# PHYSICS 465 Project: Superconditing Transmon Qubit System


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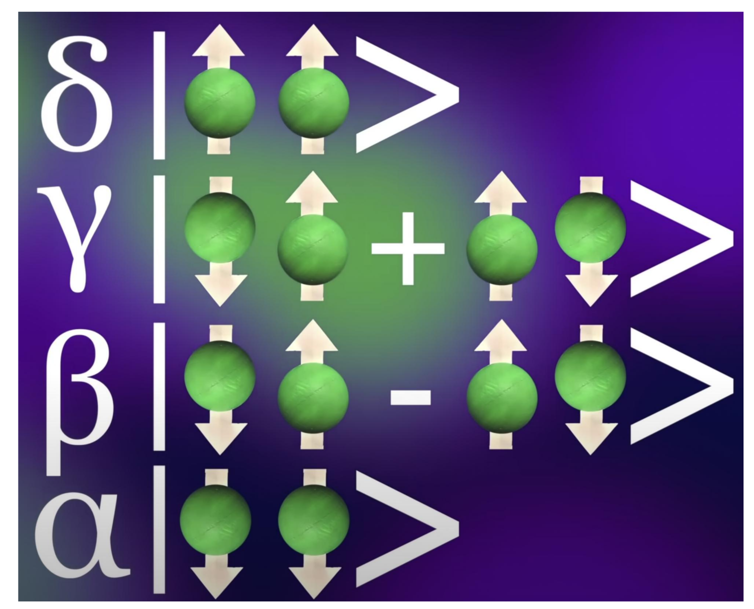
**Overview: This project focuses on the idea of Network of Quantum States Transitions. A generalized three-transmon-qubit system programming stimulation and its network visualization is created with discussions of potential useful network analysis and modifications.**


## Introduction



Qubits are the fundamental building blocks for quantum computing. It can be viewed as a quantum opposition of a traditional binary bit that is physically realized with a two-state device, such as the spin of an electron, where the two levels may be interpreted as spin up and spin down. To understand how qubit is different from classical bits, we can consider a simple example of a system with 2 bits, each with either |0⟩ for spin-down and |1⟩ for spin-up. If the bits are classical, we would have 4 possibilities, which respectively are, |0⟩|0⟩, |0⟩|1⟩, |1⟩|0⟩, |1⟩|1⟩. And this classical system would contain 2 pieces of information, which represent the value of each bit. However, in the context of quantum mechanics and qubits, we would need 4 pieces of information to describe the system as it exists in the superposition of all 4 possibles states, consequently requiring 4 complex coefficients (α, β, γ, δ) as shown on the figure to the right. With further derivation, we can conclude that a system with N qubits contains the superpositions of 2^N different states. In this project, I created a simulation of a three-qubit system, thus having 8 states in my system.

Transmon qubit is a type of superconducting charge qubit. In a two-level transmon qubit, where the ground state |0⟩ and the excited state |1⟩ form the basic states of the qubit, the states correspond to different energy levels of the system. The qubit can be manipulated by applying external control fields, such as microwave pulses, to induce transitions between the ground and excited states. Transmon qubits operate in a regime where it transitions smoothly between the charge and phase degrees of freedom of a superconducting circuit. The distinguishing feature of a transmon qubit is its design optimized for long coherence times. It is typically achieved by introducing a large shunting capacitance to reduce the sensitivity of the qubit to charge noise. This enhanced coherence makes the transmon qubit a promising candidate for various quantum computing and quantum information processing applications. In this project, I programmed a three-transmon qubit system simulation and created a network of the system.

## Methods

In this project, I used Python networkX and cytpscape to construct and visualize the network, with the help of Qiskit and IBM Quantum Lab.

**Step 1: Construct Hamiltonian of the transmon qubits:**

Define Hamiltonian parameters: Ec>>Ej for transmons (They are set as arbituary constants for simplication purposes.)

Charging Energy (Ec): the energy required to add or remove a a pair of electrons (Cooper pair)to or from the qubit island

Josephson Energy (Ej): represents the energy associated with the tunneling of Cooper pairs across the Josephson junction.

- Modeling transmon qubit Cooper-pair box Hamiltonian in the charge basis is given by $\hat{H} = 4E_C\left(\hat{n} - n_g\right) - E_J\cos\left(\hat{\phi}\right)$

where n̂: he number of Cooper pairs transferred, ϕ: the gauge-invariant phase difference between islands, $n_g$: effective offset charge of the device. The charging energy Ec, Josephson energy Ej, and offset charge can be expressed as follows:

$E_C = \frac{e^2}{2C_\Sigma}, \quad n_g = -\frac{C_d\dot{\Phi}_s(t)}{2e}, \quad E_J = \frac{\phi_0^2}{L_J}$, where Lj is the inductance of Josephson Junction and φ is the magnetic flux.

**Step 2: Solve the time-depentent Schrödinger Equation** using the constructed Hamiltonian $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

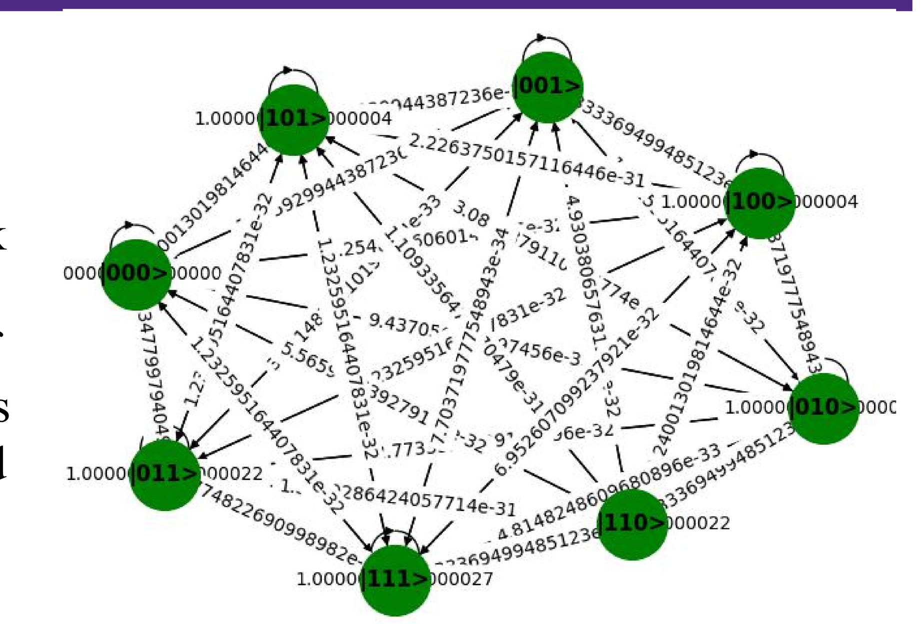
**Step 3: Calculate Transition Probabilities:** Analyze the time-dependent wavefunction derived from the previous step.

**Step 4: Perform Numerical Simulations** if analytical solutions to the Schrödinger equation if it isn't be readily available.*

**Step 5: Data visualization:** Use the theoretical calculations and simulations to construct and visualize them in the form of networks. In my three-transmon-qubit system, there will be 8 Nodes that represents the 8 superposition states, the links stands for the transition between the states with the weights of the links attributes to the probability of the transition.

Step 6: Network analysis: Peform different analysis of the network, including but not limited to, degree of nodes, clustering coefficient, community structures and resilience to perturbations for exploration of the properties and dynamics of the quantum state transition network.

## Discussion



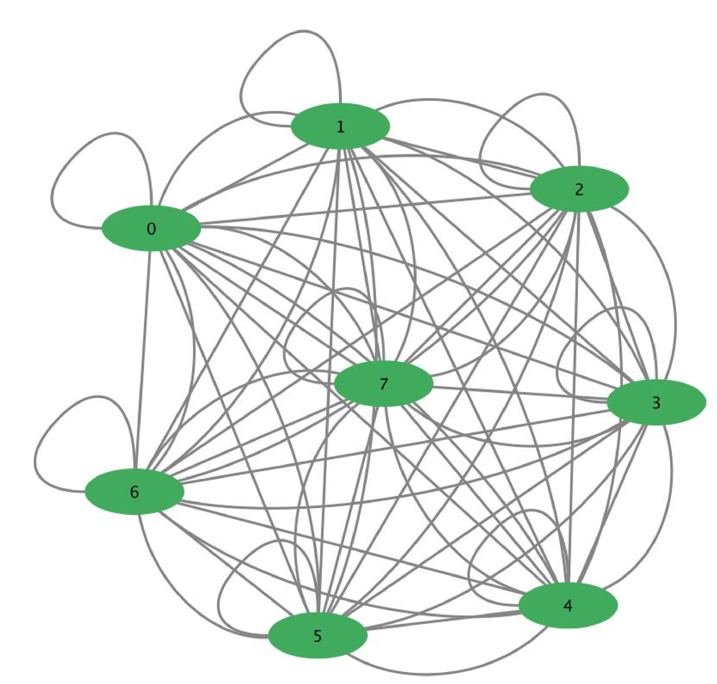
Due to the limited course and experimental experience on the topic of quantum computing as an undergraduate student, I do acknowledge that this project has certain generalization aspects as I don't have any specific transmon qubit system. With the parameters and conditions simplified as arbitrary constants, my network generated by NetworkX (shown on the right) cannot indicate any scientifically useful numerical values or conclusions. (I also provide a more concise version of the network visualization by Cytoscape below. ) However, I will discuss how this fundamental model can be modified and the potential network analysis that could be useful when applied to real-life systems.

The main modification can come from the following two parts:

**1. Hamiltonian:** Real-life qubit system can have a more complicated Hamiltonian depending on the design of the system. Main influencing factors includs: Qubit properties, Josephson junction properties, External magnetic field (Zeeman spliting and other controlled parameters.

**2. Anharmonicity** (nonlinearity of the energy levels): in this project, it is assumed that the energy levels are all equally spaced. Anharmonicity can be changed by critical current of Josephson junction and other properties like Qubit Capacitance.

## Network Analysis



**1. Degree:** The degree of a node in the network represents the number of edges (transitions) connected to that node, signifying the number of possible transitions that can occur from that energy level to other energy levels. Nodes with higher degrees indicate more interconnectedness of the state and potentially more accessible transitions.

**2. Clustering Coefficient:** The clustering coefficient measures the degree to which nodes in a network tend to cluster together. In the context of a transmon qubit system, the clustering coefficient can provide information about the likelihood of multiple transitions occurring among a group of energy levels. A higher clustering coefficient suggests that transitions between energy levels are more interconnected, indicating potential multi-qubit interactions or coherence between states.

**3. Community Structure:** Community detection algorithms can identify groups or clusters of energy levels that exhibit a higher density of transitions within the group compared to outside, revealing substructures or modules within the transmon qubit system, indicating potentially correlated or coherent sets of energy levels. Communities can represent qubit subsystems or groups of energy levels that interact more strongly with each other.

**4. Resilience to Perturbations:** Analyzing the resilience of the network to perturbations can provide insights into the robustness of the transmon qubit system. This can involve measuring how the network properties change when certain nodes or transitions are removed or perturbed. Understanding the system's resilience can help identify critical energy levels or transitions that are crucial for maintaining coherence and stability, which can be very useful for further modifications and adjustments of the qubit design.

By examining these network properties, we can gain a deeper understanding of the dynamics, connectivity, and interplay between different energy levels in the transmon qubit system. These properties can provide insights into the system's behavior, coherence, and potential applications in quantum information processing tasks.

## References & Acknowledgements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2. "A field guide: Introduction to Qubits, and Creating Superpositions and Quantum Interference." IBM Quantum. https://quantum-computing.ibm.com/composer/docs/iqx/guide/creating-superpositions.