

What to do if my quantum system is leaking?



Andy Li



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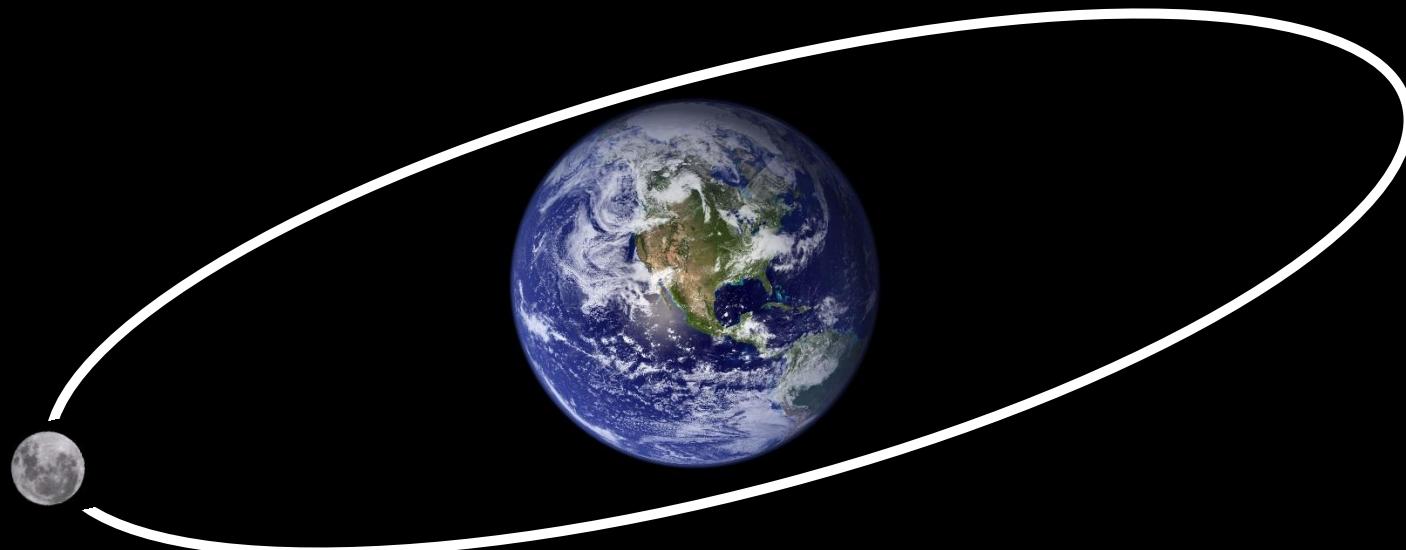
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(ETH Zurich)

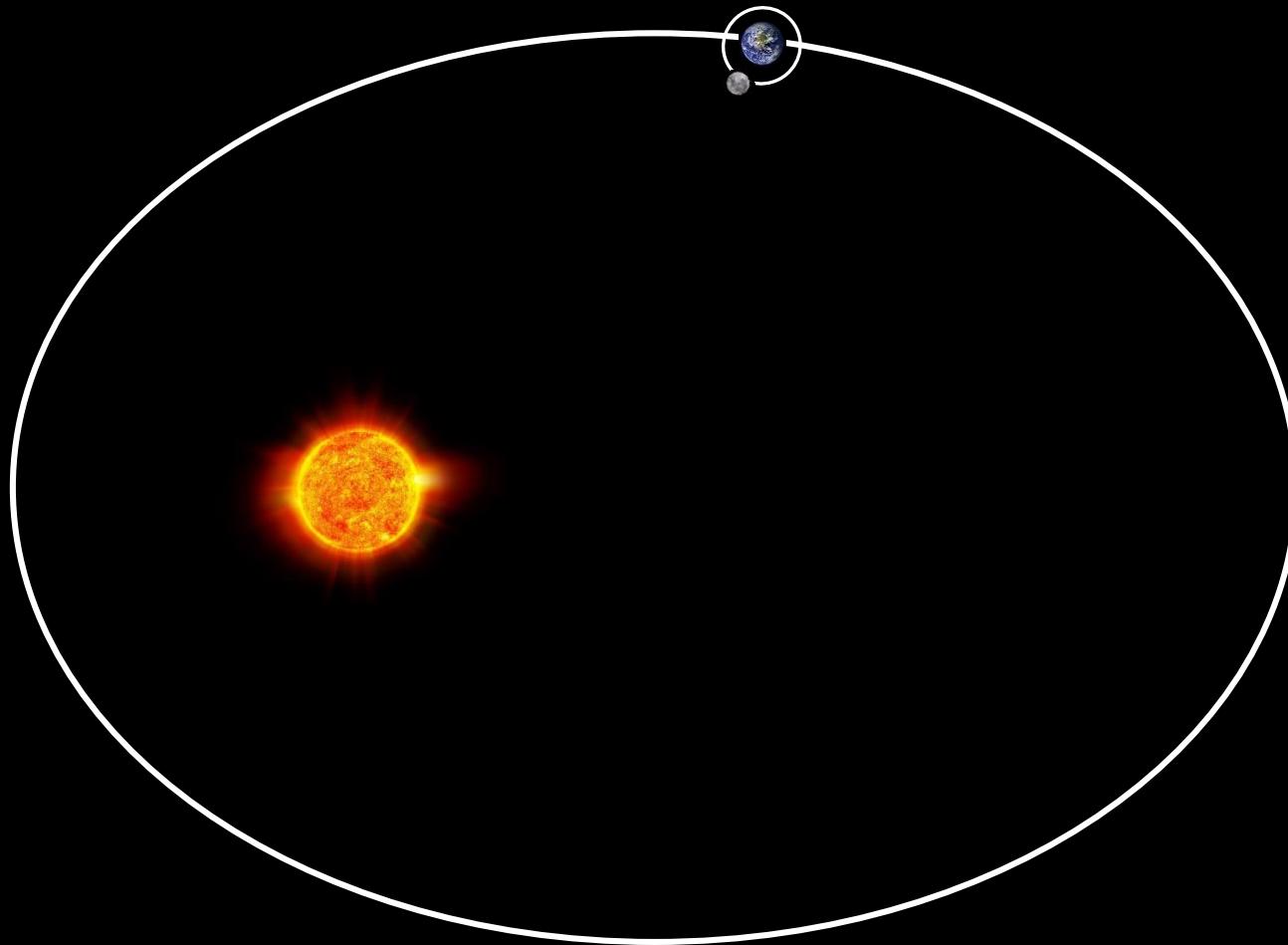


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UNIVERSITY

Closed system



Closed system



Closed system

Definition:

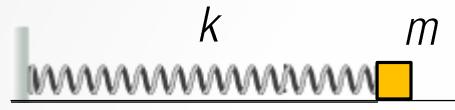
Can treat system as if it was “alone in the world”.

Closedness is an approximation

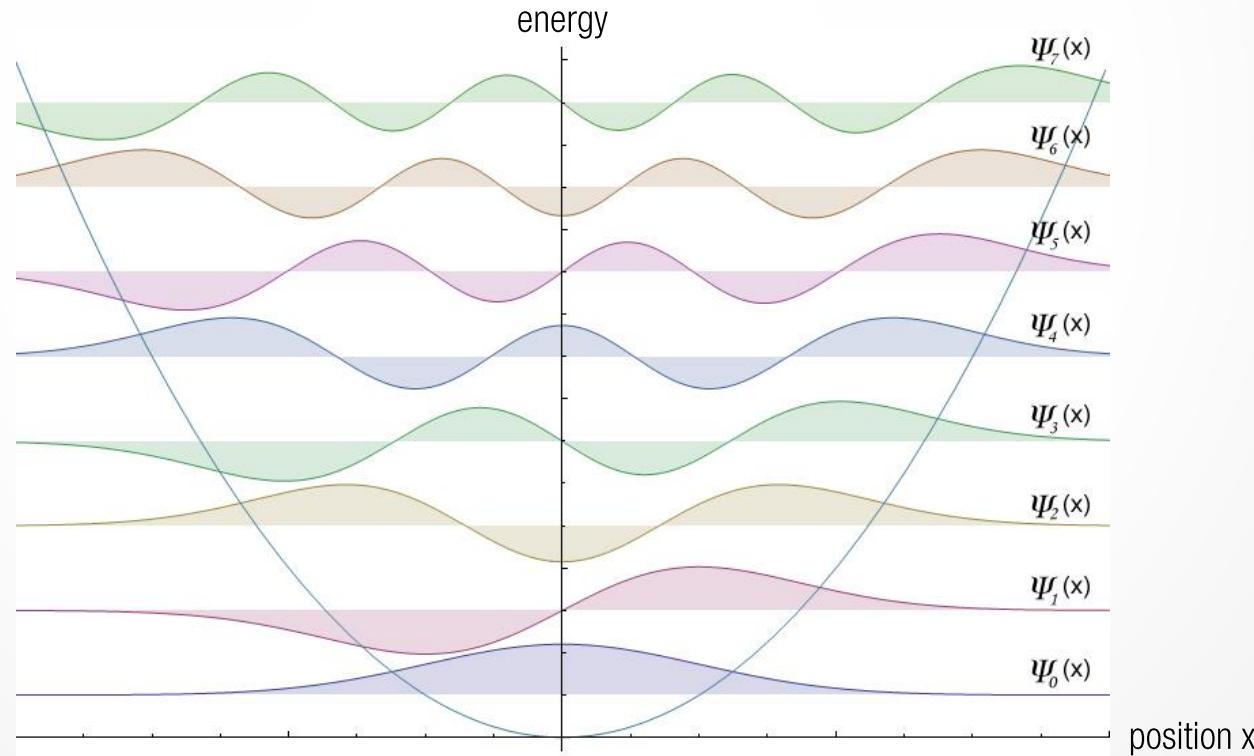
(except, perhaps, for the universe itself)

Closedness is convenient!

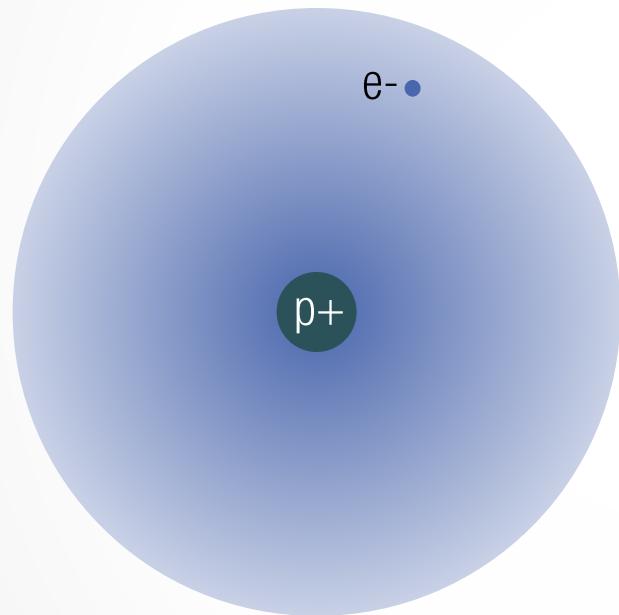
Closed quantum system



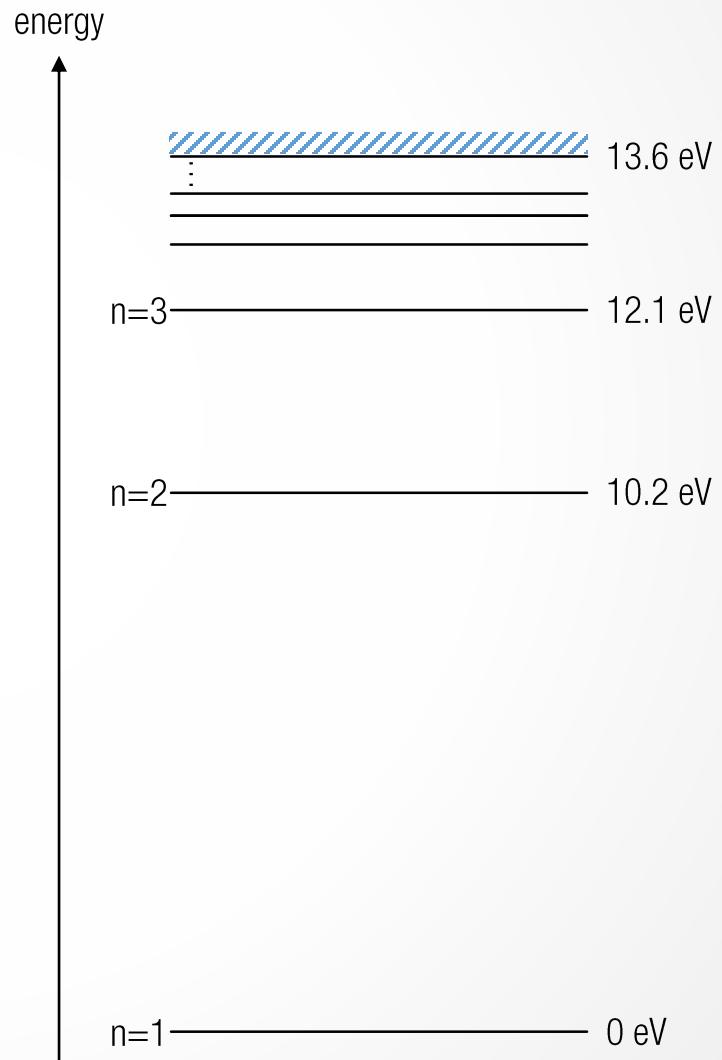
$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2}kx^2$$



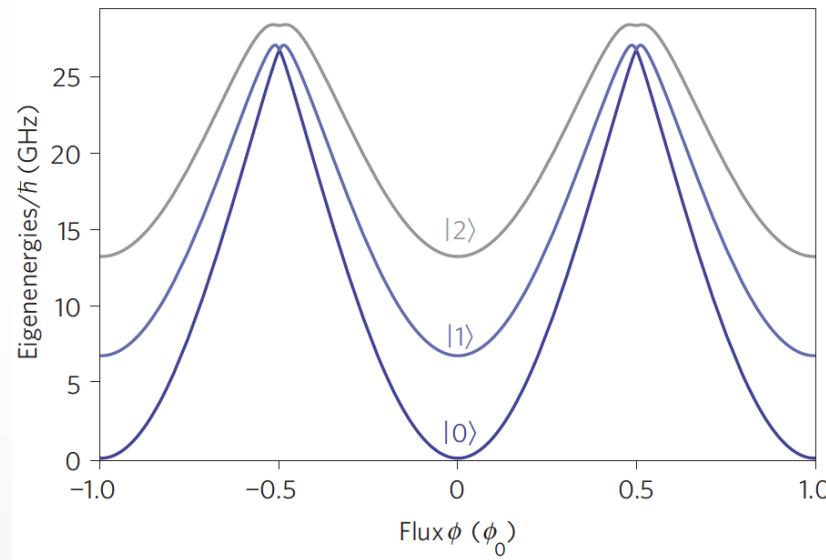
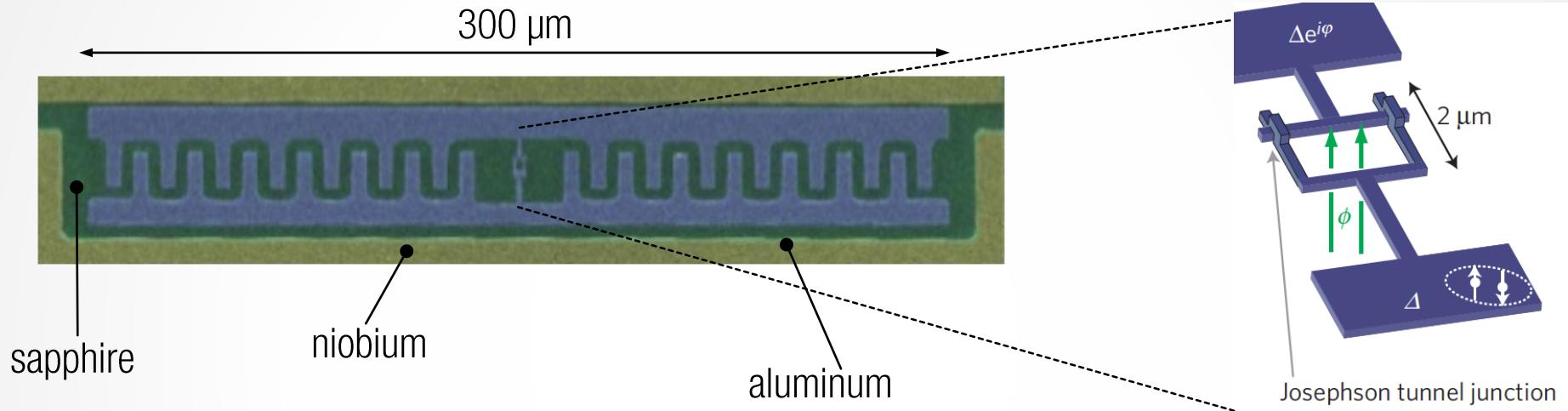
Closed quantum system



$$H = -\frac{\hbar^2}{2\mu} \nabla^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$



Closed quantum system

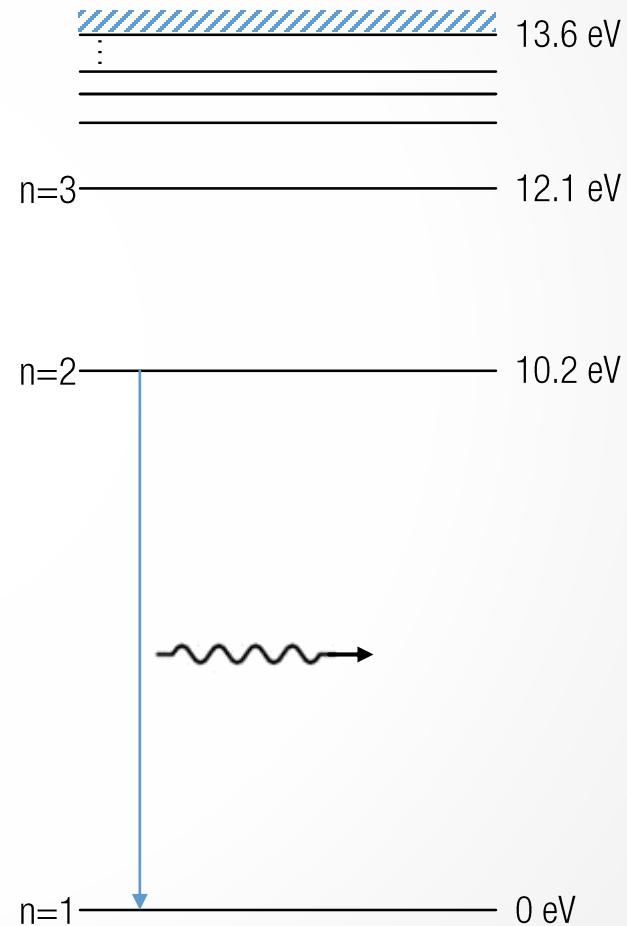
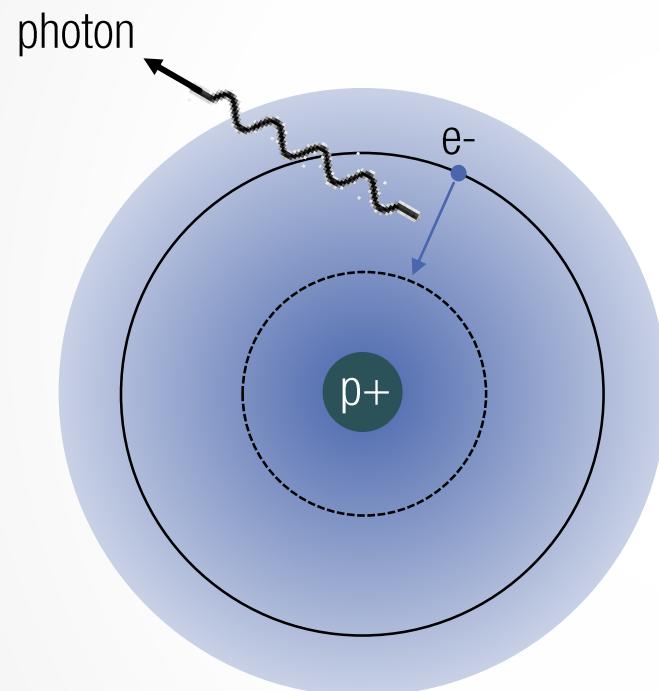


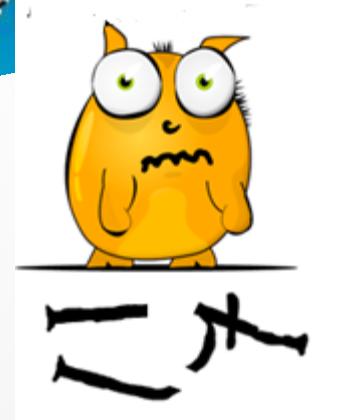
Closedness

is fragile

in quantum systems...

H-atom is always open!



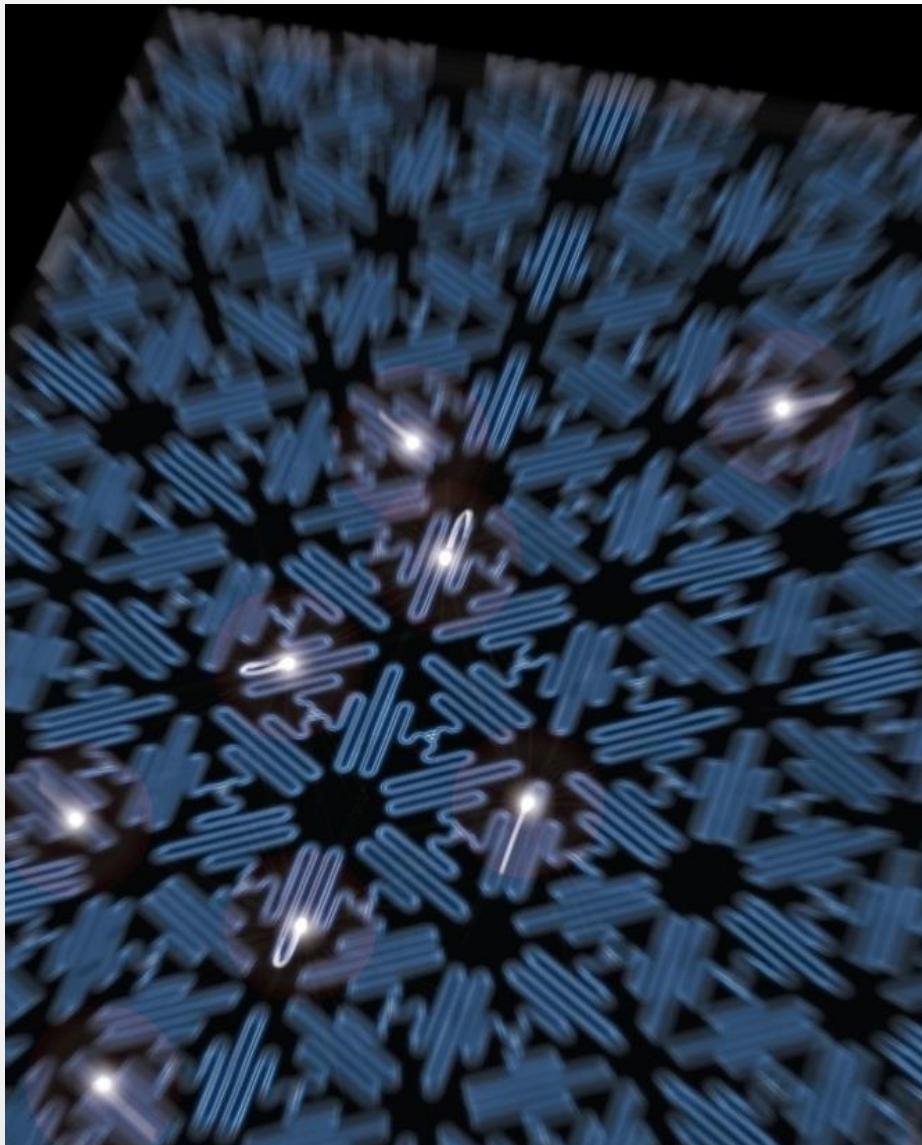


Quantum systems
tend to be
leaky . . .

Q1: How to “stop” leaks?

Q2: How to describe open QM systems?

Interacting photons in circuit QED lattices

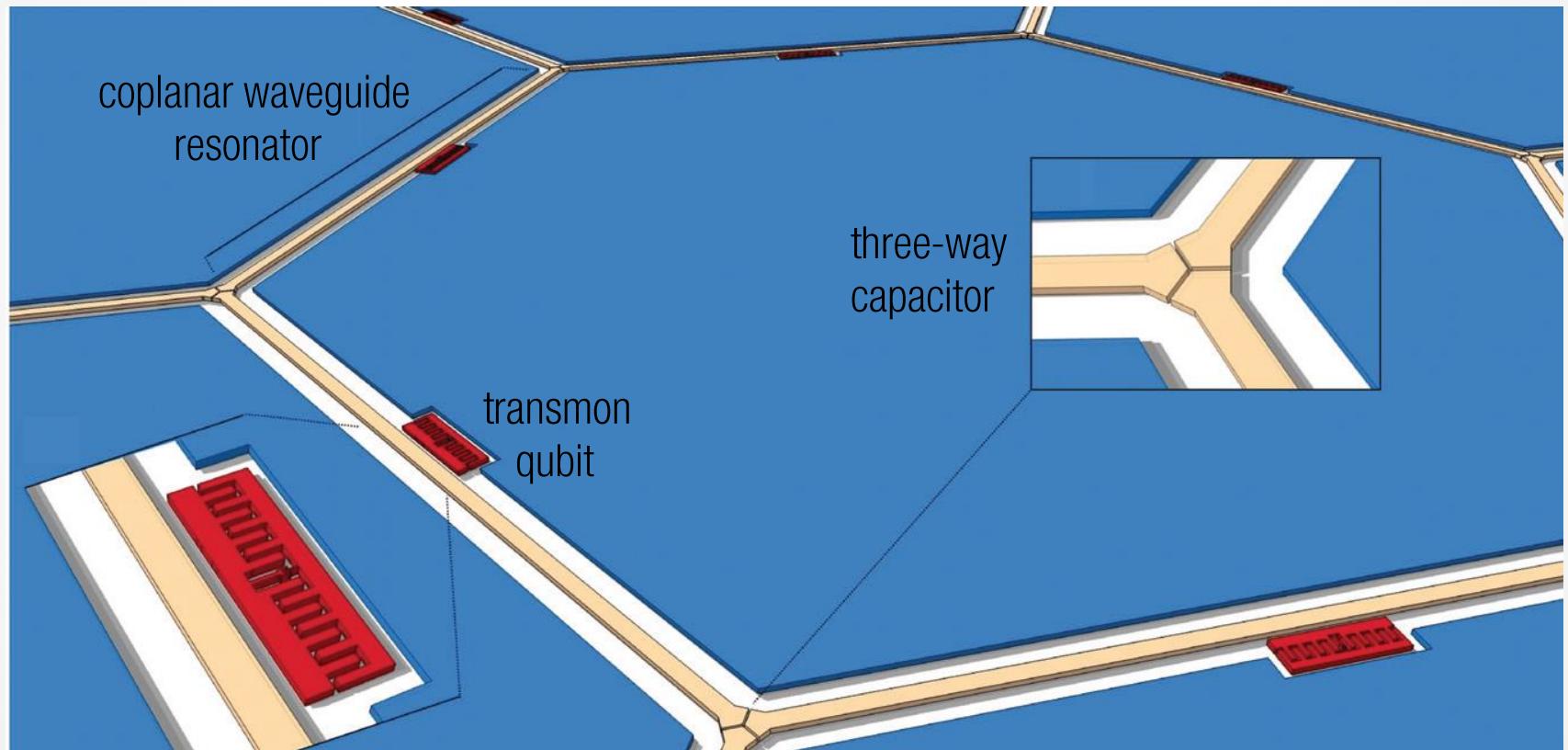


Reviews:

Houck, Türeci, JK,
Nature Phys. 8, 292 (2012)

Schmidt, JK,
Ann. Phys. 525, 395-412 (2013)

Interacting photons in circuit QED lattices

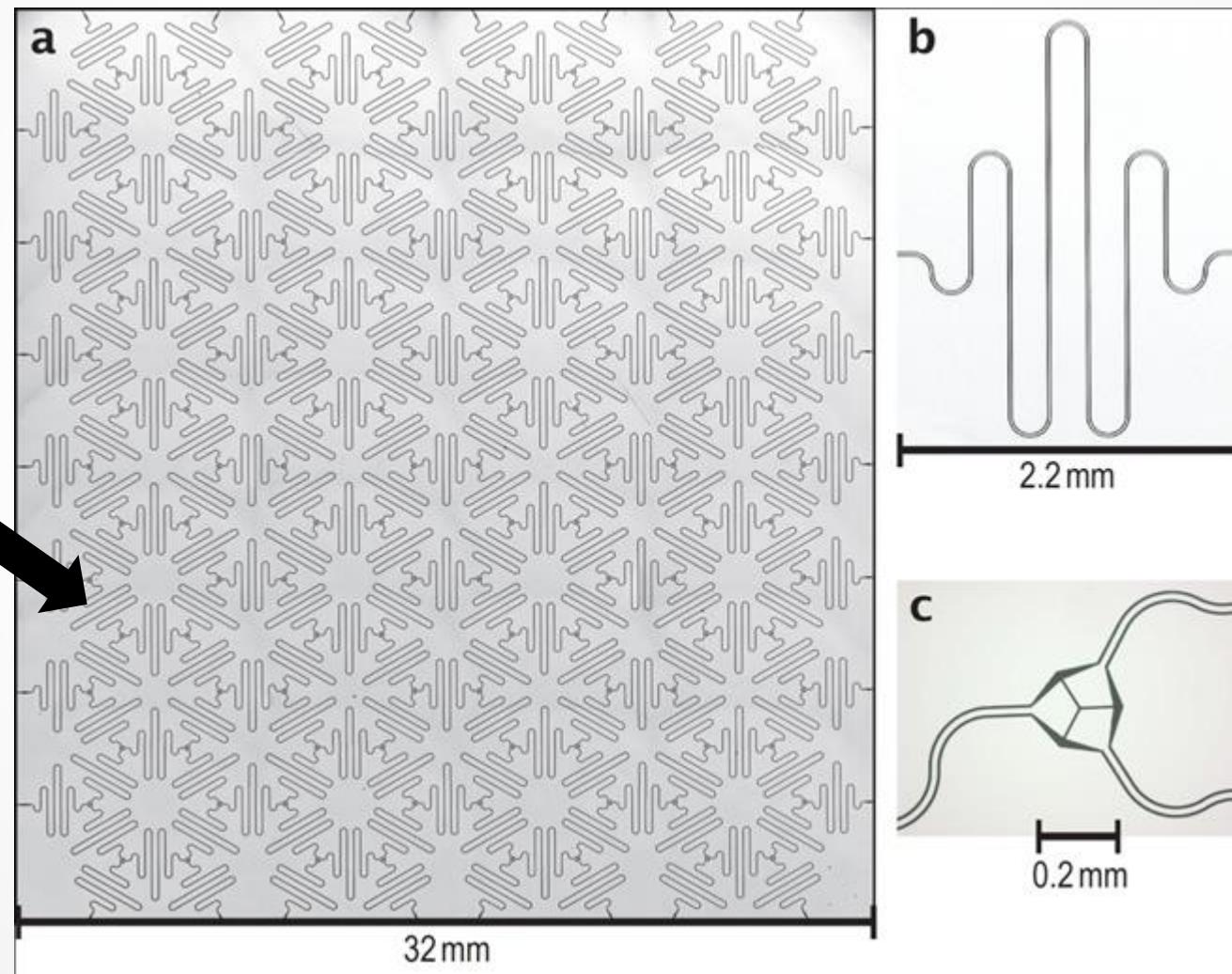




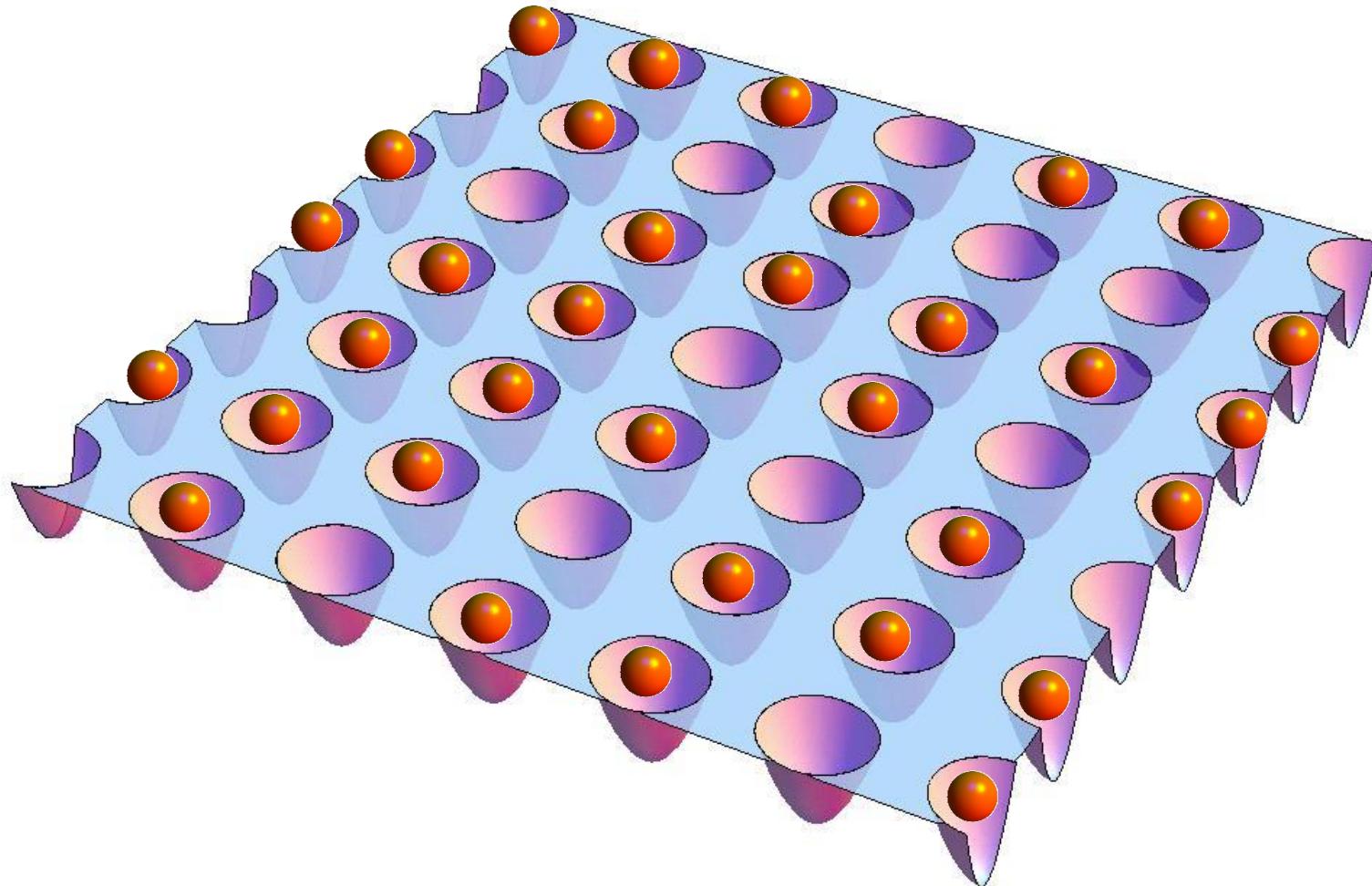
Andrew Houck
(Princeton U)

Experimental realization

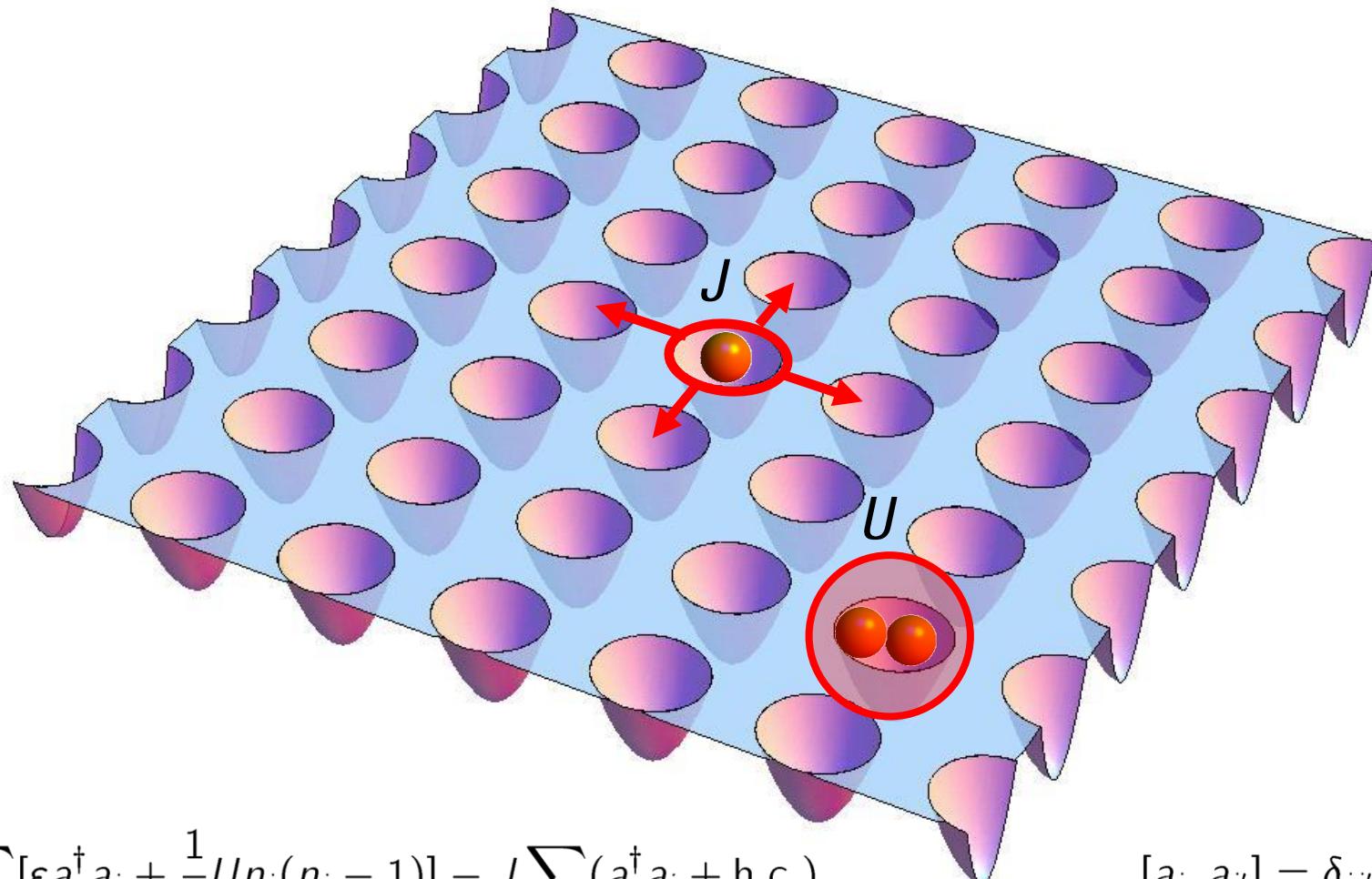
add qubits



The Bose-Hubbard model



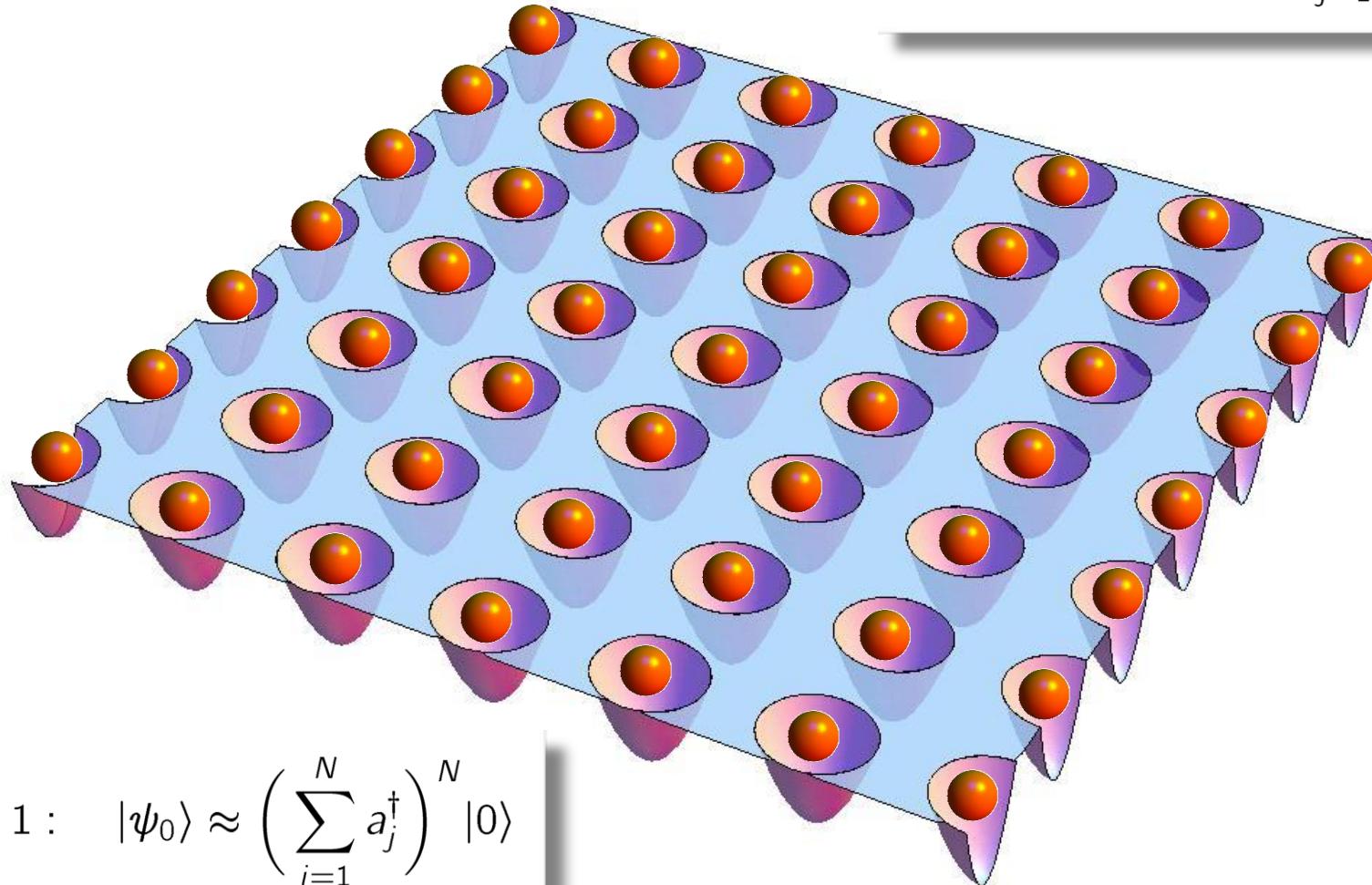
The Bose-Hubbard model



$$H = \sum_j [\varepsilon a_j^\dagger a_j + \frac{1}{2} U n_j(n_j - 1)] - J \sum_{\langle i,j \rangle} (a_j^\dagger a_i + \text{h.c.}) \quad [a_j, a_{j'}] = \delta_{jj'}$$

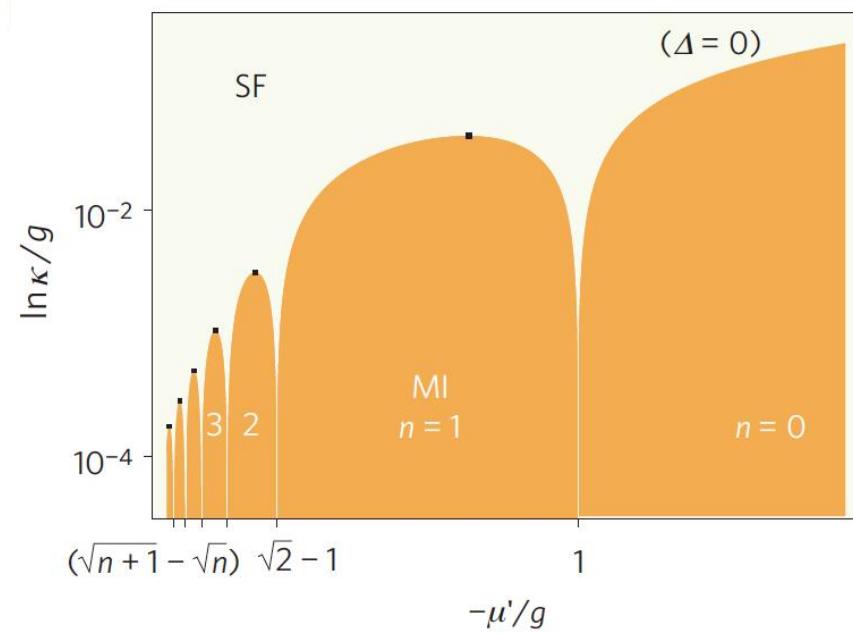
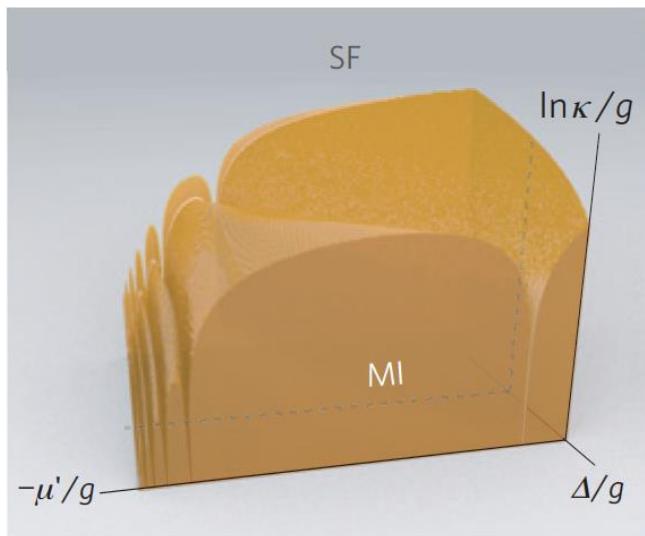
The Bose-Hubbard model

$$J/U \ll 1 : \quad |\psi_0\rangle \approx \prod_{j=1}^N a_j^\dagger |0\rangle$$



$$J/U \gg 1 : \quad |\psi_0\rangle \approx \left(\sum_{j=1}^N a_j^\dagger \right)^N |0\rangle$$

SF to MI quantum phase transition



Open-System Quantum Simulator

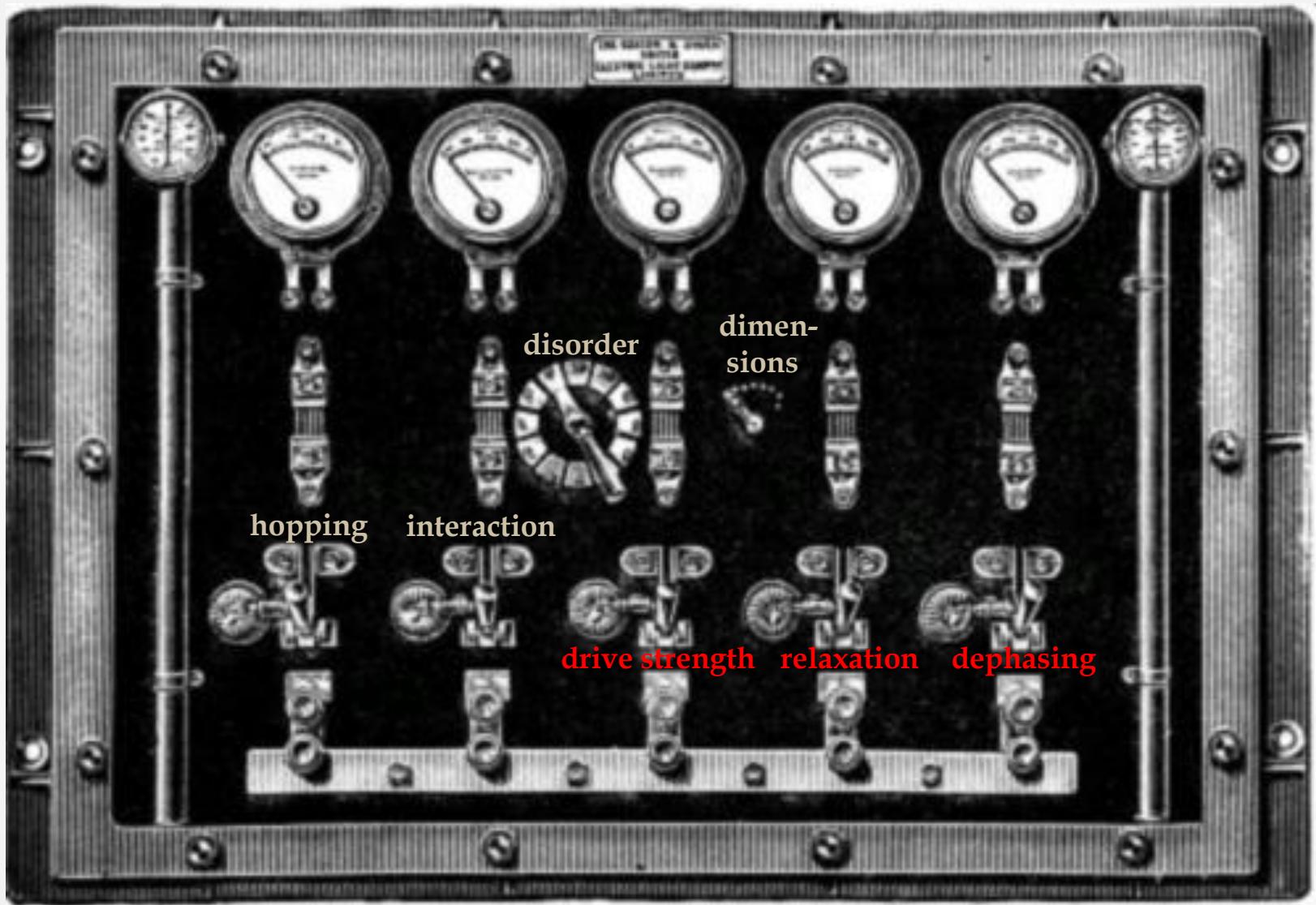


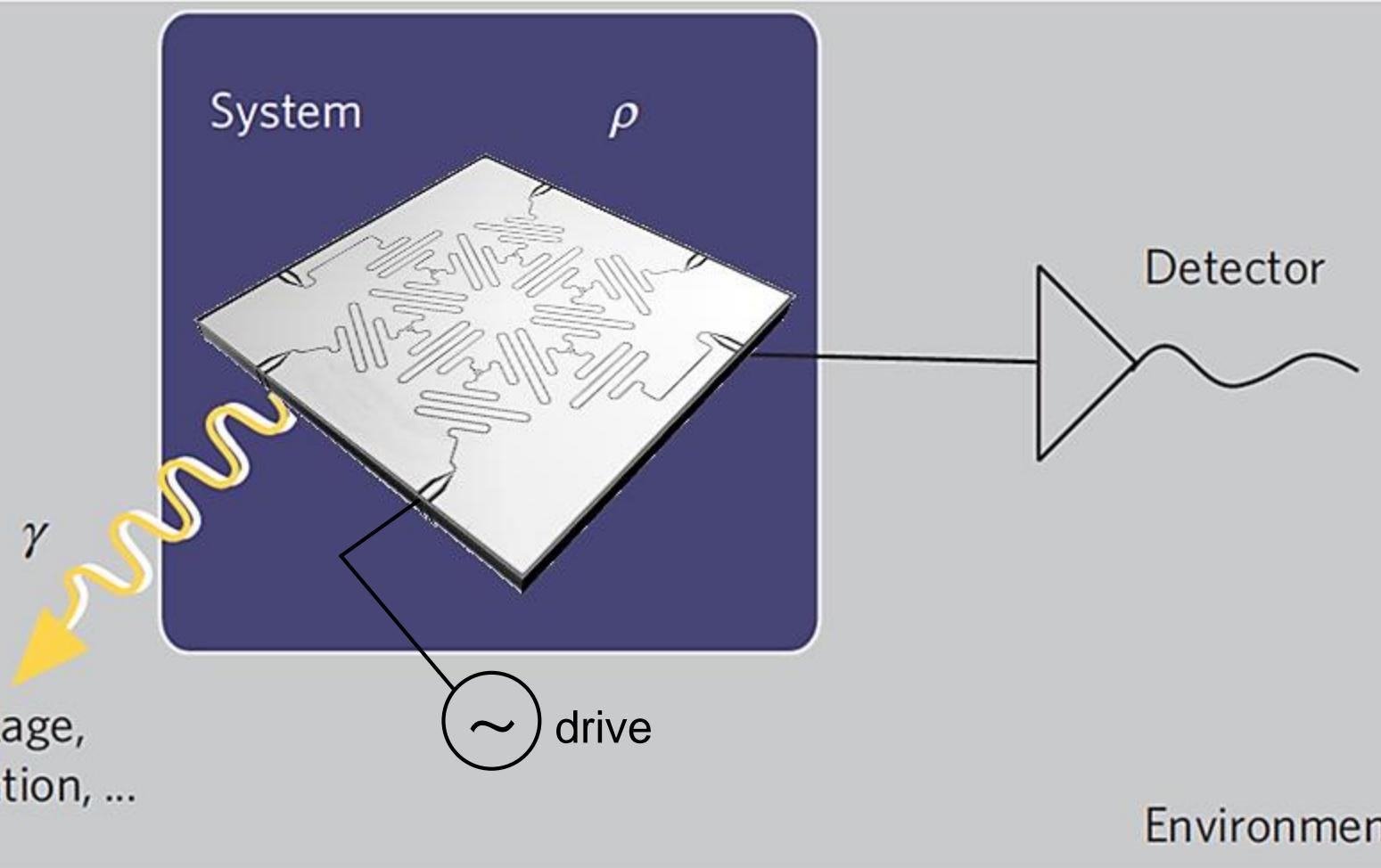
Fig. adapted from S. F. Walker

Dissipative Phase Transitions

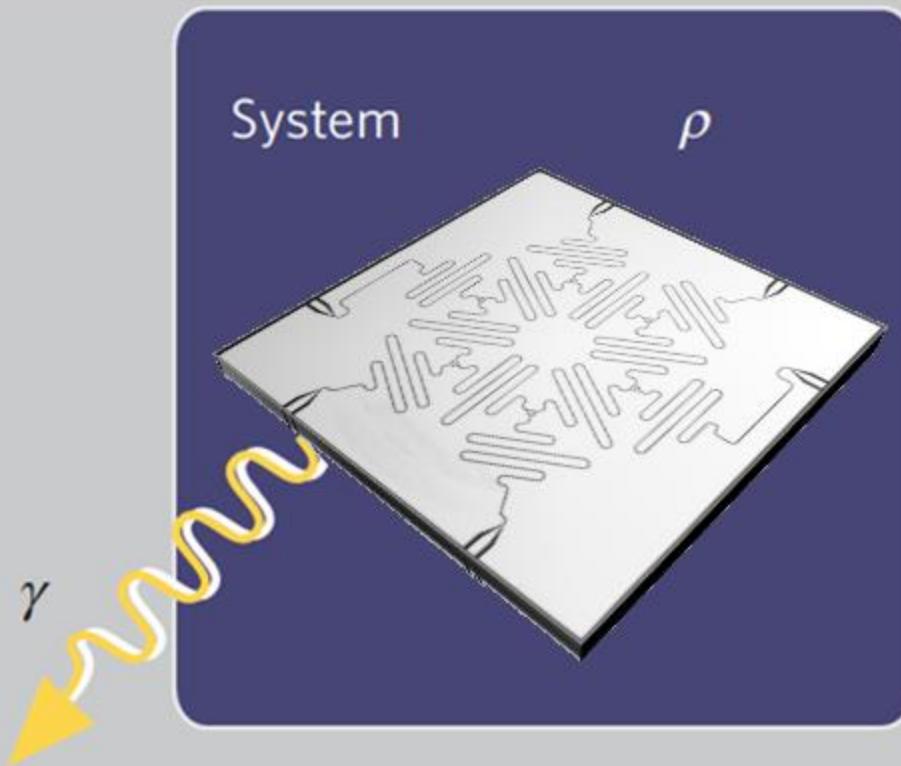
E.M. Kessler et al., Phys. Rev. A 86, 012116 (2012) [Lukin, Cirac]

	Quantum Phase Transition	Dissipative Phase Transition
System Operator	Hamiltonian H	Lindblad superoperator \mathbb{L}
Relevant Quantity	Energy eigenvalues $E_\nu : H \psi_\nu\rangle = E_\nu \psi_\nu\rangle$	Complex \mathbb{L} -eigenvalues $\lambda_\nu : \mathbb{L}u_\nu = \lambda_\nu u_\nu$
State	Ground state $ \psi_0\rangle = \underset{\langle\psi \psi\rangle=1}{\text{argmin}} \langle\psi H \psi\rangle$ $[H - E_0] \psi_0\rangle = 0$	Steady state $\rho^s = \underset{\text{tr } \rho = 1}{\text{argmin}} \ \mathbb{L}\rho\ _{\text{tr}}$ $\mathbb{L}\rho^s = 0$
Phase Transition	$\Delta = E_1 - E_0$ vanishes	$\max[\text{Re } \lambda_\nu]$ vanishes

Open quantum system



Open quantum system



Lindblad master eq.

$$\frac{d}{dt}\rho = \mathbb{L}\rho = -i[H, \rho] + \underbrace{\gamma(a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\})}_{\text{damping term}}$$

Lindblad
super-operator

damping term
(relaxation, dephasing etc.)

Open quantum system

Challenges for theory:

- worse than exact diagonalization for closed system:

$$H : N \times N \quad \Rightarrow \quad \mathbb{L} : N^2 \times N^2$$

- e.g., 4 resonators (up to 3 photons each)
 4 qubits



$\mathbb{L} : 16 \text{ millions} \times 16 \text{ millions}$
- WANTED: approximation schemes -

Lindblad master eq.

$$\frac{d}{dt} \rho = \mathbb{L} \rho = -i[H, \rho] + \gamma(a \rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\})$$

Perturbation theory: open system

Recursion relations (ρ -PT)

Eigenvalues:

$$\lambda_{\nu}^{(n)} = (w_{\nu}^{(0)}, \mathbb{L}_1 u_{\nu}^{(n-1)}) - \sum_{m=1}^{n-1} \lambda_{\nu}^{(m)} (w_{\nu}^{(0)}, u_{\nu}^{(n-m)})$$

Eigenstates:

$$[\mathbb{L}_0 - \lambda_{\nu}^{(0)}] u_{\nu}^{(n)} = -\mathbb{L}_1 u_{\nu}^{(n-1)} + \sum_{m=1}^n \lambda_{\nu}^{(m)} u_{\nu}^{(n-m)}$$
$$u_{\nu}^{(n)} = - \left(\mathbb{L}_0 - \lambda_{\nu}^{(0)} \right)^{-1} \left(\mathbb{L}_1 u_{\nu}^{(n-1)} + \sum_{m=1}^n \lambda_{\nu}^{(m)} u_{\nu}^{(n-m)} \right)$$

↑ Moore-Penrose pseudo inverse

Steady state:

$$\lambda_0^{(n)} = 0 \quad (\text{all orders})$$

$$\rho_s \simeq \rho_s^{(0)} + \alpha u_0^{(1)} + \cdots + \alpha^N u_0^{(N)} + \mathcal{O}(\alpha^{N+1})$$

Issue: no guarantee of positivity!

Amplitude Matrix Perturbation Theory

Density matrix is semi-def. positive

$$\Rightarrow \rho = \zeta \zeta^\dagger \quad \zeta : \text{amplitude matrix}$$

For positive-definite case: ρ has unique Cholesky decomposition
and expansion

$$\zeta_s = \zeta_s^{(0)} + \sum_{n=1}^{\infty} \alpha^n \zeta_s^{(n)}$$

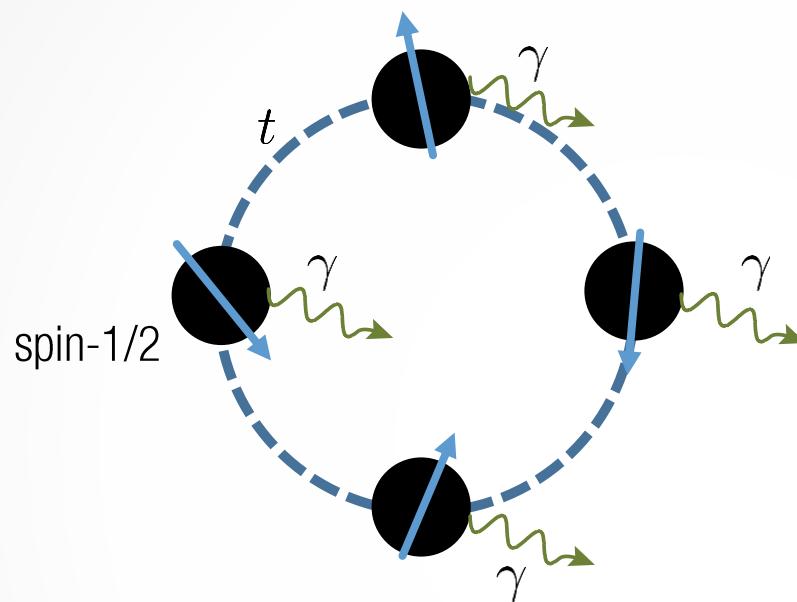
Recursion relations (ζ -PT)

$$\zeta_s^{(0)} [\zeta_s^{(0)}]^\dagger = \rho_s^{(0)} \quad (\text{by Cholesky decomposition})$$

$$\zeta_s^{(0)} [\zeta_s^{(n)}]^\dagger + \zeta_s^{(n)} [\zeta_s^{(0)}]^\dagger = \rho_s^{(n)} - \sum_{m=1}^{n-1} \zeta_s^{(m)} [\zeta_s^{(n-k)}]^\dagger$$

(sys. of linear eqs. for elements of ζ)

Test: Example System



drive $\sim \epsilon(\sigma_j^+ e^{-i\omega_d t} + \text{h.c.})$

Hamiltonian

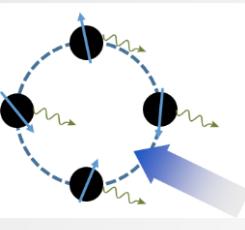
$$H = \sum_{j=1}^4 \left[\frac{\delta\omega}{2} \sigma_j^z + \epsilon \sigma_j^x \right] + \frac{t}{2} \sum_{\langle j,j' \rangle} \left[\sigma_j^+ \sigma_{j'}^- + \text{h.c.} \right]$$

(in frame co-rotating
with drive)

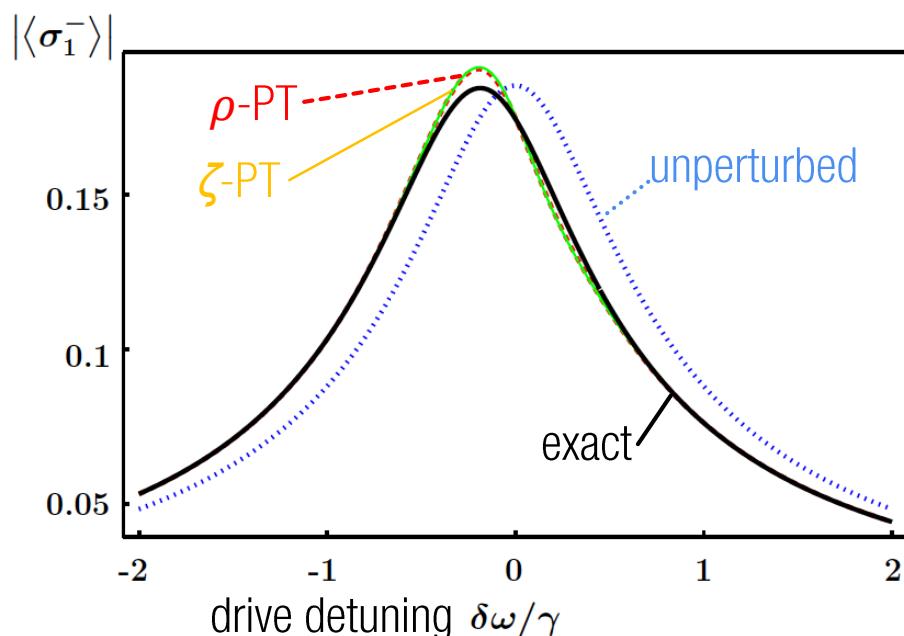
Master eq.

$$\frac{d}{dt} \rho = \mathbb{L} \rho = -i[H, \rho] + \gamma \sum_j (\sigma_j^- \rho \sigma_j^+ - \frac{1}{2} \{ \sigma_j^+ \sigma_j^-, \rho \})$$

treat as
perturbation

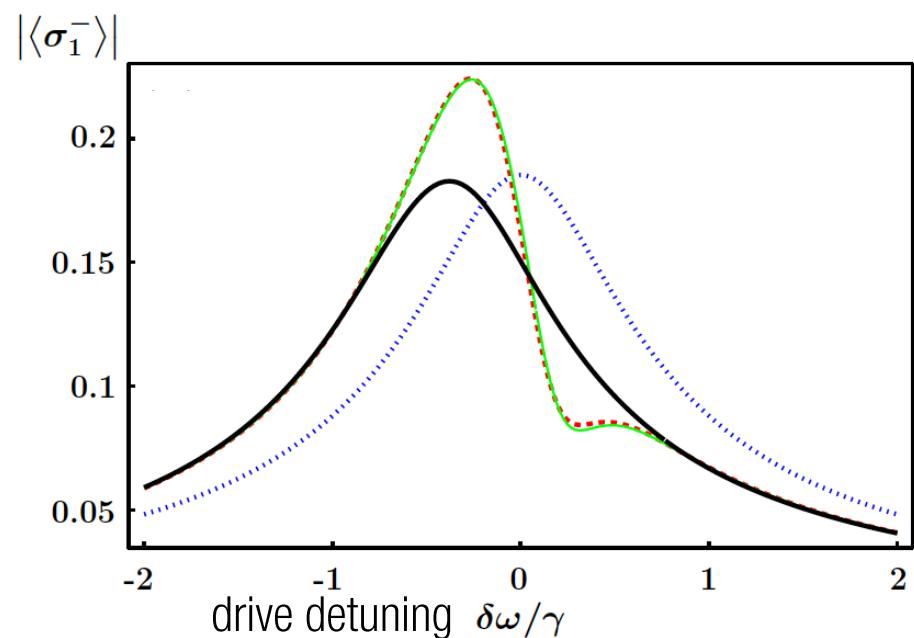


Results: exact vs. 2nd order perturbative



drive strength: $\epsilon/\gamma = 0.1$

spin interaction: $t/\gamma = 0.1$ (perturbation)

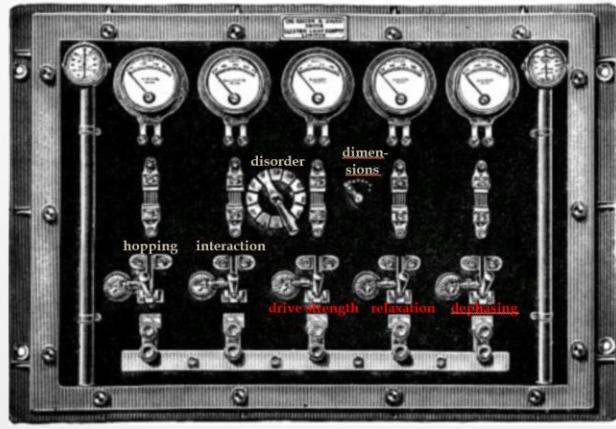


$\epsilon/\gamma = 0.1$

$t/\gamma = 0.2$ (breakdown)

Research in our group

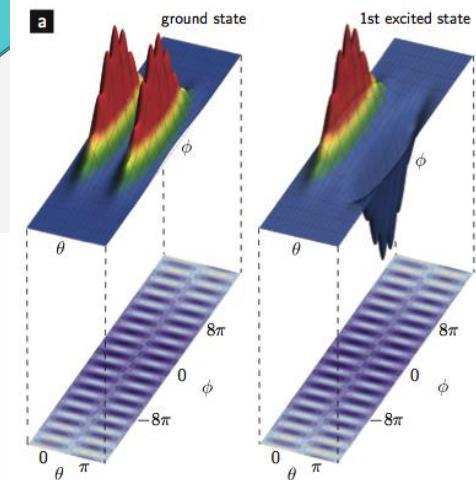
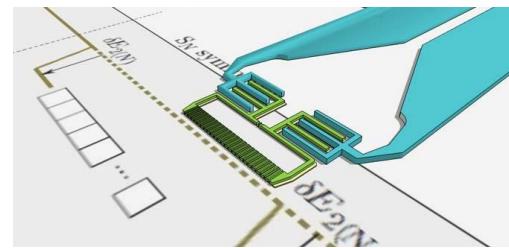
Open-System Quantum Simulator



Approximation schemes for solving
Lindblad master equations

Open circuit QED lattices

Dissipative phase transitions



Design and modeling of
new sc circuits

sc circuits with many degrees of freedom

sc circuits insensitive to noise