Supplementary Online Material for "The tipping point: a mathematical model for the profit-driven abandonment of restaurant tipping"

Sara M. Clifton[a)]

Department of Mathematics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

Eileen Herbers and Jack Chen

Department of Engineering Sciences and Applied Mathematics, Northwestern University, Evanston, IL 60208, USA

Daniel M. Abrams

Department of Engineering Sciences and Applied Mathematics, Northwestern University, Evanston, IL 60208, USA

Northwestern Institute for Complex Systems, Northwestern University, Evanston, IL 60208, USA and

Department of Physics and Astronomy, Northwestern University, Evanston, IL 60208, USA

[a)]E-mail me at: smc567@illinois.edu

S1. OPTIMAL WAITER BASE PAY

The profit optimization algorithm selects the most profitable staff pay rate for every tip rate scenario, so restaurant owners that abandon tipping must pay waiters a higher wage to compensate for the lost tips. Fig S1 shows that the relative optimal base pay when we forbid tipping must steadily increase as the tip rate increases.

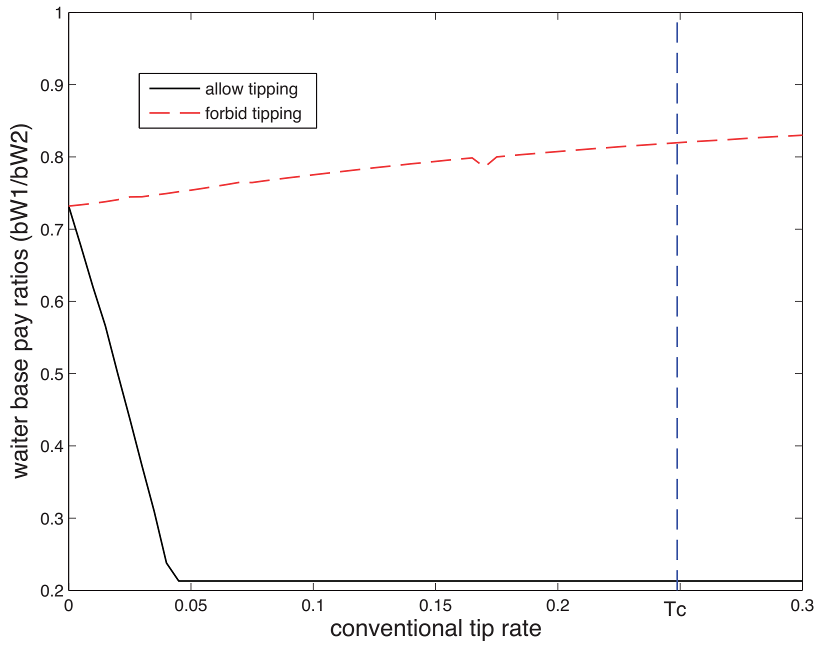

FIG. S1. Illustration of optimal waiter base pay under different tipping protocols. When our restaurant chooses to forbid tipping, the optimal waiter base pay rate increases steadily with the conventional tip rate. In other words, restaurant owners must compensate for lost tips. For this example, $m_1 = m_2 = 10, r = 4, b_{W2} = 10, b_{C2} = 25, r_{DW} = 10, r_{CW} = 1$, the minimum wage for tipped workers is 2.13, and the minimum wage for untipped workers is 7.25. Note that the base pay ratio is low when tipping is allowed because waiters at the other restaurant are paid \$10 an hour no matter what, and the optimization algorithm tells us to pay waiters minimum wage (\$2.13).

As the tip rate increases, waiters receive relatively more compensation from tips than from base pay (see Fig S1). This is expected, though it is difficult to know if our ratios are realistic because tips, often paid in cash, may be under-reported.

S2. ALTERNATIVE INTERPRETATIONS OF RESTAURANT QUALITY

Though we have framed the model as counting the literal number of waiters and cooks in the restaurant, the model could be reinterpreted as measuring the total waiter and cook quality. Because the number of cooks and waiters in our restaurant is actually the number

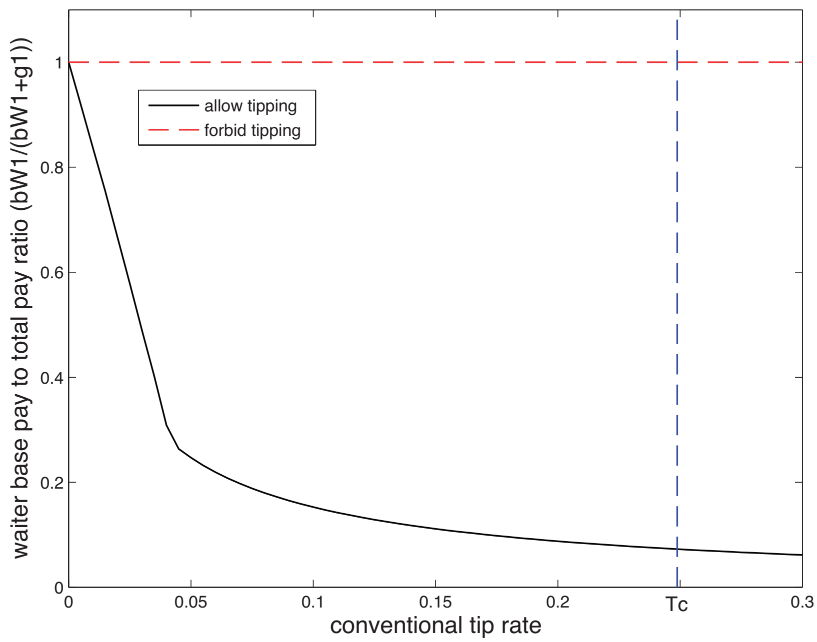

FIG. S2. Illustration of base pay as a fraction of total pay under different tipping protocols. When our restaurant allows tipping, less and less waiter pay comes from wages as the conventional tip rate increases. For this example, $m_1 = m_2 = 10, r = 4, b_{W2} = 10, b_{C2} = 25, r_{DW} = 10, r_{CW} = 1$, the minimum wage for tipped workers is 2.13, and the minimum wage for untipped workers is 7.25.

who wish to work in our restaurant, one could argue that W and C are indicators of the quality of staff we'll have when we hire the appropriate number of waiters and cooks.

Without any adjustment to the model, C could be interpreted as the fraction of cooks who wish to work in our restaurant, and $b_{C1}C$ (total pay for all cooks) could be the quality of the cooks actually hired. We simply assume that we hire the best cooks from the pool of applicants and pay them appropriately. For instance, if ten cooks want to work at our restaurant, then we could hire all ten at \$10 an hour. Equivalently, we could hire only the most qualified five cooks and pay them \$20 an hour.

We can apply the same logic to waiters, but the model will need to be adjusted slightly. As is, gratuities are split among all waiters, and a new model would need to account for the actual number of waiters working in the restaurant. For instance, if we hire half as many waiters that are twice as talented and pay them twice the salary, our profit function would not change. However, each waiter would receive double the tips because the gratuities would be split among half as many waiters.

S3. ALTERNATIVE FORMULATIONS FOR RESTAURANT QUALITY

The most difficult modeling task for this restaurant ecosystem is formulating a simple, but realistic, restaurant quality measure. The original model uses the weighted sum of waiters and cooks at our restaurant as a proxy for restaurant quality. Alternatively, a weighted sum of waiter and cook pay could be a proxy for restaurant quality:

$$q_1 = \alpha_W(b_{W1} + g_1) + \alpha_C b_{C1} \tag{1}$$

With this new definition of restaurant quality, the critical tip threshold remains (see Fig S3).

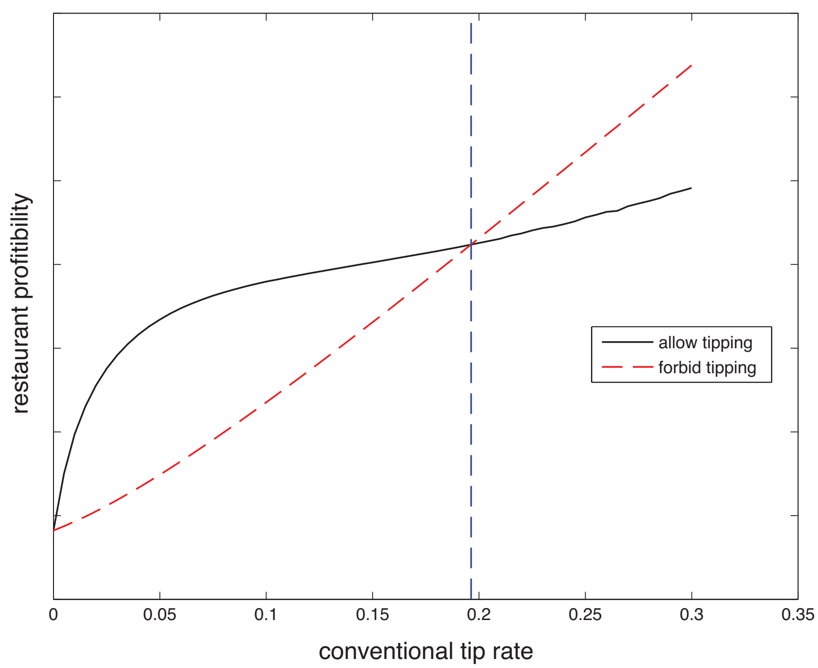

FIG. S3. Example of critical tip rate threshold with pay-dependent quality measure (1). For conventional tip rates below some critical threshold (blue dashed line), a rational restaurant owner would allow diners to leave gratuity to maximize profitability (black curve). Beyond that critical threshold, a rational restaurant owner would disallow tipping in their restaurant (red dashed curve). Both curves assume that the restaurant owner selects staff pay (within legal limits) to maximize profit. For this example, $m_1 = m_2 = 10, r = 2, b_{W2} = 10, b_{C2} = 25, r_{DW} = 10, r_{CW} = 1$, the minimum wage for tipped workers is 2.13, and the minimum wage for untipped workers is 7.25.

Another alternative proxy for restaurant quality could depend on the product of staff pay and number of staff:

$$q_1 = \alpha_W \tilde{W}(b_{W1} + g_1) + \alpha_C \tilde{C} b_{C1} \tag{2}$$

With this new definition of restaurant quality, the critical tip threshold again remains (see Fig S4). This suggests that the qualitative prediction that a critical tip rate exists is robust to the exact model formulation for restaurant quality.

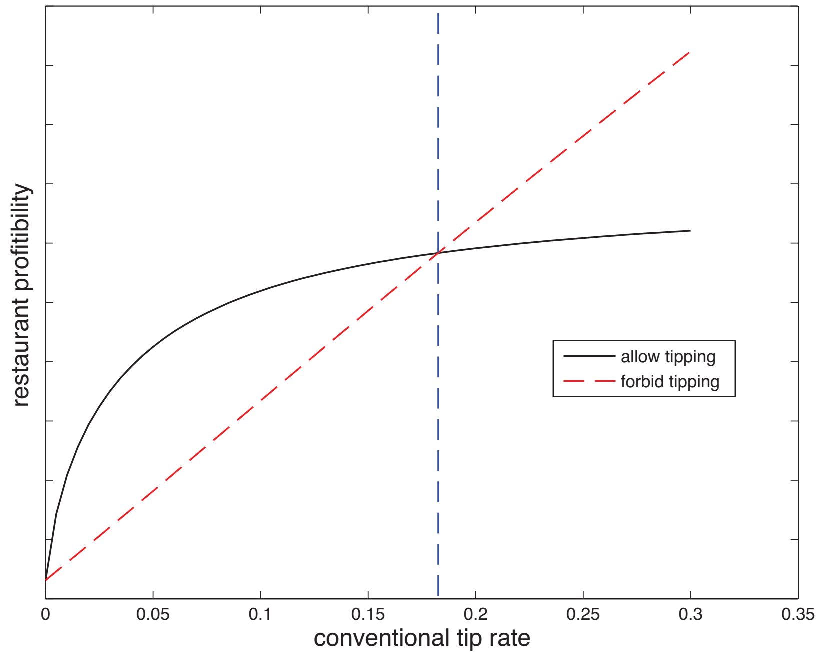

FIG. S4. Example of critical tip rate threshold with pay- and staff-dependent quality measure (2). For conventional tip rates below some critical threshold (blue dashed line), a rational restaurant owner would allow diners to leave gratuity to maximize profitability (black curve). Beyond that critical threshold, a rational restaurant owner would disallow tipping in their restaurant (red dashed curve). Both curves assume that the restaurant owner selects staff pay (within legal limits) to maximize profit. For this example, $m_1 = m_2 = 10, r = 4, b_{W2} = 10, b_{C2} = 25, r_{DW} = 10, r_{CW} = 1$, the minimum wage for tipped workers is 2.13, and the minimum wage for untipped workers is 7.25.

S4. ADDITIONAL FIGURES

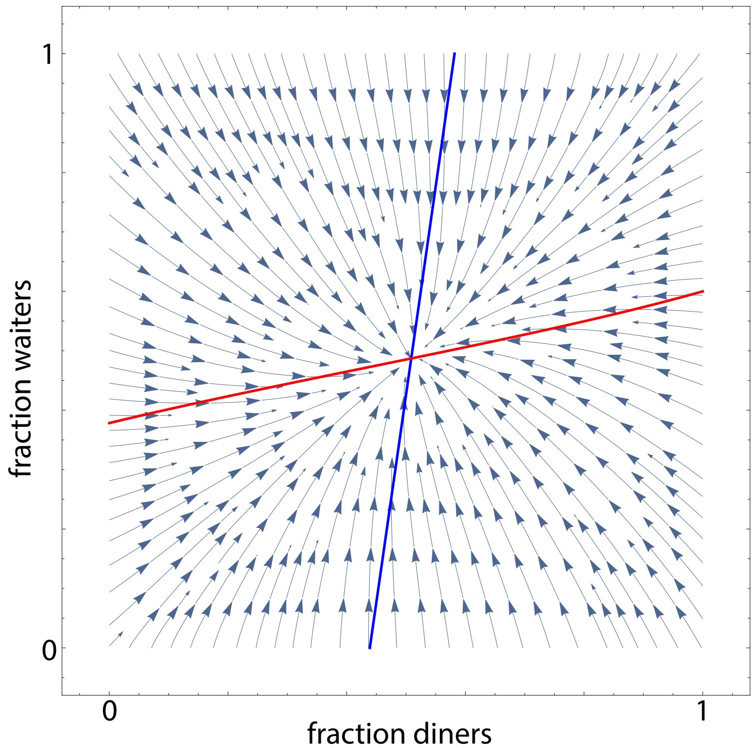

FIG. S5. Phase portrait of two identical restaurants with differing tip rates. Our restaurant (shown) enforces an automatic gratuity of $T_1 = 0.15$, and the competing restaurant allows the conventional tip rate of $T_2 = 0.2$. The steady state is $(D^*, W^*, C^*) = (0.51, 0.49, 0.50)$. Nullclines $\mathrm{d}D/\mathrm{d}t = 0$ (blue) and $\mathrm{d}W/\mathrm{d}t = 0$ (red) are superimposed. For this example, $m_1 = m_2 = 10, r = 12, b_{W1} = b_{W2} = 5, b_{C1} = b_{C2} = 10, r_{DW} = 1, r_{CW} = 1$.

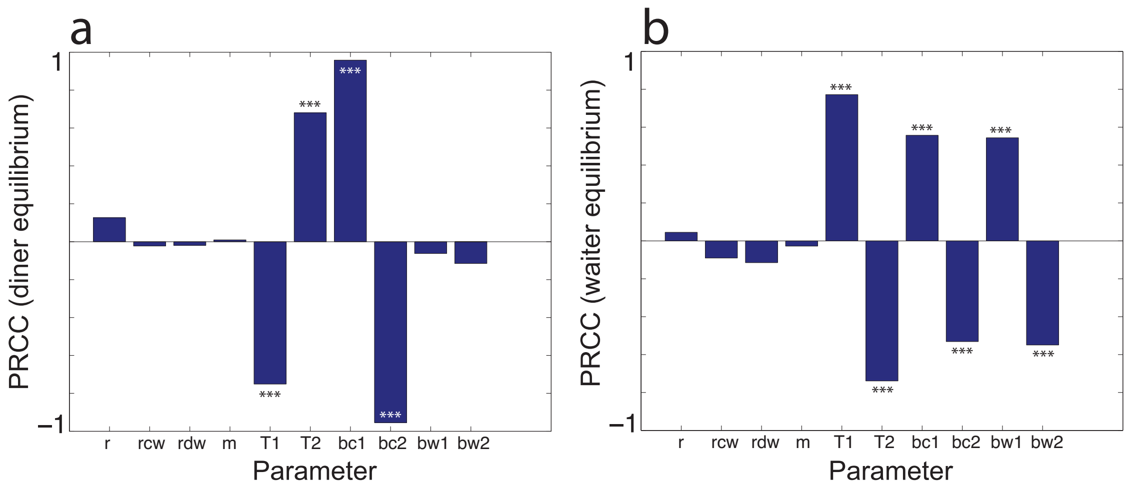

FIG. S6. Global sensitivity and uncertainty analysis for equilibrium state. **(a)** Partial Rank Correlation Coefficient (PRCC) between model parameters and diner equilibrium. **(b)** PRCC between model parameters and waiter equilibrium. Asterisks indicate that the correlation is significant ($^{***}p < 0.001, N = 100$ samples). Note that we use PRCC because numerical tests suggest that the relationships between parameters and equilibria are monotonic.