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Exploring Diverse Accounts of Teacher Knowledge

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This article explores the nature of teacher knowledge as it is portrayed by Schoenfeld's model of teaching. We attempt to situate Schoenfeld's work in the field of teacher knowledge and to elucidate the contribution that he makes to the growing body of research in this area. Towards this end, we explore two related issues. First, we distinguish between claims about the *form* of teacher knowledge and claims about the *content* of teacher knowledge. Second, we propose two families of theories of teacher knowledge, where each family shares common phenomena, methods, and theoretical forms. We argue that these two families capture much of the diversity that exists in the literature on teacher knowledge today. Our goal is to begin to develop a theoretical approach that will not only allow us to situate Schoenfeld's research, but that will also help us to compare existing theories with each other.

1. INTRODUCTION

This article is the fifth and final manuscript in a collection of articles concerned with modeling teaching. In the first article, Schoenfeld (2000) introduces a model designed to characterize a wide variety of teaching styles, and to explain, moment-by-moment, what teachers do and why. Schoenfeld describes the structure of the model, the elements that comprise the model, and the relationships between these different elements. The next two articles (Schoenfeld, Minstrell, & van Zee, 2000; Zimmerlin & Nelson, 2000) put the model to the test and offer detailed analyses of two teaching episodes. Together, the three articles provide a comprehensive introduction to Schoenfeld's model of the teaching process.[1]

Central to Schoenfeld's model is an examination of teacher knowledge. Like others, Schoenfeld argues that investigating the knowledge that teachers possess is critical to understanding the complexities of teaching. To do this, Schoenfeld draws from prior research and also introduces new terminology and new perspectives concerning the nature of teacher knowledge. The objective of our article is to situate Schoenfeld's work in the

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field of teacher knowledge and to elucidate the contribution that Schoenfeld and his colleagues make to the growing body of research in this area. However, we are attempting to go beyond simply providing a literature review to accompany the rest of this volume. Instead, we want to take some steps towards serious analysis and synthesis of the research on teacher knowledge.

Towards this end, we make two major contributions. First, we distinguish between claims about the *form* of teacher knowledge and claims about the *content* of teacher knowledge. Second, we propose two families of theories of teacher knowledge, where each family shares common phenomena, methods, and theoretical forms. We argue that these two families capture much of the diversity that exists in the literature on teacher knowledge today. Our goal is to begin to develop a theoretical approach that will not only allow us to see where Schoenfeld's research fits, but that will also help us to compare existing theories with each other. We believe this is a critical step for research on teacher knowledge, for if we want to build a common understanding, we need to know where and how theories compare—where they are saying the same thing, and where they are making competing claims.

We begin with a brief overview of research on teacher knowledge. We then review two research programs which we use throughout the article as a basis for comparison with Schoenfeld's research. We should emphasize that our purpose is not to judge or to critique these programs. Rather, we chose them because they represent diverse perspectives and therefore help us to more precisely illuminate the contributions of Schoenfeld and others to research on teacher knowledge. Following this, as mentioned above, we discuss the distinction between the *form* and *content* of teacher knowledge. Next, we introduce the two families of research on teacher knowledge alluded to above. We compare and contrast these paradigms, and specifically look at the ways in which Schoenfeld's work extends one of the paradigms. Finally, to highlight the differences between the two families, we conclude by reanalyzing the teaching episode presented in Zimmerlin and Nelson (2000) from an alternate perspective.

1.1. Research on Teaching: A Shift in Goals and Methods

The last two decades have seen an important shift in the issues that are central to research on teaching. Guided by a psychological research tradition, research prior to this shift tended to focus on identifying specific behaviors associated with effective teaching such as questioning or management techniques (Brophy & Good, 1986; Dunkin & Biddle, 1974; Shulman, 1986a). The idea was that by adopting these behaviors, teachers could influence student learning.

While this earlier strategy has not been abandoned entirely, more recent research has examined teaching from a cognitive perspective, attempting to describe how teachers think and act, and the knowledge underlying such actions (Brophy, 1991; Calderhead, 1987; Clark & Peterson, 1986). Thus, the field moved from a focus on simply identifying *what* works to one concerned with *why* and *how* teachers do what they do. Furthermore, the answers to these why and how questions were given in cognitive terms: Teachers do what they do because they do (or do not) possess certain knowledge. Thus, teaching was no longer seen as a set of isolated behaviors that could be prescribed, and instead was studied as a complex cognitive process.

Contributions to the study of teacher knowledge have been made by researchers with a variety of research concerns. Those with an interest in teacher education have often chosen to focus on the knowledge of novice teachers rather than of veteran teachers (Borko et al., 1992; Grossman, 1990). In addition, they have attempted to describe the knowledge required for successful teaching (Wilson, Shulman, & Richert, 1987). In other cases, researchers have compared the knowledge that veteran teachers use in different contexts (Fennema & Franke, 1992; Hashweh, 1987) or how teachers adapt to new materials (Heaton, 1994; Sherin, 1996; Wilson, 1994). Still other research has examined teachers' understanding of particular domains such as mathematics (Ball, 1991; Marks, 1989), science (Carlsen, 1991), and history (Wineburg & Wilson, 1991). Thus, while all of this research can be seen as dealing with teacher knowledge, these research programs have examined this knowledge under different circumstances, and with different goals.

2. THREE REPRESENTATIVE RESEARCH PROGRAMS

Ideally, we would like to develop a framework that encompasses the entire spectrum of perspectives on teacher knowledge. In this article, however, we will only take a small, but we believe significant, first step in that direction. Rather than exhaustively covering the literature on teacher knowledge, we have chosen to focus our analysis on three specific research programs that we believe are representative of the diversity that exists. Along with Schoenfeld's research, we consider Lee Shulman's study of how novice teachers acquire the knowledge needed for teaching, and the work of Gaea Leinhardt and her colleagues who have described the complexities of expert teaching. Because Schoenfeld's model of teaching is discussed in detail by the other articles in this volume, we will provide only a very brief overview of this program here. Once all three programs have been described, we will begin to explore ways to compare and contrast the different accounts of teacher knowledge.

2.1. Shulman: Knowledge Growth in Teaching

Let us first turn to the Knowledge Growth in Teaching program conducted by Lee Shulman and others at Stanford University. The focus of this research program was an investigation of how beginning teachers learn to teach. As part of this program, the researchers divided the knowledge required for effective teaching into seven separate categories that together make up what they called the "knowledge base" for teaching. These categories are subject matter knowledge, pedagogical content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners, knowledge of school contexts, and knowledge of educational aims (Shulman, 1987; Wilson et al., 1987). In addition, Shulman and colleagues discussed the major "sources" of this knowledge—the routes by which teachers come by this important knowledge. Sources for the knowledge base include the teacher's own knowledge of the subject matter, curriculum and other educational materials, materials based on the research literature, and actual teaching experience (Grossman, 1990; Shulman, 1987).

Pedagogical content knowledge. Of particular importance was the notion that teachers have "pedagogical content knowledge"—subject matter knowledge that is specialized for teaching. In simply stating that this category of knowledge exists, Shulman is actually making a very important claim. The point is that it is not sufficient for aspiring teachers to have an understanding of the domain to be taught, and to know some general pedagogy. In addition, teaching requires pedagogical knowledge that is specific to the domain to be taught. As Shulman (1986b) explains, teaching requires "ways of representing and formulating the subject that make it comprehensible to others" (p. 9). This includes the ability to choose appropriate instructional strategies and representations, anticipate student difficulties and interpret student insights. Grossman (1990) further divides pedagogical content knowledge into four subcategories: conceptions of purposes for teaching subject matter, knowledge of students' understanding, curriculum knowledge, and knowledge of instructional strategies.

Shulman believes that the development of pedagogical content knowledge is a critical element in the move from novice to expert teaching. Furthermore, he argues that a particular process drives this move, what he calls "pedagogical reasoning." Through pedagogical reasoning, a novice teacher's subject matter knowledge is used to generate pedagogical content knowledge (Shulman, 1987; Wilson et al., 1987).

In one example of this transformative process, a new teacher, Alan, adapted his understanding of the play *Julius Caesar* in order to make the play meaningful for students. Alan decided to emphasize the theme of moral conflict in his teaching of *Julius Caesar*. To help students to engage with this theme, he introduced *Julius Caesar* by describing a hypothetical situation from the television show Star Trek. Alan told his students to imagine that they were first officer on the Starship Enterprise, and a good friend of Captain Kirk. In the hypothetical situation, Kirk has been getting a little out of hand, and his rashness might endanger the fleet. In this way, Alan was able to contextualize a central issue in *Julius Caesar* in terms that were accessible to his students. "He transformed his understanding of the play, from a piece of literature that deals with the issue of moral conflict, into an activity that would allow his students to experience the emotional and intellectual struggles that are involved in moral conflict" (Wilson et al., 1987, pp. 112–113).

Shulman and his colleagues' work has had a significant influence on research on teaching. The introduction of pedagogical content knowledge was an important contribution that set the stage for much of future research on teacher knowledge. In addition, it helped to refocus the research community on the importance of the teacher's understanding of the domain and its relationship to teaching.

2.2. Leinhardt: Describing the Mental Structures of Skilled Teachers

We now turn to the research of Gaea Leinhardt and others at the Learning Research and Development Center at the University of Pittsburgh. Much of Leinhardt's research has been concerned with characterizing the knowledge of expert teachers, particularly in contrast to novices. From the mid-1980s through the early 1990s, Leinhardt produced a large number of articles on this topic. Our review here focuses on a subset of her work that has been particularly prominent (e.g., Leinhardt, 1993; Leinhardt & Greeno, 1986).

In their early work, Leinhardt and colleagues began with a move that is strongly reminiscent of the claims we attributed to Shulman. According to Leinhardt, the knowledge of teachers is divided into two related knowledge systems: knowledge of lesson structure and knowledge of subject matter (Leinhardt & Greeno, 1986; Leinhardt & Smith, 1985). Lesson structure knowledge is described as an understanding of how to plan and implement a lesson. In contrast, subject matter knowledge involves an understanding of the content to be taught.

However, unlike Shulman, the major work of Leinhardt's theory is not done by these category labels. Instead, she pushes downward into each of these categories, in order to characterize some of the particular types of knowledge found there. For example, according to Leinhardt, knowledge of lesson structure consists primarily of a collection of *schemata*, a set of knowledge elements that structure a teacher's classroom actions (Leinhardt & Greeno, 1986). The idea is that these schemata consist of sequences of goals and actions—goals and actions that correspond to what the teacher does in the classroom. Thus, Leinhardt posits a direct relationship between a teacher's knowledge of a lesson and the teacher's behavior in the classroom. There are schemata for the most mundane of activities, such as handing out papers, as well as schemata for complex subject matter-specific behavior, such as explaining a difficult concept. Furthermore, these schemata vary greatly in the time-scale at which they structure behavior. While some schemata set a broad plan for a large portion of a classroom session, others are associated with low-level, short duration activities.

Routines, agendas, and curriculum scripts. Leinhardt and her colleagues do not simply refer to all of these different types of knowledge as schemata; instead, they give names to the various types of schemata, according to the jobs they do and the time scale at which they structure behavior. For example, among the elements at the highest level are what Leinhardt refers to as *agendas*. Agendas are akin to a master plan for a lesson, they determine the overall flow of activities in a classroom session, and provide the structure within which other schemata do their work. An agenda specifies the teacher's goals for the various segments of the lesson as well as the actions the teacher can take to achieve those goals. Leinhardt describes agendas as dynamic in the sense that they can be modified as a lesson proceeds.

At a lower level are what Leinhardt calls *routines*. Routines are socially scripted sets of behaviors that allow teachers to carry out some activities in a relatively automated manner and with minimum cognitive load. For example, there are routines for handing out and collecting articles, having students share their work, and reviewing homework in class (Leinhardt, 1988).

The intermediate levels are also populated by various types of knowledge elements. Perhaps the most important are what Leinhardt, following Putnam (1987), calls *curriculum scripts*. Curriculum scripts consist of a set of ordered goals and actions for a lesson. What is particular to curriculum scripts is that they are, in general, strongly tied to the content to be taught; curriculum scripts are specific to a topic. Thus, a curriculum script might include a plan for explaining a particular concept to students.

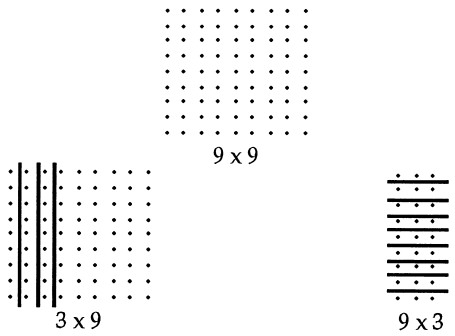

FIGURE 1. Representation of 3 × 9 and 9 × 3.

Comparing expert and novice teachers. A major goal of Leinhardt's research program has been to use this theoretical framework to contrast the knowledge possessed by expert and novice teachers. Leinhardt, Putnam, Stein, and Baxter (1991) reported that the agendas and curriculum scripts of expert teachers were more elaborate than those of novice teachers. Specifically, the expert's description of the lesson included more details and more references to both the teacher's and the students' actions during the lesson. Expert teachers also described specific check-points where they would evaluate the progress of the lesson during instruction.

In one example, Leinhardt et al. (1991) describe a novice teacher's attempt to help her students understand the equivalence of 9 × 3 and 3 × 9. The teacher, Ms. Benny, had been teaching a set of lessons on the multiplication facts, and the topic of the current lesson was multiplying by 9. Ms. Benny arranged a 9 × 9 array of counters on the overhead projector. To show the problem 3 × 9, she separated three of the nine rows with strings, thereby representing three sets of nine (Fig. 1). Next, she asked a student to come up to the overhead and show the class 9 × 3 on the array. During the student's attempt to do so, Ms. Benny realized that it would not be easy to show nine sets of three on the 9 × 9 array. She tried to abort the activity, but the student was quite insistent that he wanted to be able to do it. Together, the student and Ms. Benny struggled to complete the activity while the rest of the class lost interest.

The researchers suggest that Ms. Benny had only a partially developed curriculum script, and that an important detail had not been specified sufficiently. "The problematic aspect of the script in this case was the representation Ms. Benny was using to teach multiplication" (Leinhardt et al., 1991, p. 97). They explain that the particular representation a teacher chooses to use must match the concept or procedure under discussion in the class. "Given richer knowledge of the representation embedded in a curriculum script, Ms. Benny would probably have avoided the confusion in this lesson" (Leinhardt et al., 1991, p. 97).

2.3. Schoenfeld: Modeling the Teaching Process

We now turn to the research program that is the focus of this volume, Schoenfeld and his colleagues' work at the University of California, Berkeley to develop a model of the teaching process. The goal of the program is to explain why a teacher does what he or

she does during the moment of instruction. By developing a comprehensive model, Schoenfeld hopes to be able to account for different teaching styles and different types of lessons.

The work of modeling begins by partitioning a lesson into episodes that correspond to coherent sets of actions on the part of the teacher, what Schoenfeld calls *action sequences*. Through a series of iterations, each episode is further decomposed into a set of more fine-grained action sequences. As a result of this process of decomposition, the skeletal form of the model is produced. Schoenfeld describes several different types of action sequences including routines, scripts, mini-lectures, and simple talk.

Central to the model is the claim that each action sequence corresponds to one or more goals. Furthermore, Schoenfeld explains that teachers hold multiple goals and at multiple grain sizes. Therefore, an action sequence may be related to an overarching goal, a content and/or social goal, as well as more local goals. Similarly, the model elaborates the beliefs and knowledge that influence each action sequence, along with the triggering and terminating events.

Schoenfeld is careful to distinguish between what actually happens in the classroom and what the teacher plans to do. In particular, Schoenfeld contrasts action sequences with the idea of a *lesson image* and introduces the notion of an *action plan*. A lesson image (Morine-Dershimer, 1978–1979) is a broad vision of what the teacher expects to happen. Related to this, an action plan illustrates how the teacher proposes to achieve a specific goal. As Schoenfeld explains:

> [T]he full set of intentions and expectations concerning what will take place in a lesson is the lesson image, and the teacher's expectation that he or she will engage in a particular sequence of actions to achieve one or more particular goals is an action plan. What the teacher actually does, and what is parsed in our analyses of classroom sessions, is a nested collection of action sequences. (p. 13)

Clearly, a teacher's lesson image and his or her action plans are important determinants of action sequences. Moreover, understanding where a teacher's intended plans went awry and how the teacher responded often provide valuable insights into the teachers' beliefs, goals, and knowledge. By investigating the relationship between action plans and action sequences, Schoenfeld attempts to characterize not only what the teacher planned to accomplish but also what did happen.

2.4. How Do These Research Programs Compare?

The research programs of Shulman and Leinhardt both involve coherent and extensive studies of teacher knowledge that have contributed much to the field throughout the past decade. Schoenfeld's work, while much more recent, also addresses the issue of teacher knowledge, and we believe has the potential to impact the related literature in powerful ways. Yet, considered together, there are a number of issues that remain unaddressed. In particular, we believe that the relationship between these different accounts of teacher knowledge is not obvious and has not been carefully elaborated. Where do the different terms used by these researchers overlap? Do these different characterizations of teacher knowledge constitute competing hypotheses concerning the nature of teacher knowledge,

or are they simply different ways of saying the same thing? These are the issues that drive our analysis of these programs in the next sections of the article.

3. THE FORM AND CONTENT OF TEACHER KNOWLEDGE

We have found it useful to distinguish between claims about the *form* of teacher knowledge and claims about the *content* of teacher knowledge. This distinction is far from new. It is a basic part of the content of introductory cognitive science and artificial intelligence courses, and it has been mentioned previously in the teacher knowledge literature (Grossman, 1994; Schoenfeld, 1998; Shulman, 1987). Nonetheless, we believe that there is much leverage still to be gained by pursuing this distinction.

In brief, claims about the form of knowledge are concerned with the specific structures through which knowledge is organized and represented in a teacher's mind. Claims about the content of knowledge, on the other hand, have to do with what the knowledge is for, or what it is about. For example, imagine that someone possesses the following knowledge:

> If the baby is hungry, she will cry.

The form of this statement is an if–then rule, what is called a production rule. The content is what the rule is about, in this case babies and crying. Imagine now, a different rule.

> If you touch hot coals, you will get burned.

The form of this statement is the same, an if–then rule. But the content of the rule is different; it is about coals and fires instead of babies. This is simple and fundamental, but very important for understanding research on teacher knowledge. Some claims are only about the form of teacher knowledge, while others are about the content of this knowledge. For example, if someone says that some particular knowledge is pedagogical content knowledge, then they are making claims that are almost exclusively about content—they are telling us something about what that knowledge is about and perhaps about the types of circumstances in which it might be used—but they are not telling us anything about the form of the associated cognitive structures. In contrast, when one says that an element of teacher knowledge is a script, one is not saying anything about the content of that script. A teacher may have one script for reviewing homework and another script for introducing the quadratic equation—saying that an element of teacher knowledge is a script does not define what the script is about. Thus, when we say that teachers possess scripts, we are only making claims about the form of this knowledge. In what follows, we briefly discuss, the categories of content and form.

3.1. Comparing Claims about Content

So what do different researchers say about the content of teacher knowledge? Consider, for example, Shulman's (1987) seven categories of teaching knowledge:

subject matter knowledge, pedagogical content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners, knowledge of school contexts, and knowledge of educational aims. Shulman is not the only researcher who proposes categories of this sort. As we have seen, Leinhardt and Greeno (1986) claim that knowledge for teaching includes two major categories, lesson structure knowledge and subject matter knowledge. In other work, Elbaz (1983) provides an alternative set of categories: knowledge of self, knowledge of the milieu of teaching, subject matter knowledge, knowledge of curriculum development, and knowledge of instruction.

In examining these different claims, there are some clear places where the categories proposed by different researchers overlap (e.g., knowledge of lesson structure and curriculum knowledge). In contrast, other researchers appear to have proposed categories that include knowledge that may not be captured elsewhere (e.g., knowledge of self). While we do not attempt an exhaustive analysis of this sort, the program to make these sorts of comparisons is valid and worthwhile. This is precisely our point. It is because these researchers are all looking at the *content* of teacher knowledge that we can make these kinds of comparisons. Moreover, when we compare claims about content, we can begin to see how various researchers carve up the broad landscape of teacher knowledge.

3.2. Comparing Claims about Form

Next, we turn to a discussion of the form of teacher knowledge. To repeat, claims about the form of teacher knowledge relate to the structure of this knowledge. For example, much of the work done by Leinhardt and her colleagues involves claims about form (Leinhardt & Greeno, 1986; Leinhardt et al., 1991). Consider their statement that teacher knowledge includes agendas, curriculum scripts, and routines. Each of these types of knowledge is a variety of *schema*—a template knowledge structure with fixed or default elements and blank slots that are filled in at the moment of use. Furthermore, both curriculum scripts and agendas are schemata that are built out of goals and actions, thus allowing them to structure teacher behavior in the classroom. In addition, in a curriculum script, these goals and actions are loosely ordered. Thus, there are many claims here about the form of teacher knowledge. Notice that, for example, in saying that a curriculum script involves subcomponents that are *ordered*, we are making a claim that constrains how that knowledge is represented.

Again, our point is that as we try to make sense of the diversity within research on teacher knowledge, it is important to compare claims about the form of teacher knowledge with other claims about the form of this knowledge. For example, we can make comparisons between Schoenfeld's claim that teachers have a lesson-image and Leinhardt's description of an agenda. Both are knowledge elements that function as a plan that guides the teacher at the broadest level of a lesson. These constitute claims about form because of the implication that the structure of the plan is actually reflected in how the knowledge is represented in the teachers' mind. In fact, in many respects, they seem to have very similar forms in mind. Both consist of smaller pieces that correspond to components of the classroom activity associated with a lesson.

Terms can specify form and content. Before concluding this section, we offer one caveat. In our discussion above, we tended to emphasize either claims about form or claims about content that were associated with specific terms. However, we do not mean to imply that any term is only about the form of teacher knowledge or about the content of this knowledge. In fact, there are many instances in which a researcher makes a claim about teacher knowledge that says something about both form and content.

For example, Leinhardt et al. (1991) explain that a curriculum script involves the goals and actions for how to teach the topic of a lesson. Thus, while perhaps the main idea, in this case, was to describe the form of an element of teacher knowledge, we also find out something about its content—that a curriculum script has to do with the subject matter of the lesson. Compare Anders (1995) notion of a *classroom script.* A classroom script involves more than just subject matter knowledge, it also includes the teacher's knowledge of the students, the classroom context, and the community. Thus, while curriculum scripts and classroom scripts share similarities in form, they make different claims about content.

Furthermore, even when it appears that a researcher is making claims that seem to be primarily about the content of knowledge, there are almost always some implications for form. For example, Sherin (1996) proposes that as teachers develop expertise, elements of subject matter knowledge and pedagogical content knowledge become tightly connected in knowledge structures called *content knowledge complexes*. The idea here was to make a claim about the content of teacher knowledge, that there are larger knowledge structures that do not fit neatly into either the subject matter knowledge category, or the pedagogical content knowledge category. Yet, because she implies that strong connections exist among elements of teachers' subject matter knowledge and pedagogical content knowledge, Sherin's statement has implications for the form of teacher knowledge.

4. TWO PARADIGMS IN THE STUDY OF TEACHER KNOWLEDGE

In the previous section, we argued that distinguishing between claims about form and claims about content is a valuable first step in considering the different approaches that researchers take in the study of teacher knowledge. We now discuss another way of understanding and bringing order to this diversity. Here, we argue that to fully understand and appreciate the diversity that exists among research on teacher knowledge, we need to build some understanding of the types of theories that are proposed by various researchers.

4.1. Finding Families of Theories

Our next step, then, is to find groups of theories of teacher knowledge among which there is a family resemblance. That is, we want to organize the various accounts of teacher knowledge into groups that share some or all of a set of characteristics. The characteristics of theories we considered in our analysis related to: (1) the phenomena a theory investigates, (2) the methodology the theory uses, and (3) the epistemic form in which the theory is cast. Taken together, a particular type of phenomenon, a methodology, and an

epistemic form define a family of theories. We understand these three dimensions as strongly interdependent. For example, in a sense, phenomena do not exist separately from the associated theoretical paradigm. Nonetheless, this coarse frame can help in understanding how the various theories relate.

More specifically, we define these dimensions as follows:

Phenomena. Phenomena are a theory's empirical subject of scrutiny; theories are dedicated to explaining the phenomena that are selected for attention. Some examples of phenomena are observations of a teacher presenting a particular lesson and observations concerning how a teacher's instructional methods develop through time.

Methodology. Methodology has a lot to do with what a researcher actually does: What they do to collect observations and what they do to analyze those observations once they have them. Some typical methodologies in research on teaching are ethnographic methods and qualitative analyses of videotapes of teaching.

Epistemic forms. An epistemic form is the kind of representational structure in which the theory is expressed (Collins & Ferguson, 1993). Some typical epistemic forms in cognitive studies of education are lists of kinds of knowledge, mechanistic cognitive models, narrative explanations, and ethnographic descriptions.

4.2. Two Families: Cognitive Modeling and Knowledge System Analysis

We believe that much of the diversity is quite well captured by only two families of theories. We call these two families the *cognitive modeling* paradigm and the *knowledge system analysis* paradigm. In fact, these families are not specific to research on teaching, they are approaches to explaining many kinds of cognitive phenomena.

As we lay out these two paradigms, it is important to keep in mind that these are essentially caricatures of two research approaches. No research program has all of the characteristics of a family, and most have some of the characteristics that we will associate with both families. Nonetheless, this simple framework does capture many of the important differences among theories. To illustrate the two paradigms, imagine you are a scientist trained in one of these disciplines. How would you go about investigating teacher knowledge?

First, suppose that you were a cognitive modeler. If you wanted to investigate teacher knowledge, the first thing you might do would be to go to a classroom and videotape a teacher teaching. Once you had this videotape, you would view it carefully, making guesses about what is going on in the teacher's head while he or she is teaching. In particular, you would work toward constructing a cognitive model that is sufficient to explain aspects of the teacher's behavior. As part of its structure, this model would include various types of knowledge elements. These elements are embedded in a mechanism that is used to produce accounts of behavior.

Now, suppose instead that you were a knowledge system analyst. You would probably want to focus your attention on interesting phases of the teacher lifecycle, such as a novice teacher's earliest teaching experience. And rather than using videotapes, you might be inclined to use ethnographic methods. In that case, you would sit in the back of a teacher's classroom, on occasion, taking notes on what you saw. In addition, you would probably want to interview the teacher outside of class to add to what you observed. Finally, with these observations in hand, you would proceed to create theoretical categories that help

you in describing the aspects of knowledge that contribute to successful teaching, and how that knowledge changes and evolves through time.

In summary, cognitive modelers build models to explain relatively short-term segments of activity. They use models, which are constituted in part by knowledge elements, to create accounts of teacher actions. In contrast, a knowledge system analyst is more concerned with characterizing the knowledge possessed by teachers, and not with building constructs or mechanisms that explain actions at a detailed level. A typical knowledge system analysis approach is to put knowledge into categories. Furthermore, rather than explaining behavior in episodes, they are interested in how knowledge originates and evolves.

Let's take a moment to think about where to locate each of the researchers we have discussed within these two families of approaches. Leinhardt's research program fits quite well in the cognitive modeling family. The knowledge she posits specifies behavior at the level of individual teacher actions. And this knowledge is embedded in a particular mechanism based on goals and their associated actions.

Shulman's research, on the other hand, is largely a program in knowledge system analysis. The emphasis is not on modeling teacher actions. Instead, he is more concerned with developmental phenomena, and broad accounts of the types of knowledge required for teaching. He develops categorization schemes and crafts narrative explanations.

Fitting Schoenfeld into this framework is not quite as straightforward. However, his work is clearly closest to the cognitive modeling paradigm, and it was designed to do this type of work. Therefore, we begin by using Schoenfeld and Shulman to clarify differences between the cognitive modeling and knowledge system analysis paradigms. In particular, we do this using the three dimensions we laid out earlier.

Refining the contrast: Phenomena, methodologies, and epistemic forms. *Phenomena*. Even though they both examine teacher knowledge, there is an important sense in which Schoenfeld and Shulman are studying different phenomena. Schoenfeld's research is guided by the question: What is the teacher doing and why? In other words, he wants to understand the underlying causes of specific teacher actions. In contrast, the phenomena of interest for Lee Shulman are frequently developmental and genetic. He wants to know: "How does a teacher's behavior change through time?" and "What does a teacher know and when did he or she come to know it?" (Shulman, 1986b, p. 8).

Methodologies. Schoenfeld's and Shulman's scientific approaches also differ in the methodology they use. Schoenfeld's methodology consists of videotaping teachers and carefully examining the videotape. Shulman's methodology is ethnographic, observing in classrooms and interviewing teachers over extended periods of time. These approaches are typical of the cognitive modeling and knowledge system analysis paradigms.

Epistemic forms. Schoenfeld and Shulman also differ strongly in their target epistemic forms. Schoenfeld's theory is cast in the form of a cognitive model. It specifies different kinds of knowledge and how they function as part of a cognitive mechanism. Shulman's theory consists of a set of categories of knowledge. He also produces accounts of how these various categories originate and how they interact to develop through time.

4.3. What Does This Tell Us about Schoenfeld's Model?

While these differences highlight the ways in which Schoenfeld's research represents the cognitive modeling paradigm, we believe that it is not quite right to situate Schoenfeld exclusively in this paradigm. The situation is a bit more complicated, and to explain it we examine more closely the relationship between Schoenfeld's and Leinhardt's research. In brief, we claim that Schoenfeld's foray into cognitive modeling extends beyond the work of Leinhardt in several interesting ways. Furthermore, we believe that these extensions push in the direction of knowledge system analysis.

Schoenfeld and Leinhardt share a number of key points in their analysis of teaching. To start, they both make claims about the form of teacher knowledge. For example, both of these researchers use the term *routine*, and they use this term for similar jobs within theories belonging to the same family. This is not to say that there are not interesting differences; in fact, part of the point is that this is a place that it makes sense to look for differences. For instance, Schoenfeld emphasizes that his routines are not constrained to small sets of scripted behavior. Unlike Leinhardt's, they can also account for well-established behavior that occurs over a larger time period. Nevertheless, there are clear similarities between Leinhardt's description of agendas, routines, and curriculum scripts, and Schoenfeld's discussions of lesson images and action plans.

Two other similarities are also worth noting. First, both Schoenfeld and Leinhardt describe teaching as goal-driven. Critical to their analyses is drawing connections between teachers' goals and teachers' actions. Second, both researchers work with the assumption that a lesson can be divided into segments that cohere psychologically for the teacher. The work they do, then, is to understand how and why a teacher moves from one episode to the next, and what a teacher does within each episode. All this is typical of cognitive modeling.

At this point, however, there are interesting and important differences between the research of Leinhardt and Schoenfeld. First of all, in Leinhardt's model, a knowledge structure fully determines behavior. In contrast, Schoenfeld allows for a complex ensemble of knowledge plus the exigencies of the situation to lead to behavior. Thus, a teacher is not expected to do the same thing each time he or she teaches a particular lesson. Instead, the teacher's actions also depend on the context in which the lesson is being taught and what is happening in class at that moment.

Schoenfeld's model also draws on more varieties of knowledge than does Leinhardt. In particular, in addition to routines and scripts, Schoenfeld discusses how teachers' beliefs, subject matter knowledge, and pedagogical content knowledge influence the actions that teachers take during instruction. In doing so, Schoenfeld is including in his analyses types of knowledge not usually addressed in cognitive modeling.

These features have another important implication for Schoenfeld's research. Because of the complexity that he models, Schoenfeld is able to characterize a wide variety of teaching. In general, the mathematics lessons that Leinhardt considered can be thought of as fairly traditional lectures—homework review, introduction of new material, classwork, and then individual practice. In contrast, Schoenfeld takes a much broader view of what it means to teach mathematics and hopes to model diverse types of lessons equally well. This is evident in the different lessons and teaching styles analyzed in Schoenfeld et al. (2000)

(a) $\frac{m^6}{m^2}$ (b) $\frac{x^3y^7}{x^2y^6}$ (c) $\frac{x^5}{x^5}$

FIGURE 2. Three expressions involving exponents.

and Zimmerlin and Nelson (2000). Furthermore, because Schoenfeld models such diversity, it seems possible that the model can be used to represent the evolution of individual teachers' beliefs, knowledge, and goals over time—something that has traditionally been the interest of knowledge system analysis. In fact, Schoenfeld (1998) suggests just this, that the model could in fact be used to study change in teachers across time.

5. ANALYSIS OF A TEACHING EPISODE

To further illustrate the distinction among different accounts of teacher knowledge, we want to exercise the ideas we have laid out thus far in the context of a particular example. This will help to illustrate how an examination of a particular teaching episode would be different depending on whether it is based in a cognitive modeling approach or in a knowledge system analysis. Furthermore, it will give us an opportunity to consider how a researcher such as Schoenfeld, can draw on both perspectives in his analysis of a teaching episode.

The following example comes from the work of the Teacher Model Group at the University of California, Berkeley and is analyzed in detail in Zimmerlin and Nelson (2000). The particular episode of interest involves a short segment of class in which a beginning mathematics teacher reviews three problems about exponents with his students.[2]

Earlier, in the first part of the lesson, the class discussed two main ideas about working with exponents. The first was the notion that raising a variable (or a number) to a particular power was the same as multiplying that variable by itself the number of times indicated by the exponent. So, for instance, y^4 could be decomposed into $y \cdot y \cdot y \cdot y$. Second, and related to this idea, the class practiced dividing variables raised to different powers by subtracting the exponents. For example, $y^6/y^2 = y^4$ since $6-2 = 4$. The teacher explained to the students that subtracting the exponents could be thought of as expanding the numerator and denominator and then canceling those variables that appeared in both. Thus, he might have demonstrated that $y^6/y^2 = (y \cdot y \cdot y \cdot y \cdot y \cdot y)/(y \cdot y) = y \cdot y \cdot y \cdot y = y^4$.

Next, the teacher asked the students to work in groups to simplify each of the following three expressions (Fig. 2). The teacher circulated through the class as the students explored the problems. Then, after several minutes, he gathered the class to go over the problems together. It is here that we focus our attention.

In reviewing problem (a), the students chorused their agreement that the correct answer was m^4. The class also agreed on an answer to the second problem. Although this problem involved dealing with multiples bases, most students had solved the problem without difficulty and found that the answer was xy. The teacher then asked the students for their answer to the third problem. At this point, the class was not in

$$\frac{x^5}{x^5} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$$

FIGURE 3. A decomposition of the problem x^5/x^5.

agreement, and the students responded with several possible answers. "Zero." "One." "x^0." "Nothing."

Recognizing that the class had not reached consensus, the teacher decided to work out the problem on the board. With his students' assistance, the teacher decomposed the problem as is shown in Fig. 3.

He then asked the students, "So what do I do?" "Cancel!" came the unanimous reply. After crossing out the five x's in the numerator and denominator the teacher again turned to the class. "So what am I left with?" Here again, the class was unsure. Some students replied that an x remained, while others claimed that the answer was zero.

The teacher appeared surprised by the different answers he heard. After a moment, he continued the discussion by writing 5/5 on the board and asked the class what that fraction would equal if he were to cancel both of the fives. While the students agreed that 5/5 = 1, the solution to the other problem was still somewhat unclear to them. The teacher suggested that since 5/5 equaled 1, so would $(x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x)$. In addition, he explained that if the students solved problem (c) by subtracting exponents, as they had done in problems (a) and (b), they would come up with the answer x^0. Thus, the teacher claimed that in fact, $x^0 = 1$, and had students write this in their notes for the day.

5.1. An Analysis in Terms of Cognitive Modeling

In their fine-grained analysis of this episode, Zimmerlin and Nelson (2000) characterize the teacher's actions in terms of Schoenfeld's model of teaching. Their analysis is based on a videotape of the lesson, as well as an interview with the teacher after the teacher had an opportunity to watch the videotape. Zimmerlin and Nelson explain that the teacher had a particular vision of how the lesson would proceed in class, a lesson image. In brief, the teacher believed that the students would easily be able to solve problems (a) and (b), but would run into difficulty with problem (c). To help students solve this third problem, the teacher planned to decompose x^5/x^5 into $(x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x)$ and discuss the fact that $x^0 = 1$.

Zimmerlin and Nelson (this volume) also describe specific goals that were the basis for the teacher's actions in this excerpt, as well as the teacher's implementation of various scripts and routines. In particular, their analysis offers an explanation of what happened when the lesson did not unfold as the teacher had planned. While the teacher anticipated some student difficulty recognizing that $x^5/x^5 = 1$, he expected this problem to be resolved by reminding the students that $x^5/x^5 = (x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x)$. What the teacher had not predicted, however, was that it would simply not be obvious to the students that $(x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x) = 1$.

Faced with this problematic situation, the researchers suggest that the teacher had to revise his current goal. Instead of simply showing the class that $x^0 = 1$, the teacher

changed his focus to convincing the class that $(x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x) = 1$. In addition, the teacher did not have a preplanned script or routine to implement, and instead needed to develop a new action plan to satisfy the current goal. Specifically, the teacher decided to emphasize the idea that canceling both the numerator and denominator yields 1, hence his presentation of 5/5 to the class. Thus, the idea of introducing 5/5 was not part of the teacher's lesson image, and instead was a plan developed on the fly in response to students' confusion about whether or not $(x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x)$ was equal to 1.

Thus, Zimmerlin and Nelson's account is fundamentally a story of shifting goals. They are centrally concerned with articulating the teacher's goals at any moment during the lesson and describing how these goals change. Furthermore, they are interested in the scripts and routines that allow execution of the goals.

5.2. Knowledge System Analysis

For a different perspective, we now analyze the same teaching episode from the point of view of knowledge system analysis. The point here is not to claim that one paradigm does a better job of explaining the episode than another. Instead, our goal is simply to illustrate that these two paradigms offer different insights.

Before we begin our analysis, recall that a knowledge system analyst might not work from the same data sources as a cognitive modeler. The use of a videotape record is not too bad an approximation—we can imagine our knowledge analyst performing an ethnographic analysis on the videotape, rather than by sitting in the classroom. However, the interviews may be more problematic. Although Zimmerlin and Nelson's (2000) analysis was partly based on interview data, a knowledge system analyst would almost certainly have performed the interview in a somewhat different manner. Furthermore, they might very well have done more interviews spread over a longer time-period, including an interview prior to the lesson. Nonetheless, we will do our best to imagine what a knowledge system analyst might say about this episode.

Because knowledge system analysts are concerned with issues of development and genesis, they might choose to concentrate on changes in the teacher's subject matter knowledge and pedagogical content knowledge that occurred during this teaching segment. Based on the teacher's plan for the lesson, we can assume that the teacher had some subject matter knowledge related to the domain of exponents prior to teaching the lesson. For example, he knew the procedures for working with exponents and understood that $x^0 = 1$. In addition, the teacher understood that the expression x^5/x^5 could be simplified in two ways—by subtracting the exponents, or by decomposing the problem and canceling.

From the teacher's description of the lesson, it is also clear that he had pedagogical content knowledge related to this topic. In particular, the teacher chose to assign a problem that could be solved in two different ways, and planned to use these two methods to help students learn about a particular mathematical issue, that $x^0 = 1$. Furthermore, the teacher expected his students to simplify the expression by subtracting the exponents, and to say that the solution to the problem was x^0. He planned to then introduce the second strategy to help the students see that the answer was also 1 and that $x^0 = 1$. Thus, the teacher not only understood the subject matter he would teach in this lesson, he also had a pedagogical

understanding of the material—how he would approach the ideas with students, and how he predicted they would respond.

In practice, however, the teacher's assumptions concerning how the students would respond did not turn out to be entirely accurate. While the students understood the procedure for canceling the x's in the numerator and denominator of $(x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x)$, they did not understand that what remained was equal to 1. The teacher recognized the students' difficulty and developed a new pedagogical strategy to help the students see that $(x \cdot x \cdot x \cdot x \cdot x)/(x \cdot x \cdot x \cdot x \cdot x) = 1$. Specifically, he selected another example to share with the class, 5/5, one that he thought they would recognize immediately as having the value 1.

Thus, a knowledge system analyst might conclude that the teacher's subject matter knowledge was sufficient for teaching the lesson—he understood what he needed to about exponents. Yet, he needed additional pedagogical content knowledge, a more detailed understanding of how to teach exponents, in order to teach the lesson successfully. And, in fact, in the context of the lesson, the teacher was able to develop new pedagogical content knowledge. He realized that students were having difficulty with a specific part of the lesson, chose an appropriate representation to share with the class, and provided an explanation of the purpose of the new example. Furthermore, it is possible that in teaching the same lesson again in the future, the teacher will apply this new pedagogical content knowledge.

So, this analysis is not a story of how the teacher's goals shifted during the moments of instruction. Rather, we have tried to speculate more broadly about the knowledge that the teacher had prior to the lesson, where this was sufficient to teach the lesson, and what sorts of changes might have occurred in the teacher's knowledge that would influence future teaching. Thus, in contrast to the cognitive modeling analysis, this analysis was more concerned with seeing this episode as one in a long cycle, contributing to the teacher's development.

6. CONCLUSION

The purpose of this article has been to consider how the research of Schoenfeld and his colleagues fits into the diverse literature on teacher knowledge. To do so, we have introduced what we feel are two helpful distinctions. The first distinction is between claims concerning the form of teacher knowledge, and claims concerning the content of that knowledge. The second is a distinction between two types of theories proposed by researchers: We distinguished between what we called the cognitive modeling and the knowledge system analysis approaches. While we have not attempted to present a comprehensive analysis of the literature on teacher knowledge, we believe that this article represents an important starting point in trying to elaborate connections across varied descriptions of teacher knowledge. By continuing in this direction, future analyses can work towards a unified account of teacher knowledge that clarifies the contributions of a great many research programs.

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NOTES

1. In the fourth article, Aguirre and Speer (2000) discuss the relationship between teachers' beliefs and goals and extend the work of Schoenfeld's model in this area.

2. We find it interesting to note that Schoenfeld (1998) extends the analysis of Mark Nelson given in Zimmerlin and Nelson (2000) and includes a discussion of pedagogical content knowledge and beliefs. By doing so, Schoenfeld further illustrates how he attempts to use the tools of knowledge system analysis to aid in his work. This is precisely what we hope to illustrate here.

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