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8. INTEGRATING NOTICING INTO THE MODELING EQUATION

INTRODUCTION

Understanding teacher cognition in the moments of instruction has become increasingly important to the mathematics education community over the past two decades. Various reforms and policies make it clear that to support student learning, instruction must be based, at least in part, on the ideas that students raise in class, the reasoning that ensues, and the representations that are used (CCSSI, 2010; NCTM, 2000; NRC, 2001). Teaching, as a result, must be responsive; teachers must adapt their lessons as they unfold, often making decisions about how to proceed in the midst of instruction. But how does such in-the-moment decision- making occur? What are the cognitive processes involved as teachers carry out instruction?

One productive program of research that examines these questions is the teacher-modeling work conducted by Alan Schoenfeld (e.g., 2010). Schoenfeld asks "What is the teacher trying to achieve at the moment" (2010, p. 9), and "How did that become the teacher's goal for that moment?" Schoenfeld's research emphasizes that teachers' practices are comprised of established routines that are based in a teacher's goals, resources, and orientations. In addition, Schoenfeld explores how goals are activated and prioritized in situations that deviate from a teacher's expectations for a given lesson.

Our own work takes a different approach to studying teachers' in-the-moment actions. Specifically, we investigate the nature of teacher noticing. We ask "To what do teachers typically attend during instruction?" and "How do teachers decide where to pay attention during instruction?" (i.e., Sherin, Russ, Sherin, & Colestock, 2008). We focus on the dynamic relationship between a teacher's efforts to identify significant moments of instruction and the teacher's interpretation of those moments.

Though different, both programs contribute valuable information to the study of teachers' in-the-moment cognition. The goal of this chapter is to examine the relationship between these research programs. In particular, we consider what might be gained from integrating an explicit focus on teacher noticing into Schoenfeld's "modeling equation," that is, his procedure for unpacking the moment-to-moment actions of teachers. Similarly, we ask how Schoenfeld's advances in teacher modeling can enhance our own study of teacher noticing and our understanding of how teachers' attention guides their decision making.

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In what follows, we begin by reviewing Schoenfeld's approach to teacher modeling. We draw attention to the key components of the model and demonstrate his analytic methods with an episode from an eighth-grade classroom. Next we describe our research on teacher noticing and present two examples from our data set. We then apply Schoenfeld's constructs of goals, resources and orientations to these examples. In doing so, we illustrate how noticing serves as both a catalyst for and a product of teacher decision-making. To conclude, we reflect on how integrating noticing into models of teaching has altered our understanding of how noticing – and thus teaching – works.

MODELING THE TEACHING PROCESS

In the mid 1990s Schoenfeld turned his attention from modeling mathematical problem solving and tutoring to modeling the domain of teaching (e.g., Schoenfeld, 1998). In particular, he took up the task of making sense of teaching by modeling the moment-by-moment decision making of teachers. One of the central ideas of Schoenfeld's work is that if we understand a teacher's goals, resources, and orientations, then we can construct a coherent explanation of a teacher's actions during instruction. Doing so involves identifying established routines that a teacher relies on, routines that are based in those goals, resources, and orientations. Furthermore, when unforeseen events occur, Schoenfeld maintains that an analytic focus on goals, resources, and orientations allows us to make sense of a teacher's responses by considering how that teacher reprioritizes his or her goals based on existing resources and orientations. But what exactly does Schoenfeld mean by *goals*, *resources*, and *orientations*?

Key components of Schoenfeld's model

Goals

Broadly speaking, goals are the conscious or unconscious objectives that a teacher hopes to attain. According to Schoenfeld, teachers hold multiple goals at multiple grain sizes. At any given time, for instance, a teacher might hold an overarching goal, a content and/or social goal, as well as several local sub-goals. Furthermore, different goals may become activated (and deactivated) at different points throughout a lesson. For example, a teacher's overarching goal may remain in play throughout a lesson, while the local goals shift as the teacher moves the class through particular segments of the lesson. Goal prioritization is based on whatever the teacher considers to be most important at a given moment. In Schoenfeld's model, teachers make decisions that are consistent with their goals, and teachers draw on their resources to achieve them.

Resources

In using the term resources, Schoenfeld refers primarily to the cognitive resources, or knowledge, that an individual brings to a situation. Schoenfeld emphasizes that there are a range of types of knowledge that individuals possess. These include

procedural and conceptual knowledge, as well as knowledge of isolated facts and problem-solving strategies, all of which have the potential to influence the decision-making process. Furthermore, Schoenfeld considers knowledge to be associative. We come to recognize familiar situations and draw on established "knowledge packages" (2010. p. 27) to respond to such events. In addition, new connections among knowledge elements are established as we engage in new experiences. Along with cognitive resources, Schoenfeld notes that teachers may also draw on material and social resources during instruction. It is this collection of resources to which Schoenfeld refers, and which teachers draw on to achieve their goals.

Orientations

Schoenfeld uses the term orientations to incorporate the notions of disposition, belief, and value. He explains that, in particular, a teacher's attitudes towards teaching and learning shape how the teacher interacts and responds to students. Thus, for example, the beliefs a teacher holds concerning what it means to learn mathematics, how a classroom should be organized, and who (or what) should hold the place of authority in the classroom can play a key role in how resources are applied and which goals are activated at the moment. Moreover, a description of a teacher's orientation should specify the conditions under which a particular orientation is likely to be activated.

In his recent book, *How we think* (Schoenfeld, 2010), Schoenfeld details the modeling process through three examples of teaching. In doing so, he demonstrates that, through the lens of goals, resources and orientations, what at first glance might seem like random behavior on the part of the teacher, is instead behaviour that is quite coherent. The implications of this work are particularly noteworthy because he effectively models not just a single type of teaching, but a variety of types of teaching practices. Furthermore, based on his model, Schoenfeld makes suggestions concerning effective levers for productively influencing teacher practice.

A mini-example of Schoenfeld's modeling process: Crowd Estimation problem

Modeling a lesson involves first partitioning the lesson into segments that correspond to the main activities that took place in class. Next, each of these segments is decomposed into sub-segments that reflect a finer-grained parsing of the lesson activities. This iterative process continues until the entire lesson is decomposed into small segments of activity called "action sequences." As a result of this process of decomposition, a skeletal form of the model for a given lesson is produced. Each segment is delineated by an initial triggering event and a final terminating event. Furthermore, for each segment corresponding goals, orientations, and resources are identified to justify the teacher's actions in that segment. This process often leads to the discovery of patterns of goal activation and corresponding actions on the part of the teacher.

To illustrate this modeling process, we consider a 12-minute whole-class discussion from an eighth grade mathematics lesson taught by David Louis. While

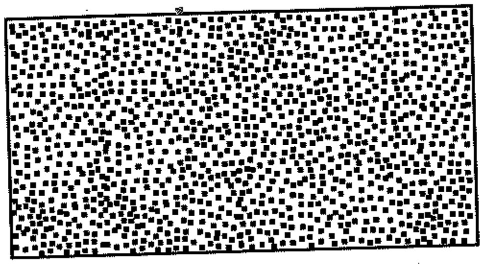

Figure 1. Estimate the population of the crowd shown in the picture.

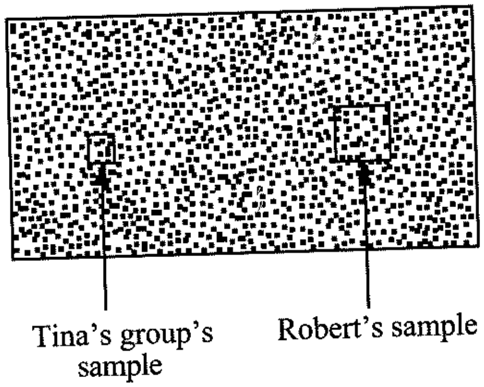

Figure 2. Two proposed solutions to the Crowd Estimation problem.

Schoenfeld typically models entire lessons, we use this mini-example as a way to give the reader a taste of the kind of modelling that Schoenfeld undertakes. (Our description of the following lesson draws on Russ, Sherin, and Sherin, 2011.)

The lesson comes from a unit on comparing and scaling (Lappan et al., 1997). Students were given a picture of a rectangle densely filled with dots (Figure 1) and told to imagine that the picture was an aerial photograph of a crowd with each dot representing a person. Working in small groups, students estimated how many people are in the photo. Tina's group shared their solution, explaining that they divided the original rectangle into 126 small squares, counted 17 dots in one of the small squares and estimated the total population by multiplying 17 by 126.

Mr. Louis then asked the class, "What do people think about this group's method?" Several students responded, including Robert who suggested using bigger squares to establish a more accurate estimate of the population. Robert explained that "with smaller squares there may be a bunch of dots packed into a small area. In just that particular area or something. Or there might have been not a lot of dots" (see Figure 2).

Mr. Louis turned to the class for comments: "What do you think about what Robert just said?" Some students voiced their agreement with Robert but Jeff suggested they find the average number of dots in 10 small squares. "It would have been better if instead of … one small square … they took ten squares from all random spots that were small size and divided the total of all the groups by 10."

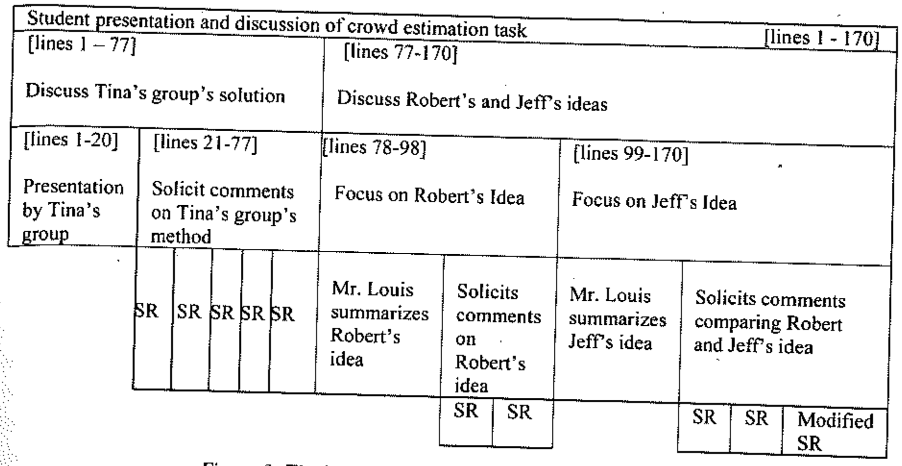

Figure 3. The iterative partitioning of a lesson segment.

After a few minutes, Mr. Louis drew the class' attention specifically to Robert's and Jeff's ideas. "We have two competing ideas here." He drew a diagram to illustrate the different approaches and encouraged the students to compare and contrast the two methods. "Which way do you think would produce the most accurate estimate of the population?"

As the class discussed Robert's and Jeff's methods, students raised a number of issues, including the role of averaging ("[For] a better estimate you have to have an average."), the context in which the sample was drawn ("Robert's methods would be better if … the big squares had the same number of dots each time") and the relationship between the samples ("Is Jeff's method just … making the square ten times larger?").

Parsing the lesson segment

This portion of the lesson can be partitioned into two main episodes: one in which the class explores Tina's group's solution and a second in which the class discusses two additional strategies, that of Robert and Jeff (Figure 3). The first episode can be further divided into two smaller episodes, Tina's group's initial presentation of their solution strategy, and then a whole-class discussion of the strategy. During this discussion, Mr. Louis uses a particular discourse routine in which he first solicits a student's idea, then asks another student to rephrase the idea, and finally asks for comments on the idea. This discourse routine is used five times during the episode as noted in Figure 3. (SR is used to refer to this "solicitation routine.")

In the second part of the episode, Mr. Louis explicitly focuses the class on strategies offered by Robert and by Jeff. For each strategy, Mr. Louis first summarizes the student's approach, and then asks members of the class to elaborate. In doing so, he again uses his familiar solicitation routine. In the final cycle of the solicitation routine, Mr. Louis modifies the routine somewhat, as he pursues a student's comment about sampling.

Resources, goals, and orientations

Mr. Louis' teaching during these episodes is guided by two overarching goals: to use students' ideas to structure lessons, where possible, and second, that students should comment substantively on each other's ideas. These goals are guided by Mr. Louis' orientation that learning mathematics should be a sense-making activity for students, and that talking about one's thinking and the thinking of others is a key component of an effective learning environment. Mr. Louis has strong pedagogical and subject matter knowledge. He often structures his lessons similarly – with a student presentation and discussion, followed by Mr. Louis choosing select methods for the class to discuss further (see Sherin, 2002 for more information on this approach). While Mr. Louis had not precisely anticipated the methods raised by Robert and Jeff, he was in familiar territory and recognized these two methods as central to the mathematical goals he wanted students to examine. Late in the discussion, when one student asked about a situation in which the two sampling methods might reveal different results, Mr. Louis modified his familiar discourse approach. Rather than asking students to respond the question, he provided an explanation of the issue that had been raised to the entire class. The question that Schoenfeld's modeling process answers is: What drives Mr. Louis' decision-making in this episode of instruction? For example, what goals and orientations does Mr. Louis' have that led to his decision to have students comment on Tina's solution? Or, what resources does Mr. Louis draw on when deciding to compare Robert and Jeff's idea?

In our work we are interested in a different, but related, set of questions. When we look at Mr. Louis' instruction in this episode we wonder not just about what drives the decisions he makes at any moment, but also what led him to interpret those moments of instruction as requiring a decision. Given a particular orientation and set of goals, the field of what a teacher might attend to is still fairly large. Our question then is why and how any particular moment stands out to the teacher.

The classroom is a complex environment, with many things happening simultaneously. A teacher cannot notice everything with equal weight; instead the teacher must choose where and to what to attend in the midst of this complexity. For this episode of Mr. Louis' teaching, we wonder how, amongst all those things that were happening, did Mr. Louis come to understand (perhaps tacitly) Tina's presentation as a "decision point" – a time to decide among various pedagogical moves? What did he "see" in that solution that led him to decide to have students comment extensively on it?

To answer these types of questions we investigate teacher noticing, that is, where and how teachers decide to focus their attention during instruction. A teacher might attend, for example, to the level of noise in the classroom, to students' solutions to a particular problem, or to how students respond to each other's questions. In the episode with Mr. Louis, we saw him attending to the particular solutions of his students. We can go further to say that we saw him noticing how students were making sense of the affordances of the different solution methods. We can imagine another teacher who might have noticed something else – perhaps the clarity of

the presentations or even the correctness of the student solutions. Had he noticed something different, Mr. Louis may not have made the pedagogical decisions that he did when he did.

TEACHER NOTICING

For the past 15 years, we have been engaged in a program of research designed to examine the nature of teacher noticing. We argue that teacher noticing is a key component of teaching expertise, particularly in the context of current mathematics education reforms. The idea that noticing is a component of expertise is not a new claim. Experts in diverse domains have been found to be able to recognize meaningful patterns in their areas of expertise. For example, chess experts are better able to identify layouts on a chess board than novice players (Chase & Simon, 1973). Of course, chess layouts consist of static pieces while the classroom represents a much more dynamic situation. Thus, it seems likely that the act of noticing during instruction is more complex than what has been studied previously. In addition, current reforms call for teaching that is responsive and flexible, in which teachers respond to student ideas as they arise during instruction. This approach towards teaching seems to rely strongly on teachers' in-the-moment noticing abilities.

While noticing is used in everyday language to indicate the general observations that a person makes, here we use the phrase *teacher noticing* to refer to the processes through which teachers manage the "blooming, buzzing confusion of sensory data" with which they are faced (Sherin & Star, 2011, p. 69). In particular, we understand noticing to involve two main processes: *attending to particular events in a classroom* and *making sense of those events* (Sherin, Jacobs, & Philipp, 2011). As stated above, teachers must decide what to pay attention to in the classroom, as well as what not to pay attention to. Furthermore, teachers are not just passive observers of those events to which they do attend. Instead, they interpret what they notice and therefore make sense of the situation in light of the ongoing lesson.

Examining the nature of teacher noticing has highlighted the consequential nature of noticing on teaching. A teacher can only respond to what he or she notices. Returning to the mini-example from Mr. Louis' class, a teacher who did not notice that Tina's method would provide a reasonable estimate presumably would not have decided at that moment to open the class to discussion about the affordances of her method. Thus, one aspect of our work considers how shifting a teacher's notice might serve as a catalyst for changing that teacher's instruction. We would therefore like a model of teaching that accounts for our intuitive sense that noticing impacts teachers' in the moment decision-making.

Studying teacher noticing

We have recently taken a novel approach to exploring teacher noticing. With the use of new digital technologies, we have asked *teachers* to identify moments of instruction that stand out to them as interesting, thereby capturing what teachers

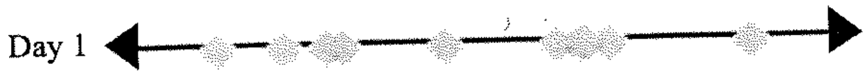

Figure 4. Temporal distribution of teacher-captured clips over one class period.

notice in the moment of instruction. Our methodology involves the use of a small wearable camera attached to a hat. The camera features selective archiving, which allows teachers to record 30 seconds of video immediately after the event has taken place. Thus teachers can capture an event to record immediately after it occurs. We have thus far given the camera to a range of high school math and science teachers and asked them over a period of several days to "capture what's interesting." We follow up with an interview of the teachers so that they can describe to us why they chose to capture each of the selected moments.

We have found this approach to be fairly effective. Teachers can successfully use the camera to capture moments and they seem to do so discerningly. We do not, for example, see teachers capture moments only at the beginning or end of lesson, or in regular intervals. This leads us to believe that teachers are tagging in a selective manner, much as we suspect their noticing operates. We think this indicates that they are somewhat aware of their noticing during teaching; this is not a wholly unconscious process. Furthermore, while some teachers seem to be on the lookout for certain kinds of information that they expect to tag, for other teachers, noticing is unplanned; they simply wait to see "what stands out" as a lesson proceeds (Sherin, Russ, & Colestock, 2011). Through this work, we have begun to characterize moments that stand out to teachers as interesting. We find that teachers notice a range of different kinds of issues in the classroom, some that relate to students, others to their teaching, to subject matter, to organization, and to school context.

Two examples of teacher noticing

To demonstrate the types of analyses we undertake in our study of teacher noticing, we elaborate with two examples from a high school algebra class. The teacher, Ray Bryant, was in his fifth year of teaching at an urban public high school in a large Midwestern city. Mr. Bryant used an integrated curriculum, covering topics from algebra, geometry, and statistics in his class. The school day was organized in a block schedule, with class periods of 90 minutes, meeting three times a week. In the class Mr. Bryant selected for this study, students were arranged in six groups of five. He typically organized instruction with students first working in their groups to prepare presentations on the previous nights' homework or in-class problems. Next a student from each group would present the group's solutions to the class. This was followed by a whole-class discussion of the problem, as well as the introduction of concepts and methods by Mr. Bryant.

Our standard analysis starts with looking at what the teacher notices overall during the course of the class. Figure 4 presents a timeline of the noticed moments for the day. As one can see, Mr. Bryant selected moments to capture throughout the lesson. These moments reflected many different topics of interest: of the 10 clips, 5

were related to issues of student thinking, 2 to student characteristics, 2 to climate, and 1 to pedagogy. In addition, we examined the types of classroom participant structures in which the clips were selected; and again a variety was found, with half taking place during whole-class activities and half during group work.

Example 1. Early in the lesson, Mr. Bryant was planning to move rather quickly through an example when a student asked a question about the absolute value problem the class was working on. Mr. Bryant chose to capture this moment with the camera. In his after-class interview about this moment, Mr. Bryant explained "[This] was one of those critical moments where … I had just planned on brushing right through that … but … I made a decision to stop and see where this was going to go because this one student obviously had something he wanted to share with the class." In terms of the focus of the teacher's attention, we would code this instance as taking place during whole-class work and being related to student mathematical thinking.

Example 2. Later in the lesson, students were working in groups while Mr. Bryant circulated around the room. When he approached Clarissa's group, Mr. Bryant's attention was drawn to the fact that the students were talking and doing calculations out loud, but they were not recording their joint work. He captured this moment with the camera and explained in the interview, "I [captured] that because I walked up to that group and they were clearly all discussing what was going on … [they] were talking about the [problem] … all five of them … but nobody's writing anything down." In terms of the focus of the teacher's attention, we would code this instance as taking place during group work and being related to classroom management.

APPLYING SCHOENFELD'S MODEL TO EPISODES OF NOTICING

Our approach to studying teacher noticing, while useful for providing an overall sense of what teachers pay attention to, provides somewhat limited information about what drives teacher noticing in a particular moment. We find that applying Schoenfeld's modeling process can add to our understanding of teachers' in-the-moment noticing. To illustrate, let us once again consider the moments Mr. Bryant captured, but now with the features of Schoenfeld's model in mind. We skip over the partitioning work, however, and treat each 30-second tagged moment as a single episode in his model.

Revisiting example 1

Schoenfeld's modeling approach provides us with tools to answer the question: What drives Mr. Bryant's instructional decision to stop and explore the student's question? When coupled with our attention to noticing, that question becomes: What drives Mr. Bryant's attention to moments in which stopping and exploring student questions is an appropriate instructional decision? An overarching goal of

Mr. Bryant's instruction is that students' ideas will drive the mathematical learning of the class and he structures his classroom with this in mind. Students regularly present their ideas in class and multiple solutions are typically welcomed. In this instance, Mr. Bryant did not anticipate the student's question but he had the resources (pedagogical content knowledge) that allowed him to understand that the students' question was a significant one. Thus, because of his knowledge of mathematics and his goal of student sense making, this is the kind of moment that will stand out to him as significant.

Revisiting example 2

As with example 1, Schoenfeld's work allows us to think about what goals, resources, and orientations Mr. Bryant might have that drive or constrain Mr. Bryant's noticing of students' failure to write down their ideas. Presumably, Mr. Bryant has several goals in mind for his students during class. One overarching key goal is for students to work together to learn mathematics. Mr. Bryant believes that students learn best when they are talking and working with peers, explaining and justifying their ideas to one another. This orientation and goal is evidenced in the way that Mr. Bryant has arranged his classroom and the extended class time he devotes to group work. Further, Mr. Bryant applies his knowledge of mathematics teaching in support of this goal. For example, he generally circulates during group work in order to advance students' thinking through questioning (Smith & Stein, 2011). When he approaches Clarissa's group however, a new goal is activated. He notices that students are engaged productively with the mathematics but he realizes that is not enough – they are not recording their ideas, and given the discussion he wants to have in class tomorrow, students will need a record of the work they have done so far. His overarching goal is therefore still in play, but a new local goal is prioritized by what he notices; having student record their thinking.

These examples highlight the way that Schoenfeld's model adds depth to our understanding of specific moments teachers captured as interesting. In particular, they tell us something about why the moment, or more generally this kind of moment, is likely to stand out to a specific teacher.

INTEGRATING NOTICING INTO SCHOENFELD'S MODEL

Thus far we have illustrated that we can learn more about what teachers notice, and particularly why they notice what they do, by drawing on Schoenfeld's modeling approach. At the same time, it seems reasonable to us that, given our assertion that noticing is a key component of teaching expertise, we should expect a model of teaching to account for teacher noticing. So where is the construct for noticing in Schoenfeld's model? How does it fit in with the existing model components?

We suggest that noticing is an important part of the teacher decision-making process that is currently implicit in Schoenfeld's model. For example, Schoenfeld writes about teachers behaving along the lines of implicit flow charts where if/then statements are asked (e.g., Does a student response require clarification? Do circumstances require further discussion?). These questions require teachers to notice in order to make decisions about an appropriate response. Similarly, he describes the case of a teacher having to decide whether a student's statement is in line with the teacher's agenda for the lesson. We propose that this kind of reflection necessarily involves the teacher noticing what the student said.

Furthermore, we maintain that the relationship between noticing and the model's existing components are bi-directional. On one hand, teacher noticing can be a product of the teacher's existing goals, orientations, and resources. Thus, the teacher's overall goals for student learning will influence what the teacher notices in the classroom. This is precisely what Schoenfeld's model illustrates if we think about "noticing" as a "decision" that teachers make. Going back to Mr. Bryant, the first example illustrates this relationship. It is because of Mr. Bryant's belief in the importance of students' ideas that the student's question captured his attention.

In addition, noticing can serve as a catalyst for cuing particular goals, orientations, or knowledge. We believe this was the case in the second example from Mr. Bryant. Noticing that his students were not writing down their work activated particular goals and knowledge for the teacher. It was likely in noticing that students were engaging in class in a particular way (not writing down their work), that Mr. Bryant came to realize that it was a goal he had for the students in class in that moment. To be clear, we are not equating noticing with a trigger or a triggering event. Instead, noticing is, to us, an awareness that allows events and ideas to "trigger" in the first place.

Revisiting the mini-example: Crowd Estimation problem

We have now used examples from our data to demonstrate how the construct of noticing can be integrated into Schoenfeld's modeling approach. However, that data was collected using a procedure specifically designed to tease out moments of teacher noticing. As such, it might be said, "Of course the idea of integrating noticing into the modeling approach makes sense for data about teacher noticing. But does it also make sense for the more traditional data of classroom instruction that Schoenfeld typically analyzes?" Obviously, we would like the answer to be yes.

To explore that question, we return to the small episode of Mr. Louis' instruction that we analyzed at the start of the chapter in order to show that supplementing Schoenfeld's modeling analysis with an explicit focus on noticing can help make sense of teacher-decision-making.

For Mr. Louis, his goal of using student ideas as the central mathematical content of the lesson kept him "on the look out" for potentially rich student thinking. Still, with this goal in mind there was quite a bit of student thinking during the discussion that could have been "noticed." To Mr. Louis, Robert and Jeff's ideas appeared as particularly consequential. Thus, while his decision-making is driven by his resources, goals, and orientations, noticing plays a central role as well. In addition, the fact that Mr. Louis noticed the affordances of the various solution methods may have caused a shift in his goals away from facilitating discussion among student ideas to a more teacher-led discussion of the different solutions. We do not suspect that Schoenfeld would disagree with our analysis in terms of noticing (in fact he acknowledges the role of teacher noticing in Schoenfeld, 2011). However, his analysis does not highlight what we consider to be an essential, dynamic part of the teacher decision-making process.

CONCLUSION

Modeling the complex phenomena of teaching and learning has long been a goal of education research. Scholars have attempted to develop models using constructs that give a balance of explanatory power and parsimony as well as intuitive appeal and novel insight. In this chapter, we have described and illustrated one of our field's predominant models of teaching – Schoenfeld's model of teacher decision-making during instruction. While we (and he!) believe this model highlights several important aspects of teaching expertise, we also raise the subject of what and how teachers notice during instruction influences – or interacts with – their decision-making process. In particular, we use several examples to suggest that teacher noticing can be understood as both a catalyst for and product of mathematics teachers' decision-making. In doing so we suggest how noticing might be productively and explicitly integrated into Schoenfeld's model of teaching.

Stepping back from the specifics of our examples, we might ask what this modeling exercise has bought us. Part of the value of developing models is that it not only allows us to better understand the model as a whole, but that it also gives us insight into the individual component constructs that make up the model. That is, knowing how the constructs interact with one another – how they fit together – gives us some information about the character and nature of the constructs themselves. In our case, articulating how teacher noticing could be integrated into Schoenfeld's model of teaching has highlighted for us what type of "thing" teacher noticing is.

Initially, we may have thought of the things teachers notice merely as "triggering events." Thinking of teacher noticing in that way leads us to ask questions such as: What events do teachers notice? What does the activity of noticing entail and how can someone get better at that activity? In this conceptualization of noticing, noticing is an activity that can be isolated, performed, and possibly even practiced.

However, as we began to integrate noticing into Schoenfeld's model, we realized that other conceptualizations of noticing were possible. Rather than understanding noticing as a localized activity, we began to see teacher noticing as a kind of heightened awareness that constantly underlies teacher practice. In Schoenfeld's model, noticing might then be one of the pre-existing conditions that gives rise to various decisions (one of the "ifs"), or it might be part of the background that dictates how likely particular rules within the model are to be cued. Such conceptualizations lead to questions such as: How conscious is this noticing awareness? If a teacher notices some aspect of classroom activity that is (in)consistent with his goals, how likely is he to be aware of it and decide to pursue it? When mismatches between knowledge and noticing happen, what takes priority?

We are just beginning to understand the implications of this shift in how we understand the nature of noticing. However, we are confident that the exercise of placing noticing within Schoenfeld's model of teaching will be a key step in moving forward with that understanding.

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ILANA SEIDEL HORN

9. TEACHING AS PROBLEM SOLVING

Collaborative Conversations as Found Talk-Aloud Protocols

INTRODUCTION

Alan Schoenfeld uncovered critical aspects of problem solving, identifying the way that learners use resources, heuristics, control, and beliefs to guide their activities around non-standard mathematical problems. In his groundbreaking research, he used talk-aloud protocols during problem solving sessions with undergraduates and audio recorded them to analyse their thinking. His investigation of students' talk and choices led him to develop his now well-known problem-solving framework (1985). As Schoenfeld's student, I share his deep curiosity in how people make sense of the world – only for me, the people were mathematics teachers and the problems were instructional.

Using ideas from ethnomethodology (Hymes, 1974; Garfinkle, 1967), I have spent the last 10 years analysing teachers' collaborative conversations, viewing them as naturally occurring talk-aloud protocols. From this perspective, I examine teachers' problem solving by looking at how they identify and articulate challenges in their work, as well as how they make progress on understanding these problems of teaching.

While Schoenfeld posed problems to his study participants, the teaching problems I examine emerge during interactions. In this way, I look at how teachers formulate as well as solve problems during collective work. This broader view necessitates an analysis of how teachers' knowledge and understandings of their work contribute to problem formulation and modelling as they represent, diagnose, and pursue problems of practice through their conversations.

In this chapter, I illustrate some key findings of my research on teachers' collaborative talk, demonstrating the places where "found" problem solving episodes corroborate and extend Schoenfeld's framework for mathematical problem solving. Like Schoenfeld, I find differences in how participants' beliefs, resources, and strategies influence their progress. Because I begin my analyses at the level of problem formulation, my work highlights the socially negotiated nature of problem solving. By articulating to and extending Schoenfeld's framework, this chapter contributes to a more general framework of human problem solving.

WHY STUDY TEACHERS' COLLABORATIVE CONVERSATIONS?

In the United States, teaching is an isolated profession. Teachers tend to work in their classrooms with little collegial interaction. Typically, other adults in the school only visit to evaluate performance. Even then, such visits are infrequent.

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