Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, England: Cambridge University Press.

Lewis, J. (2007). "Through the looking glass": A study of teaching. In Alan H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views,* Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 1–12). Reston, VA: National Council of Teachers of Mathematics.

Paley, V. G. (1986). On listening to what the children say. *Harvard Educational Review, 56,* 122–131.

Posner, T. (2007). Looking at equity in mathematics classrooms. In Alan H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views,* Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 127–172). Reston, VA: National Council of Teachers of Mathematics.

Rogoff, B. (2003).*The cultural nature of human development.* New York: Oxford University Press.

Schoenfeld, A. H. (2007). On modeling teachers' in-the-moment decision making. In Alan H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views,* Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 46–96). Reston, VA: National Council of Teachers of Mathematics.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34,* 14–22.

Chapter 6

Commentary 2: Moving From Shared Data to Shared Frameworks

Miriam Gamoran Sherin
Bruce L. Sherin
Northwestern University

Too often in educational research, the standards for what counts as scientific consensus are lamentably poor. We agree that "constructivism" is a good idea, that learning should be "authentic," and that teachers' "pedagogical content knowledge" is important. But rarely are we precise about what our beliefs mean, nor about the data that, as a community, we believe stands as solid evidence for those shared beliefs.

We suggest that the work presented in this volume holds the promise of taking the field a step forward. Specifically, an examination of a common data set by a group of researchers appears to be a useful context for considering how we might move toward a well-articulated and well-supported consensus among the researchers involved. Although sharing data is certainly not a necessary condition for forging consensus, we believe that much is to be learned by considering how different accounts of the same short video excerpt might be unified.

To be clear, the authors of this volume did not set out to produce a shared theory of teaching. In fact, their stated purpose is quite the opposite. In her introduction to this monograph, Lewis explains that "we intentionally seek alternative and competing perspectives on problems of practice because no one theory will sufficiently illumine what is by nature a complex object of study" (Introduction, p. 5). In the long run, Lewis may be proved to be correct; no one theory may turn out to be sufficient to explain classroom practices. We nonetheless believe that the "alternative perspectives" orientation is weaker than necessary. Instead, we argue that an attempt to forge consensus around a common set of theories or models is crucial, no matter how difficult that task might appear to be at present.

PROBLEMS AND POTENTIAL

To begin, we want to briefly lay out why a volume of this sort calls out for a synthesis based on shared models and theories. First, despite different purposes and analytic approaches, many commonalities are evident across the analyses—all examine, albeit to different degrees, argumentation, justification, and a

notable interaction between Sean and Sheena. But although the various chapters in this monograph look at the same episode, and even attend to similar phenomena, whether they are in agreement is difficult to tell. For example, are the elements of mathematical talk identified by Ball, Lewis, and Hoover (chapter 1) consistent with the description of accountable argumentation given by Horn (chapter 3)? Are they complementary? And what is the relationship between Horn's discussion of discourse as shifting from accountable argumentation to peer dispute and Posner's (chapter 4) characterization of discourse as moving from being conceptually based to being interpersonal (Engle & Greeno, 1994)? Similarly, we wonder about explicit and implicit connections between Ball and her colleagues' description of the processes involved in evaluating claims and Schoenfeld's (chapter 2) discussion of the multifaceted routine through which the teacher responds to students' mathematical ideas.

The need for an encompassing framework can also be seen when looking closely at the individual articles in this volume. Consider, for example, the chapter by Ball and others (chapter 1). In discussing the particular practices that are the focus of their chapter, the authors state that their analysis led them "to posit three essential elements that undergird the nature of mathematical talk in the segment." Their list of elements clearly has intuitive appeal—the students' talk does seem to be atypical in the ways that mathematical terms are used, in the students' offering of claims, and in the persistent probe for justifications of claims offered. Not clear, however, is the extent to which Ball and her colleagues are suggesting that those elements comprise the entire landscape of classroom discourse in the focus episode. Are those elements *the* three elements? Do other foundational elements of the classroom talk exist? And if so, how would they be identified? In other words, how seriously are we supposed to take the particular decomposition of mathematical talk embodied in this list? Do two different kinds of naming, or two different kinds of mathematical assertions, actually exist?

Our point, of course, is that none of those questions can be answered absent a broader framework in which all the theoretical entities are embedded. That is, if those entities are *elements,* being so implies that they should somehow be components of something larger. And if they are components, being so suggests a particular kind of decomposition, according to some overarching logic. What is that logic?

Our intent is not, in particular, to be critical of the contribution by Ball and her colleagues (chapter 1). Indeed, in our view their work is representative of the state of the art in our field. Our point, instead, is to attempt to make clear what is possible if researchers come together with the sort of shared focus attempted in this monograph. Real progress will not be achieved, we believe, only—or even primarily—by looking at the same data. Instead, it will be achieved when researchers forge consensus on frameworks and models.

In what follows, we examine the extent to which the chapters in this monograph foster insights about the nature of such consensus. Specifically, we describe two approaches to synthesizing the work of the researchers represented here. First,

however, we briefly introduce each of the chapters in the monograph. Our intention in doing so is not to summarize the main points of each chapter, but rather to point to particular perspectives that will be relevant for the syntheses that we present later on.

THE INDIVIDUAL CHAPTERS

In Lewis's introduction to the monograph, she emphasizes the complex nature of teaching. It is an activity filled with uncertainties, in which the same lesson plan is likely to play out differently in multiple enactments. As Lewis explains, this uncertainty poses challenges for teachers as well as for those who wish to study teaching. How can we make sense of such a complicated endeavor? What would constitute understanding an episode of mathematics teaching? The four remaining chapters in the monograph examine this issue by considering the same classroom episode, the first 6 minutes in a lesson that has come to be known as "Sean numbers" (Ball, 1990, 1993).

Ball and her colleagues (chapter 1) investigate the mathematical work in which students are engaged and the role of the teacher in establishing and supporting that work. The task is challenging, particularly because the teacher in the episode, Ball herself, can seem "invisible" at first glance. In contrast with the central position often held by teachers during whole-class discussion, in this episode, a great deal of student-to-student talk takes place without interjections from the teacher. Furthermore, the teacher's stated agenda is to hear from students; she asks them for the comments they have about the previous day's meeting with the fourth graders. Thus, the specific mathematical focus of the lesson is dictated, in large part, by the students (Hufferd-Ackles, Fuson, & Sherin, 2004).

In examining the classroom talk in this segment, Ball and coauthors (chapter 1) focus on three elements: (a) "naming," that is, the use of words and phrases to describe mathematical content and the practice of learning mathematics; (b) an orientation to making claims about mathematics and about doing mathematics; and (c) the evaluation of mathematical assertions. The authors propose that each element supports the work of doing mathematics, and they provide detailed evidence of students' participation in those discourse practices during the 6-minute episode under consideration. In addition, a central goal of Ball and colleagues' chapter is to make visible how the teacher's own discourse is used to establish and reinforce those discourse patterns. Thus, we understand Mei's reply, "I'm going to listen more to the discussion and find out," not just as a response from an extraordinary third grader but as a response to her teacher's request to "listen to one another's comments, so that we can benefit from what other people say." Ball has explicitly named the activity of listening and elevated it to a position of significance in supporting one's own learning of mathematics.

The second chapter concerns Schoenfeld's model of the teaching process. His approach involves an attempt to describe, at the moment-by-moment level, the decision-making process of a teacher. In prior work, Schoenfeld (1998, 1999)

proposed models of two secondary level teachers, both of whom maintained a strong presence in whole-class discussion. Attempting to model the teaching of Deborah Ball in this episode was therefore an important test case for examining the limits of the model.

Schoenfeld's (chapter 2) model encompasses a teacher's goals, beliefs, knowledge, and actions. Because a teacher may hold multiple goals simultaneously, the model allows for the shifting of goals over time and for different sets of goals to be given priority at particular moments. To model Ball's teaching in the episode under investigation, Schoenfeld identifies a "flexible, interruptable routine," in which comments made by students are considered in light of Ball's current goals, thereby resulting in corresponding actions on the part of the teacher. Ball cycles through the routine five times in the 6-minute episode. And although her responses to students appear spontaneous and context-dependent, Schoenfeld's model suggests that they are not arbitrary. In contrast, Schoenfeld argues that Ball's responses in this episode can, for the most part, be predicted on the basis of her goals, beliefs, and knowledge. For example, the model draws on particular beliefs about students and about mathematics to account for why Ball does not intervene in Sean and Sheena's conversation, but yet soon after, takes a detour from her stated purpose to explore Nathan's ideas about the composition of even numbers.

The third chapter, by Horn, examines the following teaching dilemma: How can classroom discourse promote productive discussion of competing mathematical claims (Ball, 1996; Sherin, 2002; Silver & Smith, 1996; Wood, 1999)? To investigate this issue, Horn introduces a discourse structure she calls "accountable argumentation." According to Horn, accountable argumentation "organizes the public disagreements among students" (chapter 3, p. 104) in such a way that the mathematical focus of discussion is maintained and social discomfort is minimized. Horn presents warrants for classifying a disagreement as an instance of accountable argumentation, noting specific norms and expectations, roles for participants, and the use of historical information as distinguishing elements. Furthermore, she illustrates that mathematical learning can and does take place during such disagreements. As an example, Horn deconstructs Sean and Mei's discussion about the parity of 10. In an unforgettable moment from the "Sean numbers" discussion, Mei responds to Sean's claim that "six can be an odd and an even number" by asking, "Why do you not call ten [an] ... odd number and an even number?" Horn uses this example to illustrate that accountable argumentation can be both sustained by students and mathematically rigorous. Toward that end, she highlights that this conversation moves from the particulars of six to more generalized claims about the oddly even numbers. (A discussion of the oddly even and evenly even numbers can be found in Posner's contribution to this volume.)

In the fourth chapter, Posner explores the social nature of mathematics learning in the classroom episode, with a particular focus on issues of equity. Posner looks closely at two segments and attempts to explicate both the mathematical meaning of students' comments as well as the interactional meaning of students' statements and actions in those segments. She explains that doing so "can provide a lens to view the roles, social relationships, and power relationships among participants" (chapter 4, p. 136).

For each segment, Posner (chapter 4) brings a variety of lenses to her analysis. For example, she considers (a) the nature of the mathematical ideas raised by students in the discussion and how those ideas have been treated by the class previously, (b) the history of students' participation in the classroom, (c) the use of inscriptions and the positioning of different inscriptions in this classroom, and (d) the degree to which different students are willing to assume authoritative roles in this discussion. Thus, to explore Sean and Sheena's discussion about the parity of zero, Posner suggests what each understands about even and odd numbers, the kind of authority they turn to for justification of those ideas, how much they typically talk in class, and more. We are to understand, for instance, that although Sheena "seemed to have a clear grasp of the nature of even and odd numbers," she tended to attribute "the justification of her positions to third parties (mainly boys)" (p. 145). This tendency on Sheena's part raises questions, then, about her comment about zero, "I'm not saying that it has to be even. I meant that it could be," (p. 143). As Posner asks, "Did Sheena change her understanding about the nature of zero through the work of the disagreement?" (p. 143). Or did she back off to avoid a confrontation with Sean? Posner's analysis clearly portrays the complexity of classroom interactions as well as the complexity involved in interpreting such interactions.

TOWARD A SYNTHESIS

What, then, might constitute a synthesis of these diverse approaches? And what would a grand synthesis tell us about the nature of teaching? We are limited—by both space and our own ability—in what we can do in this commentary. As a start, however, we illustrate two types of syntheses that we believe are useful. Both approaches shed light on the unique contributions of the chapters in this volume while suggesting important next steps for the future.

Our first approach consists of an ontological synthesis of the articles in this volume. In other words, we look closely at the kind of theoretical entities that these researchers make reference to as they make sense of the teaching episode. In doing so, we ask how those elements might fit together in a broader framework. Because all four chapters use classroom discourse as a central lens through which to examine the teaching episode, we have chosen discourse as the starting point for this synthesis. To be clear, albeit having a common focus, the chapters explore different components of the discourse that takes place, for different purposes.

Consider the work of Horn (chapter 3), for example. In making claims that accountable argumentation is a discourse structure, Horn is in fact making claims about what constitutes a classroom discourse structure more broadly. In particular, she suggests that disagreements can be classified by norms, expectations, and interactional roles. Moreover, Horn identifies a specific set of expectations that distinguish instances of accountable argumentation from other forms of whole-class discussion. Similarly, she presents the range of roles that participants take on

during accountable argumentation. In these ways, Horn offers a detailed characterization of accountable argumentation and also identifies what she sees as essential dimensions of participant structures during whole-class discussion. Recall, also, that Horn gives an example in which an instance of accountable argumentation is transformed into a "peer dispute." Thus, Horn implicitly states the existence of multiple kinds of discourse structures that one might find.

Ball and her colleagues (chapter 1) take a different approach to identifying the substance of classroom discourse during the episode under consideration. We believe that the three elements that are the foci of Ball and her colleagues' analysis are not discourse structures in the sense that we have discussed previously. Rather, we believe that they are best thought of as constituent elements of a range of discourse structures. For example, "naming" might take place within an instance of accountable argumentation as well as during a peer dispute. In fact, juxtaposing Ball and coauthors' and Horn's analyses, we find explicit evidence of "making claims" (the second of Ball et al.'s discourse elements) across both of those activities. Specifically, in the accountable argumentation portion of Sean and Sheena's conversation, Sheena states, "I could show you it" and proceeds to use the number line to demonstrate the reasoning behind her claim that zero is an even number. Later, as their conversation moves into a peer dispute, we find Sheena continuing to make claims, as in her statement "But that doesn't mean it always is even." We suspect, in fact, that the discourse elements described by Ball and her colleagues might be found across a broad range of discourse structures.

Some other ontological features of Ball and coauthors' (chapter 1) analysis deserve mention here. In their analysis, Ball and her colleagues have chosen to reify—to treat as first-class theoretical entities—discourse *processes,* such as naming. But their analysis is populated, at least implicitly, with a variety of other kinds of entities; for example, with the process of naming are the names themselves. Also present are the particular *assertions* that students make.

Decisions about what we choose to treat as first-class theoretical entities will be significant as we attempt to move toward consensus. Ball and colleagues' (chapter 1) decision to foreground the processes rather than some of the entities that participate in those processes is potentially crucial. On the one hand, the foregrounding of processes might be precisely the appropriate move if we want to get a theoretical handle on complex classroom events. On the other hand, the reification of processes may contribute to an ontological vagueness that causes more difficulty than necessary in comparing analyses across research projects.

Posner (chapter 4) can be seen, at times, as using approaches similar to those of both Horn (chapter 3) and Ball and colleagues (chapter 1) to make sense of the classroom discourse. In particular, like Horn, she describes discourse structures; she explicitly contrasts conceptually based disagreements with interpersonal disagreements (Engle & Greeno, 1994). Rather than cite specific warrants that distinguish those two types of conversations, however, Posner relies on broad characterizations of differences in participants' position, motivation, and understanding of mathematics and of classroom norms.

Later in her article, Posner (chapter 4) focuses on the act of attributing names to mathematical ideas that arise in the classroom. Thus, the oddly even numbers become referred to as "Sean numbers," and the definition of even numbers as objects in which "two things make it ... without using halves" comes to be known as "Sheena's definition." This idea seems clearly related to Ball and others' (chapter 1) discussion of "naming" as a process that directs students' attention to specific objects, tools, and ways of learning and doing mathematics. The crucial aspect for this synthesis, however, is that both can be understood as constituent elements of the classroom discourse and are likely to be found within a range of discourse structures.

A SECOND ATTEMPT AT SYNTHESIS

Schoenfeld (chapter 2) takes an approach that is quite different from those in the other chapters in this volume. Rather than attempt to categorize and deconstruct kinds of discourse, Schoenfeld focuses on elucidating some of the mechanisms that *generate* particular patterns of interaction. His goal is to describe, at the level of mechanism, what a teacher does and why. In our search for a grand theory, we found that Schoenfeld's chapter contains, most explicitly, a *model* of a phenomenon. Because his focus and approach are somewhat different from those of the other articles, we discuss what would be involved in forging a synthesis in which we start from Schoenfeld's model.

Despite the fact that Schoenfeld's (chapter 2) model addresses teaching at a broad level, it would nonetheless need to be significantly expanded if it is to address the types of insights contained in the other articles in this monograph. To begin, note that Schoenfeld's model constitutes a strongly teacher-centered analysis of classroom events. It clearly sees the unfolding of classroom events as being largely determined by factors outside the teacher. In Schoenfeld's model, those factors are modeled as inputs to the flowchart at various points. Thus, Schoenfeld's model is consistent with the observation that extra-teacher factors are involved, but those factors are not themselves modeled. That omission is not necessarily a problem with Schoenfeld's approach; when we model, we must narrow our focus and simplify. But if we want to make contact with the other analyses in this volume, we need to push Schoenfeld's model outward to encompass those extra-teacher factors.

Specifically, the other three articles in this volume all take, as their unit of analysis, interacting units that are larger than the teacher. For example, Ball and her colleagues (chapter 1) consider how students respond in light of discourse norms established previously by the teacher. This focus is, in a sense, the reverse of Schoenfeld's approach to looking at how the teacher responds to students. Moreover, Horn (chapter 3) and Posner (chapter 4) look closely at how students respond to one another. That kind of interaction is not captured by Schoenfeld's current model. One productive place to consider expanding might be in the practice of evaluating claims. Ball and her colleagues present a three-phase structure

involved in the evaluation of claims—moving from clarifying a claim, to providing examples to support or refute a claim, and finally, to considering whether a claim is true in general. One way to expand Schoenfeld's model to encompass those insights might be to enrich the teacher model so that it is more explicit about some of the classroom contingencies on which the teacher's decisions are based. More elaborately, we could attempt to build a model of student behavior in those classroom discussions, patterned after the model for teachers, that interacts with the teacher model. The resulting interaction among a "teacher model" and a set of "student models" raises questions, however. For example, is the whole truly greater than the sum of its parts—would a characterization of each individual voice in the classroom capture the workings of the entire class?

Schoenfeld's (chapter 2) model also needs to be expanded in another respect as we attempt to create a synthesis across the chapters in this volume. So far, we have noted that we can expand our unit of analysis to more thoroughly model the unfolding of classroom events. But we can also imagine including, in our modeling efforts, more of the larger history of a classroom— the sort of unfolding that happens over days, weeks, and months rather than over the minutes of individual classroom events. That sort of expansion is likely needed if we want to begin to encompass the sort of phenomena that are central in the article by Posner (chapter 4).

Certainly, the nature of the endeavor we have mapped out here is potentially mammoth; it is equivalent to the full task of understanding teaching and learning. But if we want to fully reap the benefits that can be extracted from multiple analyses of the same classroom episode, then we believe we must at least attempt that sort of synthesis.

THE VALUE OF SHARED VIDEO

In the preceding sections of this commentary, we called into question the assumption that the study of teaching necessarily requires work from different perspectives. In particular, we argued that sharing models and theoretical frameworks is a crucial goal that should underpin any attempts at analysis of common data. However, this view does not mean that we believe that sharing video data is not useful; on the contrary, we believe that sharing video data can advance the work of our community in a number of important ways.

First, sharing our video data may be helpful even if other researchers do not pursue their own competing analyses. In most studies, other researchers have access to our "raw" data only through the transcripts that we embed in our published articles. But having access to the raw data at some level is absolutely necessary for other researchers to truly evaluate the claims that we make. To date, only a few studies have published video data alongside written research reports (Carraher & Nemirovsky, 2005; Sfard & McClain, 2002). This monograph is certainly noteworthy in that respect.

Second, shared video episodes are also valuable for the field because they provide common reference points for discussion, even if researchers do not provide full-fledged competing analyses of the video episodes. They can become part of the shared vocabulary of our disciplines, and they can become shared touchstones of the ideas we generate. Thus, aside from the potential of shared video data as the focus of research, we have benefited from being able to call on the widely known "Sean numbers" video in discussions with colleagues.

Third, at the highest end, we might want to work toward the collection of true shared video libraries that are the focus of analysis by many researchers. Certainly, quantitative research has benefited from the existence of shared national databases. But the creation of a shared video archive would present numerous difficulties. Matters of privacy and the protection of human subjects would need to be carefully considered. And whether such a video archive would prove useful is far from clear. The very richness of video can present difficulties in using the data without a great deal of supporting knowledge of the context in which those data were collected. In our own research, we have noted that an analysis of video data can be difficult when we have not been present at the time the data were collected.

Hiebert, Gallimore, and Stigler (2003) describe an instance in which they were using videos from the 1999 TIMSS project with a group of mathematics teachers at a professional development event. One of the teachers in the audience happened to have participated in the TIMSS project and had agreed to have her videos shared publicly. When the teacher was introduced to the group, the participants spontaneously applauded. As Hiebert and colleagues explain, "[T]hose assembled were not applauding the lesson Ms. Lancour had taught. They had not seen her lesson. They were applauding her courage in allowing others to view the lesson as a means of improving their own mathematics teaching" (p.56). Hiebert and his colleagues refer to that teacher as one of the "new heroes of teaching" for her willingness to share her teaching in that way. We similarly applaud Deborah Ball and thank her for the opportunity not only to enter her classroom but to study it.

We remember vividly our first viewing of the "Sean numbers" video at the NCTM conference in April, 1991. The room was packed with teachers, teacher educators, and researchers. At one point Ball asked us to focus on Ofala, and yet Miriam could not get her mind off Mei's comment that "if all numbers were odd and even, we wouldn't be even having this discussion" while Bruce kept thinking of questions *he* wanted to ask Sean and Sheena. We have viewed the video many times since then, at conferences and meetings, and, with Ball's permission, with teacher education students at Northwestern University. And each time, we continue to be captivated. The video prompts researchers, policy makers, and teachers to recognize the depth at which students can engage with mathematics and to want to understand how this outcome is possible. The chapters in this volume provide important perspectives on that question. By pushing forward from common data to common frameworks, we believe the field can move even closer to understanding the nature of teaching.

REFERENCES

Ball, D. L. (1990). *Sean numbers: Transcript from Deborah Ball's classroom, January 19, 1990.* East Lansing, MI: M.A.T.H. Project, College of Education, Michigan State University. Used with permission.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*(4), 373–397.

Ball, D. L. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan, 77,* 500–508.

Ball, D. L., Lewis, J., & Hoover, M.. (2007). Making mathematics work in school. In Alan H. Schoenfeld (Ed.), A study of teaching: Multiple lenses, multiple views, Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 13–44). Reston, VA: National Council of Teachers of Mathematics.

Carraher, D., & Nemirovsky, R. (Eds.) (2005). *Medium and meaning: Video papers in mathematics education research* (Neil A. Pateman, Series Ed.). Monograph no. 13 of the *Journal for Research in Mathematics Education*). Reston, VA: National Council of Teachers of Mathematics.

Engle, R. A., & Greeno, J. G. (1994) Managing disagreement in intellectual conversations: Coordinating interpersonal and conceptual concerns in the collaborative construction of mathematical explanations. In *Proceedings of the Sixteenth Annual Conference of the Cognitive Science Society.* Hillsdale, NJ: Erlbaum.

Hiebert, J., Gallimore, R., & Stigler, J. (2003). The new heroes of teaching. *Education Week, 23*(1), 56, 42. Retrieved December 13, 2005, from www.edweek/org/ew/ewstory.cfm?slug=10hiebert.h23

Horn, I. (2007). Accountable argumentation as a participation structure to support learning through disagreement. In Alan H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views,* Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 97–126). Reston, VA: National Council of Teachers of Mathematics.

Hufferd-Ackles, K., Fuson, K., & Sherin, M. G. (2004). Describing levels and components of a math-talk community. *Journal for Research in Mathematics Education, 35* (2), 81–116.

Lewis, J. (2007). "Through the Looking Glass": A study of teaching. In Alan H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views,* Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 1–12). Reston, VA: National Council of Teachers of Mathematics.

Posner, T. (2007). Looking at equity in mathematics classrooms. In Alan H. Schoenfeld (Ed.), A study of teaching: Multiple lenses, multiple views, Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 127–172). Reston, VA: National Council of Teachers of Mathematics.

Schoenfeld, A. H. (1998). Toward a theory of teaching in context. *Issues in Education, 4*(1), 1–94.

Schoenfeld, A. H. (1999). Models of the teaching process. *Journal of Mathematical Behavior, 18*(3), 243–261.

Schoenfeld, A. H. (2007). On modeling teachers' in-the-moment decision-making. In Alan H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views,* Monograph no. 14 of the *Journal for Research in Mathematics Education* (pp. 45–96). Reston, VA: National Council of Teachers of Mathematics.

Sfard, A., & McClain, K. (2002). Analyzing tools: Perspectives on the role of designed artifacts in mathematics learning [Special issue]. *Journal of the Learning Sciences, 11*(2 & 3), 153–161.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education, 5,* 205–233.

Silver, E. A., & Smith, M. S. (1996). Building discourse communities in mathematics classrooms: A worthwhile but challenging journey: In P. C. Elliott (Ed.), *Communication in mathematics, K–12 and beyond: 1996 yearbook* (pp. 20–28). Reston, VA: National Council of Teachers of Mathematics.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30,* 171–191.

Appendix 1

Transcript of Lesson Segment

Deborah Ball's Third-Grade Class
Friday, January 19, 1990
Spartan Village School, East Lansing, Michigan

Line no.	*Speaker*	*Transcript*	*Time*
1	Ball:	**[A]**[1] Okay. A few delays, but I think we're ready to start now. **[B]** I'd like to open the discussion today with um—I have a few questions about the meeting yesterday that I'd like to ask. **[C]** So, to begin with, I would just like everybody to put pens down, there's nothing to take notes about or do right now. **[D]** But I'd like you to be thinking back to yesterday and to the meeting that we had on even and odd numbers and zero. **[E]** And I have a few questions. First—my first question is, I'd just like to hear some comments about what you thought about the meeting, what you noticed about the meeting, what you learned at the meeting, just what kinds of comments you have about yesterday's meeting? **[F]** And could you listen to one another's comments, so that we can um, benefit from what other people say? **[G]** See what y— what you think about other people's comments? Sheena, do you want to start?	12:59:15
2	Sheena:	I—I—I liked it because, well, I like talking to other classes and, and when you talk to other classes sometimes it helps.	1:00:06

[1] For ease of reference in the narrative, long statements made by Ball are broken into segments labeled [A], [B], [C], ...