Leviathan: An Agent-based Model of Primitive Society

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I. Hobbes' Leviathan

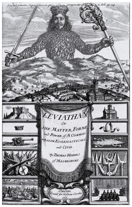
Thomas Hobbes (1588–1679), the author of the book Leviathan, is a philosopher and royalist.

Leviathan's proposal:

- the natural basic state of humankind is one of anarchy, with the strong dominating the weak.
- In order to remove that basic fear between individuals or groups, people should 'contract' with a protector as their sovereign. Under this social contract, individuals give up all rights, while those of the protector are absolute.

Our goal

- Simulate the Leviathan society with the basic assumption: People in the society may either conflict or support others.
- Study the structure and robustness of the system



II. Evolutionary Game Theory (EGT) [1]

EGT is a game theory but constantly evolving: Multiple agents are engaged in interactions with potential conflicts, and selective pressures affect the evolution of the strategies of them. Such dynamics can be modeled numerically by a finite group of individual carrying its own genes that determine its decisions. [2]

Example: Rock-Paper-Scissors (RPS) with mixed strategy. In a RPS game, an agent chooses one of the actions in $(Rock, Paper, Scissors)$ according to a probability distribution (p_R, p_P, p_S), or in another word, the gene. Such decision making strategy based on a probability distribution is called mixed strategy. Different agents have different genes. After two agents' interaction, the payoff of one player can be found in a matrix, whose rows and columns are labelled by the player and their opponent's action:

$$\begin{array}{c} \\ Rock \\ Paper \\ Scissors \end{array} \begin{array}{c} \begin{array}{ccc} Rock & Paper & Scissors \end{array} \\ \begin{bmatrix} 0 & -1 & x \\ x & 0 & -1 \\ -1 & x & 0 \end{bmatrix} \end{array}$$

Now we evolve the system with selective pressures – agents will be randomly eliminated and replaced with the "clones" of the agents with the largest payoff. Figure 2 shows the average strategy made by agents in each iterations.

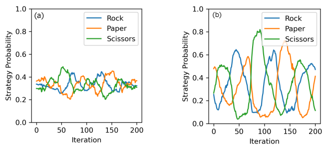


Figure 1: Average strategy among the population. (a) With some parameters ($x = 2$), Evolutionary Stable Strategy (ESS) exists: It's a move (or play) that would assure the type of agents that wields it an evolutionary (that is, Darwinian) advantage over the opponent. (b) With different system parameters, the system exhibits different dynamical phases. By setting $x = 0.5$ in the payoff matrix, the ESS is no longer stable and the dynamics becomes oscillatory.

III. Models and Results

In Leviathan, an agent has two decisions when involved in the competition over finite amount of resource ($2r$): Hawk and Dove [3]. Hawk (labelled by 0) means the one fighting for food and Dove (labelled by 1) means one trying to avoid conflict and cooperate. The payoff matrix is given by

$$\begin{array}{c} \\ Hawk\ (0) \\ Dove\ (1) \end{array} \begin{array}{c} \begin{array}{cc} Hawk\ (0) & Dove\ (1) \end{array} \\ \begin{bmatrix} r - c & 2r \\ 0 & r \end{bmatrix} \end{array}$$

Here we choose to study the case when $r = c = 1$.

Our goal is to look for dynamics that can't be described by a simple ESS. As people play different roles in the society, various strategies must exist at the same time. In the network perspective, we are also looking for scale free properties, hubs (protectors) and communities (sources of survival pressure). We'll present three models, from simple to complex, to build a Leviathan.

1. Agents with Mixed Strategy

The study of Leviathan starts from the simplest case: the agents make decisions out of $(Hawk, Dove)$ randomly, according to their only gene $(p_0, p_1 \equiv 1 - p_0)$. After a few iterations, we plot the average strategy versus iterations. We also try to construct the collaboration network – link represents two agents both play Dove in their last interaction [4]. See figure 2

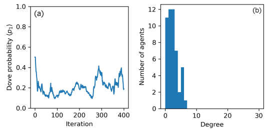


Figure 2: (a) Average strategy $\langle 0p_0 + 1p_1 \rangle = \langle p_1 \rangle$. In this model, the system reaches ESS in the first few iterations. Later the average strategy fluctuate around ESS. (b) A typical degree distribution of the collaboration network when the system reaches ESS. The collaboration network is constructed by adding links when two agents both play Dove in their last interaction. The figure indicates the random nature of the model and the non-existence of hub.

Such a model is not able to realize a structured society as the network is clearly random. Besides, the average cluster coefficient is as low as 0.05. The result also explain the intuition: given no any external information, agents are not able to form a stable structure.

2. Agents with Conditional Mixed Strategy

Now we gradually add complexity in our model and see what'll happen. In this model, agents have the **temporal** memory of their last interaction with others. For each pair of agents, A and B, there are 5 possible interaction history:

$$(0,0), (0,1), (1,0), (1,1), (None, None),$$

where (m, n) means agent A (B) plays m (n), and $(None, None)$ means they never met before. Their decision will condition on those 5 situations, which can be described by 5 genes:

$$p(0|(m,n)), \quad p(1|(m,n))$$

We study the average strategy as the system evolves and construct a network – link represents two agents both play Dove in their **last three interactions**.

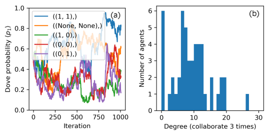
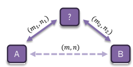


Figure 3: (a) Average strategy. In this model, the system does not reach ESS and oscillates between several states. (b) A typical degree distribution of the collaboration network. Though we can't determine whether the degree distribution is under a power law or a Poisson law, compared to last model, agents has developed a more stable and cooperative relationship to survive in this competitive environment. Hubs are seen in this model.

With various initial condition and parameters, we don't see an ESS, which may or may not be due to the limited population and iteration number. But stable cooperative relationships are observed with average cluster coefficient ~0.3.

3. Agent in a Network

In this model, agents not only know their interaction history, but also are aware of the **spatial** network structure near them. On the right, we consider a triangular subgraph in the network, agent A and B have a shared neighbor. According to the chain rule, we feed the effective "special" interaction information to A and B for decision use. Now those agents' decision will condition on in total 25 situations, controlled by $5\ \text{temporal} \times 5\ \text{spacial}$ genes.



$$m = m_1 + m_2 + 1 \bmod 2$$
$$n = n_1 + n_2 + 1 \bmod 2$$

However, it turns out that the extra complication makes convergence extremely difficult. Though mutation rate can be manually reduced, but we can't draw concrete conclusion from such a chaotic state.

IV. Reference

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[2] C. Adami, J. Schossau and A. Hintzed, "Evolutionary game theory using agent-based methods," *Physics of Life Reviews*, vol. 19, no. 1, 2016.

[3] J. Maynard-Smith and G. R. Price, "The Logic of Animal Conflict," *Nature*, vol. 246 , no. 5427, pp. 15-18, 1978.

[4] A.-L. Barabási, Network Science, 2023.