social group competition: Modeling the decline of religious affiliation, Phys. Rev. Lett., 107 (2011), art. 088701. (Cited on p. 647)
[2] C. Andris, D. Lee, M. J. Hamilton, M. Martino, C. E. Gunning, and J. A. Selden, The rise of partisanship and super-cooperators in the US House of Representatives, PLoS One, 10 (2015), art. e0123507. (Cited on p. 646)
[3] R. J. Aumann, Rationality and bounded rationality, Game. Econ. Behav., 21 (1997), pp. 2-14. (Cited on p. 647)
[4] D. Baldassarri and A. Gelman, Partisans without constraint: Political polarization and trends in American public opinion, Amer. J. Sociol., 114 (2008), pp. 408-446. (Cited on p. 646)
[5] E. Ben-Naim, P. L. Krapivsky, and S. Redner, Bifurcations and patterns in compromise processes, Phys. D, 183 (2003), pp. 190-204. (Cited on p. 647)
[6] J. Borge-Holthoefer and Y. Moreno, Absence of influential spreaders in rumor dynamics, Phys. Rev. E, 85 (2012), art. 026116. (Cited on p. 647)
[7] R. Carroll, J. Lewis, J. Lo, N. McCarty, K. Poole, and H. Rosenthal, "Common space" DW-NOMINATE scores with bootstrapped standard errors (joint House and Senate scaling), http://voteview.com/dwnomin_joint_house_and_senate.htm, 2015. (Cited on pp. 647, 648, 652)
[8] J. L. Carson, G. Koger, M. J. Lebo, and E. Young, The electoral costs of party loyalty in Congress, Amer. J. Polit. Sci., 54 (2010), pp. 598-616. (Cited on p. 655)
[9] C. Castellano, S. Fortunato, and V. Loreto, Statistical physics of social dynamics, Rev. Modern Phys., 81 (2009), p. 591. (Cited on p. 647)
[10] A. Clauset, M. Young, and K. S. Gleditsch, On the frequency of severe terrorist events, J. Conflict Resolut., 51 (2007), pp. 58-87. (Cited on p. 647)
[11] A. Downs, An economic theory of political action in a democracy, J. Polit. Econ., 65 (1957), pp. 135-150. (Cited on pp. 647, 650, 654)
[12] M. P. Fiorina and S. J. Abrams, Political polarization in the American public, Annu. Rev. Polit. Sci., 11 (2008), pp. 563-588. (Cited on p. 646)
[13] K. E. Fisher, S. Erdelez, and L. McKechnie, Theories of Information Behavior, Information Today, Inc., 2005. (Cited on p. 647)
[14] S. Galam, Application of statistical physics to politics, Phys. A, 274 (1999), pp. 132-139. (Cited on p. 647)
[15] H. Gintis, Towards the unity of the human behavioral sciences, Polit. Philos. Econ., 3 (2004), pp. 37-57. (Cited on pp. 648, 655)
[16] M. J. Hetherington, Review article: Putting polarization in perspective, Brit. J. Polit. Sci., 39 (2009), pp. 413-448. (Cited on p. 646)
[17] S. J. Hill and C. Tausanovitch, A disconnect in representation? Comparison of trends in congressional and public polarization, J. Polit., 77 (2015), pp. 1058-1075. (Cited on pp. 646,647 )
[18] G. Kernell, Strategic party heterogeneity, J. Theor. Polit., 28 (2015), pp. 408-430. (Cited on p. 655)
[19] M. Levendusky, The Partisan Sort: How Liberals Became Democrats and Conservatives Became Republicans, The University of Chicago Press, 2009. (Cited on p. 655)
[20] X. Lu, J. Gao, and B. K. Szymanski, The evolution of polarization in the legislative branch of government, J. Roy. Soc. Interface, 16 (2019), art. 20190010.. (Cited on p. 647)
[21] W. Mao, When one desires too much of a good thing: The compromise effect under maximizing tendencies, J. Consum. Psychol., 26 (2016), pp. 66-80. (Cited on p. 647)
[22] N. McCarty, K. T. Poole, and H. Rosenthal, Polarized America: The Dance of Ideology and Unequal Riches, MIT Press, 2016. (Cited on p. 646)
[23] K. T. Poole and H. Rosenthal, The polarization of American politics, J. Polit., 46 (1984), pp. 1061-1079. (Cited on pp. 646, 648)
[24] B. Schwartz, A. Ward, J. Monterosso, S. Lyubomirsky, K. White, and D. R. Lehman, Maximizing versus satisficing: Happiness is a matter of choice, J. Pers. Soc. Psychol., 83 (2002), p. 1178. (Cited on p. 647)
[25] H. A. Simon, Theories of bounded rationality, Decision and Organization, 1 (1972), pp. 161176. (Cited on p. 647)
[26] H. A. Simon, Human nature in politics: The dialogue of psychology with political science, Amer. Polit. Sci. Rev., 79 (1985), pp. 293-304. (Cited on p. 647)
[27] V. Sood and S. Redner, Voter model on heterogeneous graphs, Phys. Rev. Lett., 94 (2005), art. 178701. (Cited on p. 647)
[28] J. Xie, S. Sreenivasan, G. Korniss, W. Zhang, C. Lim, and B. K. Szymanski, Social consensus through the influence of committed minorities, Phys. Rev. E, 84 (2011), art. 011130. (Cited on p. 647)


[^0]:    *Received by the editors April 5, 2019; accepted for publication (in revised form) March 10, 2020; published electronically August 6, 2020.
    https://doi.org/10.1137/19M1254246
    Funding: This work was supported by the James S. McDonnell Foundation through grant 220020230, the Northwestern University's Data Science Initiative, Suzanne Hurst and Samuel Peters, and the Santa Fe Institute Omidyar Fellowship.
    ${ }^{\dagger}$ Santa Fe Institute, Santa Fe, NM 87501 (vcy@santafe.edu).
    $\ddagger$ Department of Engineering Sciences and Applied Mathematics, and Northwestern Institute on Complex Systems, Northwestern University, Evanston, IL 60208 (dmabrams@northwestern.edu).
    ${ }^{\S}$ Departments of Communication and Political Science, University of California, Los Angeles, Los Angeles, CA 90095 (gkernell@ucla.edu).
    ${ }^{\top}$ Department of Physics and Astronomy, and Northwestern Institute on Complex Systems, Northwestern University, Evanston, IL 60208 (motter@northwestern.edu).

[^1]:    ${ }^{1}$ While a single dimension historically performs well at explaining the vast majority of legislative voting behavior, there are several notable time periods - such as during the civil rights movementduring which a second dimension is salient [7].

[^2]:    ${ }^{2}$ The result that the trivial solution $\mu^{\star}=0$ is the only fixed point for $\sigma>\sigma_{c}$ holds even for two parties with $\mu_{1} \neq \mu_{2}$, as can be seen with a series expansion for large $\sigma$ retaining only leading order terms.

[^3]:    ${ }^{3}$ Note that this measure is relative in nature-it reflects the similarities and differences among members' voting records across a large number of bills, not their positions on any specific issue.

[^4]:    ${ }^{4}$ This result can be immediately seen for the symmetric case $\mu_{1}=-\mu_{2}$. In that case, each party is drawn toward the center to increase votes, despite having split the number of votes equally with the opposing party.

