THE SMALL WORLD PROPERTY IN QUANTUM NETWORKS

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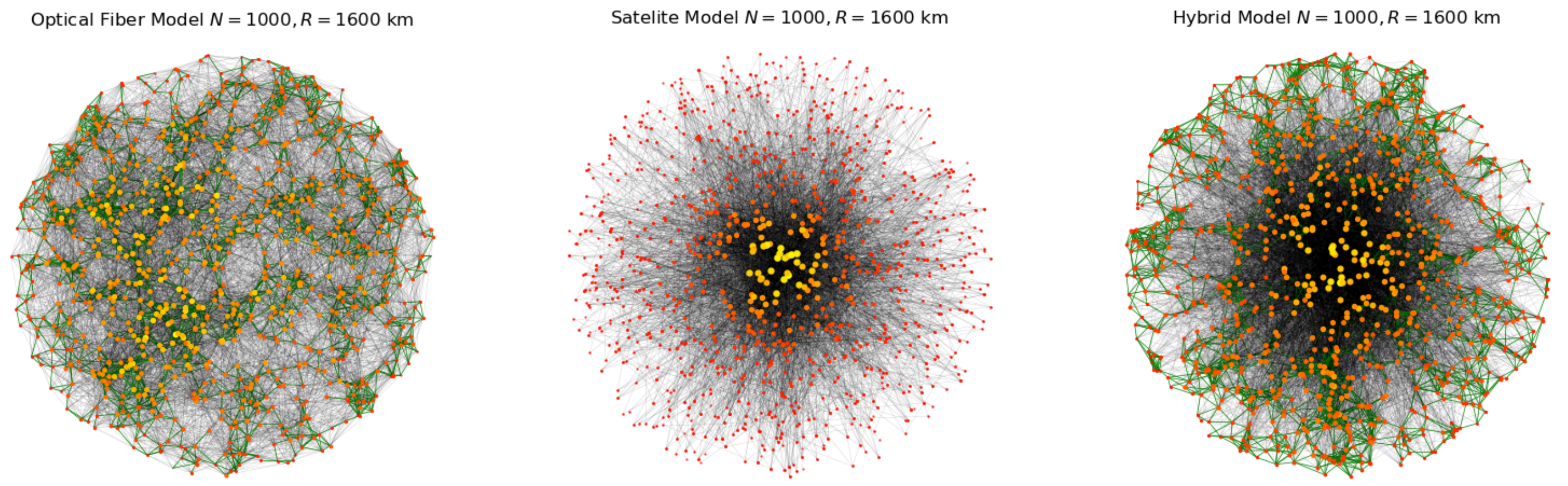
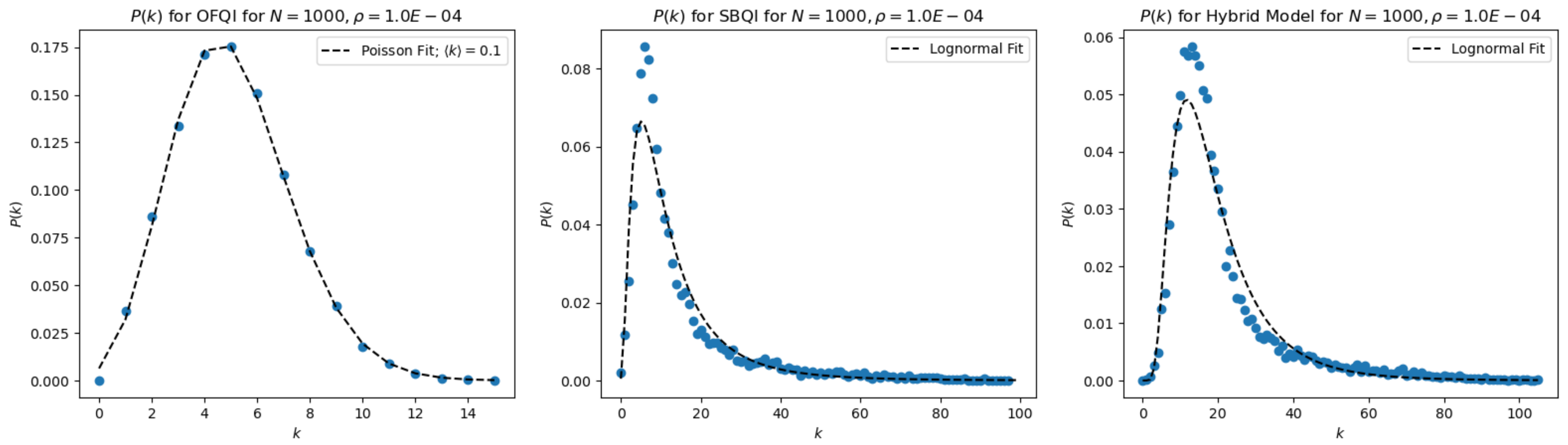
Abstract

Given the recent experimental strides being made towards long-distance quantum communication [1] and the many proposed models for the network structure of a quantum internet, we seek to evaluate the efficacy of two network models keeping in mind the inherent destruction of links that comes with entanglement swapping. Taking this constraint into account we come up with a more meaningful definition of a small-world network and evaluate it on the theoretical quantum internet models outlined in [2] and [3]

1. Introduction

Following the models outlined in [2] and [3] we construct the network as N uniformly distributed nodes on a disk of radius R. The visualizations of the different models alongside their degree distributions are shown below for the optical fiber quantum internet (OFQI), the satellite-based quantum internet (SBQI), and the hybrid model combining the two. Note that the OFQI behaves like an Erdős–Rényi network with a Poissonian degree distribution, lacking hubs, while the SBQI has a log-normal degree distribution with a larger presence of hubs towards the center.



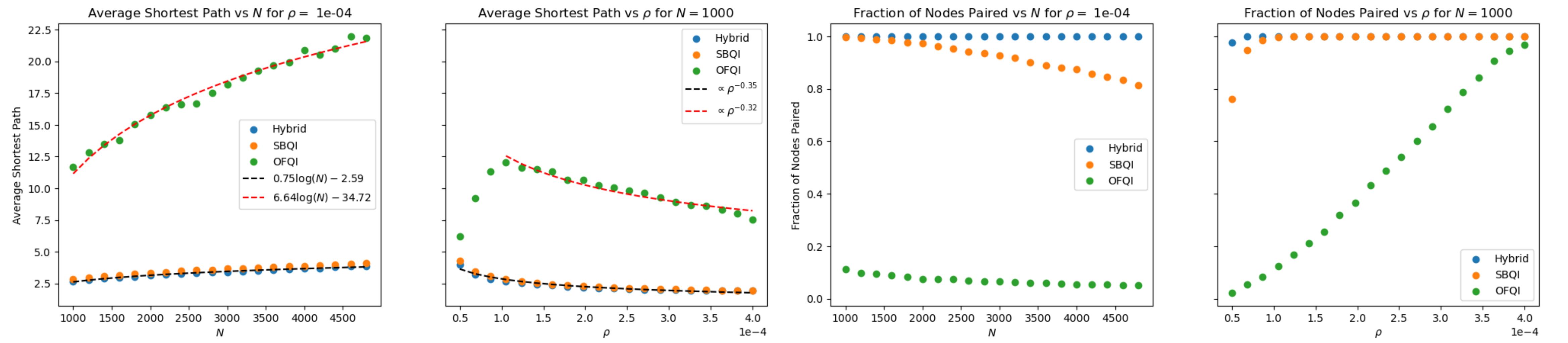


To connect any pair of nodes (A, B) in the network, entanglement swapping must be performed at all of the nodes in the path from A to B. In doing so all of the edges in that path are destroyed. Therefore, the shortest path between two nodes that need to be paired will depend heavily on what order the nodes are paired in. It is clear we need a way to probe the network structure to give us information about the shortest paths and the possibility of pairing nodes.

2. A modified definition of the shortest path

Probe the structure of each of these networks by pairing randomly assigned nodes and noting the path length according to the following procedure: Given a graph $G = (V, E)$ with $|V| = N$ We randomly assign a set of $N/2$ pairs P.
For every $(A, B) \in P$ we find the shortest path between A and B, record its length and remove the edges of that path from the graph. We keep doing this until every node is paired, or it is no longer possible to pair nodes. We record the average shortest path distance as well as the number of nodes that were paired.
We note that if the network is small-world but not robust to edge removal, the number of nodes paired will drop indicating a poor result. If the network is robust to edge removal but not small-world, the average shortest path would not scale well with N. Therefore it is necessary to look at both quantities

3. Results



4. Analysis

We note that the presence of hubs helps both the SBQI and Hybrid models as we see the shortest path scales slower than $\log(N)$ for a fixed $\rho = \frac{N}{\pi R^2}$. We observe that the OFQI grows faster than $\log(N)$ and furthermore seems to be able to pair its nodes past some critical density, this is corroborated by [2] as well as the green curve in the fraction of nodes paired vs density graph. We see that the multilayered hybrid model wins overall in terms of nodes paired and its small-world property. It is clear that in the case of the SBQI and OFQI alone increasing N for a fixed density leads to a sparser and more disconnected network.

5. Conclusion

When it comes to efficient network structure for quantum communication it is clear that quantum networks need to be small-world while also being robust to edge removal. In the models outlined in [2] and [3] we see that the latter model is clearly better in this regard due to the presence of hubs. However, the best model seems to be a multilayered hybrid network, combining both models discussed in [2] and [3] we see a network with a prevailing small-world property as well as more robustness when it comes to edge removal.

References

[1] Ji-Gang Ren et al. "Ground-to-satellite quantum teleportation". In: *Nature* 549.7670 (Aug. 2017), pp. 70–73. ISSN: 1476-4687. DOI: `10.1038/nature23675`. URL: `http://dx.doi.org/10.1038/nature23675`.

[2] Samuraí Brito et al. "Statistical Properties of the Quantum Internet". In: *Phys. Rev. Lett.* 124 (21 May 2020), p. 210501. DOI: `10.1103/PhysRevLett.124.210501`. URL: `https://link.aps.org/doi/10.1103/PhysRevLett.124.210501`.

[3] Samuraí Brito et al. "Satellite-Based Photonic Quantum Networks Are Small-World". In: *PRX Quantum* 2 (1 Jan. 2021), p. 010304. DOI: `10.1103/PRXQuantum.2.010304`. URL: `https://link.aps.org/doi/10.1103/PRXQuantum.2.010304`.