Market Concentration and the Productivity Slowdown

Jane Olmstead-Rumsey*

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Abstract

Since around 2000, U.S. aggregate productivity growth has slowed and product market concentration has risen. To explain these facts, I construct a measure of innovativeness based on patent data that is comparable across firms and over time and show that small firms make innovations that are more incremental in the 2000s compared to the 1990s. I develop an endogenous growth model where the quality of new ideas is heterogeneous across firms to analyze the implications of this finding. I use a quantitative version of the model to infer changes to the structure of the U.S. economy between the 1990s and the 2000s. This analysis suggests that declining innovativeness of market laggards can account for about 40 percent of the rise in market concentration over this period and the entire productivity slowdown. Strategic changes in firms' R&D investment policies in response to the decreased likelihood of laggards making drastic improvements significantly amplify the productivity slowdown.

*Department of Economics, Northwestern University, 2211 Campus Drive, Evanston, IL 60208. e-mail: janeolmsteadrumsey2015@u.northwestern.edu. I especially thank Matthias Doepke, Guido Lorenzoni, and David Berger for helpful comments and discussions. I also thank Ben Jones, Martí Mestieri, Kiminori Matsuyama, Nico Crouzet, Dimitris Papanikolaou, Matt Rognlie, Bence Bardóczy, Egor Kozlov, Ana Danieli, Jason Faberman, Francois Gourio, Gadi Barlevy, Jonas Fisher, B. Ravikumar, Yongseok Shin, Amanda Michaud, Andrea Raffo, and seminar participants in the Northwestern macro lunch, Chicago Fed, St. Louis Fed, Fed Board pre-job market conference, Economics Graduate Student Conference, CREI student macro lunch, and Midwest Macroeconomics meetings.

1 Introduction

After a boom in the 1990s, U.S. productivity growth began to decline in the early 2000s and industry leaders began capturing an increasingly large share of sales across many sectors of the economy. In this paper I illustrate a new mechanism that explains both of these trends, along with patterns of increasing profitability, increasing productivity differences, and a declining rate of market leadership turnover in U.S. industries. This mechanism is the declining ability of laggard firms to catch and overtake market leaders through innovation.

To support the existence of this mechanism in the data, I show that patent quality, one measure of innovativeness, has fallen sharply among smaller firms since 2000 after a boom in the 1990s. This fact is robust to using market-based measures of patent value or measures of the social value of firms' patents using citation counts. It is also a broad based phenomenon, spanning many sectors of the economy, though it is most pronounced in high tech sectors. I also show that there has been less turnover in market leadership in many industries since 2000.

To understand how leaders (the largest firm in terms of sales in each industry) and laggards (other, smaller firms) respond to diminished opportunities for laggard firms to grow through innovation, and the effect of these responses on aggregate growth, I develop and estimate a general equilibrium, quality ladder model of directed innovation along the lines of Aghion et al. (2001). There are a continuum of industries populated by two incumbent firms producing differentiated goods. Incumbent firms can improve their varieties through innovation. Each industry also contains a competitive fringe of firms with the ability to imitate whichever incumbent has the lower quality variety. Market concentration, measured as the market leader's share of industry sales, is high when the quality difference between the two incumbents' product varieties is large.

Unlike Aghion et al. (2001), the model accommodates the possibility that laggard firms have an "advantage of backwardness," allowing them to improve their variety more drastically than market leaders do when they innovate. A model parameter governs the extent of laggard firms' advantage of backwardness. Other model parameters capture alternative explanations for rising market concentration, slowing productivity growth, or both, that have been suggested in the literature such as slowing knowledge diffusion or entry rates, increasing product differentiation, rising market power, and declining real interest rates.

To infer the relative importance of different changes to the U.S. economy in explaining these trends, I estimate the model parameters for two steady states to match data on concentration, productivity growth, the profit share, patent quality, the rate of turnover in market leadership, aggregate R&D expenditures, and R&D expenditures at the firm level for the U.S. in two separate periods, the 1990s and the 2000s. This exercise suggests a dominant role for the parameter governing laggard firms' advantage of backwardness to explain rising concentration and slowing productivity growth compared to other explanations.

I then use the quantitative model to explore the channels through which laggards' declining patent quality alone can explain trends in concentration and productivity growth. When laggards firms' advantage of backwardness declines, these firms respond to a lower chance of attaining market leadership by investing less in research and development. Facing a diminished probability of being overtaken, market leaders invest slightly more. Together, these decisions lead to larger average quality differences between leaders and laggards in steady state. The model has a direct mapping from quality differences to leaders' market shares and markups such that sales concentration and the profit share of total output also rise. This change explains about 40 percent of the observed rise in concentration between the 1990s and the 2000s, and about 25 percent of the increase in the profit share.

Laggards' declining patent quality also has implications for productivity growth in the model. The source of endogenous growth in the model is quality improvements to the differentiated products that firms produce. Making laggards' quality improvements more incremental generates a productivity slowdown through two channels. One is direct: even if firms devoted the same resources to research and development, the economy would grow more slowly because average quality improvements are smaller than before. Second, there is a strategic effect that amplifies the productivity slowdown: because of the relocation of R&D expenditures in the economy towards market leaders, whose innovations tend to be more incremental, the economy grows even more slowly than before. According to a growth decomposition exercise, this latter force accounts for roughly half of the productivity slowdown in the model. Quantitatively, the estimated decline in laggards firms' advantage of backwardness generates a productivity slowdown in the model of a similar magnitude to the slowdown observed in the U.S.[1]

[1] The fact that changing innovativeness alone can explain the entire productivity slowdown does not rule out other explanations that have been proposed, since there may be forces working to increase productivity

Finally, I discuss how the model predictions are consistent with patterns of widening productivity gaps within sectors, rising markups, sector level correlations between the productivity slowdown and rising concentration, and the fact that industry leaders conduct a larger share of total industry R&D in the 2000s than in the 1990s.

Related Literature and Contribution This paper contributes a novel mechanism to the large and growing literature linking trends in concentration, productivity growth, and business dynamism using models of endogenous growth. Many of these papers emphasize the increasing importance of intangible assets and information and communications technology (ICT) as a possible explanation (Aghion et al. (2019a), de Ridder (2020), Corhay, Kung, and Schmid (2020)). Non-technological explanations include demographic changes (Hopenhayn, Neira, and Singhania (2018), Peters and Walsh (2019), Karahan, Pugsley, and Sahin (2019), Engbom (2019), Eggertsson, Mehrotra, and Robbins (2019), Bornstein (2018)) or declining real interest rates (Liu, Mian, and Sufi (2019), Chatterjee and Eyigungor (2019)). The most closely related explanation is the one in Akcigit and Ates (2020) and Akcigit and Ates (2019) that diffusion of knowledge from leaders to laggards is slowing down, either because of ICT and the increasing importance of data in firms' production processes or because of anti-competitive use of patents.

Rather than emphasizing particular features of information technology, the theory presented here instead hypothesizes that general purpose technologies (GPTs) may affect firm dynamics and market structure in addition to raising aggregate productivity growth. Past fluctuations in patent quality and productivity growth have been attributed to waves of innovation due to the arrival of new GPTs (Kelly et al. (2018); Kogan et al. (2017)). Bresnahan and Trajtenberg (1995) note that GPTs are applicable in a wide range of sectors and exhibit innovational complementarities, meaning that they increase the productivity of downstream research and development efforts.[2] Given the new evidence presented here on heterogeneity in patent quality across firms and time, I argue that these innovational complementarities appear to be stronger for smaller firms than for market leaders.[3]

Most neo-Schumpeterian growth models assume goods within sectors are perfect substitutes so that each sector has just one producer in each period (see Klette and

growth that the model does not capture such as population growth, entry, improvements in human capital, and globalization.

[2]See Brynjolfsson, Rock, and Syverson (2018) for further discussion.

[3]See Section 2.2 for further discussion.

Kortum (2004), Lentz and Mortensen (2008), Acemoglu and Cao (2015), and Akcigit and Kerr (2018), for leading examples). Because of this, these models are not well-suited to address industry-level moments such as sales concentration. Introducing a duopoly (plus a competitive fringe) allows me to make unified predictions both about market concentration at the industry level and firm-level innovation rates, and makes not only markups but also sales concentration within sectors an endogenous outcome of the innovation process.

The duopoly formulation also brings together previously distinct strands of literature in macroeconomics concerned with (i) slowing growth (ii) changes in market structure and potentially market power and (iii) superstar firms. Strands (ii) and (iii) typically rely on opposing assumptions. According to the literature on rising market power, incumbent firms exercise greater pricing power now than in the past and this is reflected in rising markups and profitability (de Loecker, Eeckhout, and Unger (2020), Barkai and Benzell (2018)). On the other hand, the literature on superstar firms contends that greater import competition and greater consumer price sensitivity due to better search technology like online retail have *increased* competitive pressures and reduced the market power of incumbent firms, resulting in reallocation to the most productive (superstar) firms (Autor et al. (2020)). The model resolves this seeming contrast by demonstrating how markups can rise at the same time as there is reallocation to relatively more productive firms without any changes at all to consumer preferences or the aggregate production function. The model is also consistent with the finding of Kehrig and Vincent (2020) that being a superstar firm is a temporary rather than permanent status. In the model, the relative advantage of high value added firms grows in the 2000s and the average duration of these "shooting star" spells increases, but these firms are eventually displaced by competitors.

The model's industry structure with imperfect substitutes makes it possible to quantitatively compare explanations for increased markups and profits in recent years to the superstar firm hypothesis that greater price sensitivity has sparked reallocation to large, productive firms. Within the model, neither story matches the data as well as a decline in laggards' patent quality, though I show that the static superstar firm experiment generates a productivity slowdown alongside rising concentration in the estimated model. To my knowledge, this is the first dynamic version of Autor et al. (2020) with endogenous productivity growth.

The finding that laggards' patent quality has declined since 2000 is consistent with Bloom et al. (2020), who show that despite increasing inputs (expenditures, workers) to R&D, outputs in terms of productivity improvements have declined using a variety of case studies. Anzoategui et al. (2019) also identify a decline in R&D productivity using indirect inference in a DSGE model with endogenous productivity growth. Several empirical papers have also documented that laggard firms are less likely to overtake market leaders in recent years (Bessen et al. (2020), Pugsley, Sedlacek, and Sterk (2020), Andrews, Criscuolo, and Gal (2016)). This paper sheds more light on the channel through which this happens: I estimate a mild *decrease* in the cost per patent to explain rising expenditures on R&D over this period, but also a large decrease in the average contribution of a new patent to the value of the firm for laggard firms.[4]

Finally, many papers have studied rising concentration and the productivity slowdown (Hall (2015), Syverson (2017)) in isolation from one another. Rising concentration is mainly a within-sector phenomenon (Hsieh and Rossi-Hansberg (2020)) that is occurring at the national/product market level rather than at the local level.[5] The finding that market concentration is rising is robust to the inclusion of foreign firms (Covarrubias, Gutierrez, and Philippon (2019)) or more sophisticated methods of identifying firms' direct competitors (Pellegrino (2020), using data from Hoberg and Phillips (2010)).

A variety of explanations for rising sales concentration have been proposed, from the introduction of ICT that creates winner-take-all markets and enables the growth of superstar firms (Bessen (2017), Crouzet and Eberly (2018), van Reenen (2018)), to excessive regulations that erect barriers to entry and create unnatural monopolies (Covarrubias, Gutierrez, and Philippon (2019)), to increased mergers and acquisitions activity, possibly due to weak antitrust enforcement (Grullon, Larkin, and Michaely (2019)). This paper complements these hypotheses by contributing a novel mechanism that, according to the quantitative exercise, explains around 40 percent of the rise in concentration.

[4]Contemporaneous work by Cavenaile, Celik, and Tian (2020) estimates an endogenous growth model with incumbents and a competitive fringe with step by step innovations and finds that declining R&D productivity of small firms can explain a large share of the rise in concentration and the productivity slowdown. The advantage of allowing for patent quality heterogeneity and including new data on patent quality as a target for the estimation is that I can separately identify changing costs and changing output of R&D.

[5]In fact Berger, Herkenhoff, and Mongey (2019) and Rossi-Hansberg, Sarte, and Trachter (2020) find evidence that local sales concentration has fallen over this period.

2 Empirical Motivation

I first review aggregate trends in productivity growth and market concentration to motivate the analysis. I then show that innovativeness has declined relative to the 1990s along various metrics, particularly for laggard firms, and discuss potential causes. Finally, I show that laggard firms are less likely to catch up to the leading firm in their industry to become the sales leader now than in the past.

2.1 Market Concentration and Productivity Growth

To establish the main empirical motivation for the paper, Figure 1 plots the average market leader's share of total industry sales in Compustat and the total factor productivity growth rate. Among U.S. public companies, market concentration has risen significantly since the late 1990s.[6] The average market leader's sales share within narrowly defined 4-digit Standard Industrial Classification (SIC) industries has risen from around 40% in the l990s to over 50% in 2017. Total factor productivity growth averaged about 1.7% between 1994 and 2003, but slowed to about 0.5% between 2004 and 2017.

According to the standard Olley and Pakes (1996) decomposition, aggregate total factor productivity growth could be slowing down for two reasons. First, average TFP growth across all firms could be slowing down. Second, reallocation to the most productive firms (i.e. the covariance of sales share and productivity) could be slowing down. Baqaee and Farhi (2020) show that within-firm growth has contributed very little to aggregate TFP growth since the late 1990s. Broad-based below-trend productivity growth, not increasing misallocation among U.S. firms, seems to be driving the aggregate slowdown, lending support to explanations focusing on the incentives of existing firms to improve productivity, like the hypothesis I propose here.

2.2 Trends in Patent Quality

Economists have long relied on patents as an observable proxy for innovativeness (Griliches (1998)). The most commonly used measure of patent quality, counting the number of forward citations a patent receives from future patents, shows substantial

[6] See Grullon, Larkin, and Michaely (2019) and Council of Economic Advisers (2016) for overviews of trends in market concentration. More than 75% of U.S. industries have experienced an increase in the Herfindahl-Hirschman index.

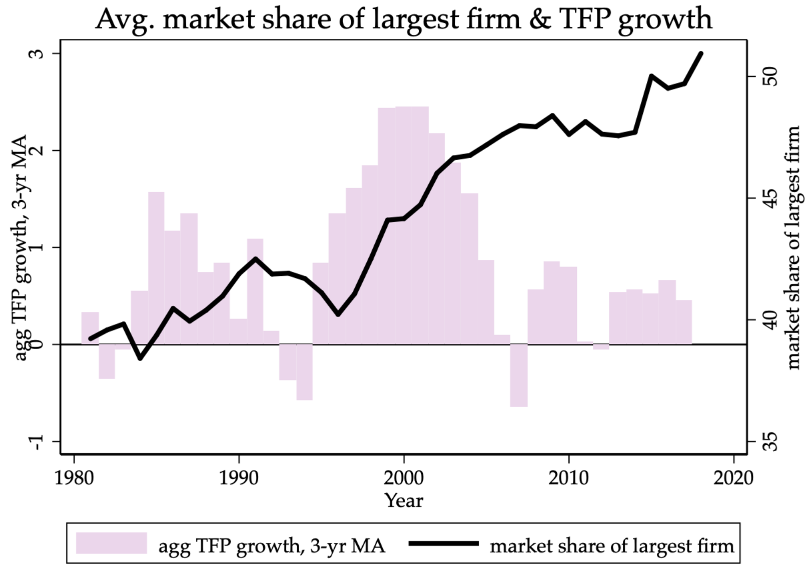


Figure 1: Average market share of largest firm (by sales) in 4-digit SIC industries from Compustat (weighted by industry sales); utilization-adjusted total factor productivity (TFP) growth from Fernald (2014), three year moving average.

heterogeneity in quality in the cross section of patents, with a few patents receiving many citations and most receiving none or just a few (Akcigit and Kerr (2018)).

Recent evidence using alternative measures of patent quality also points to substantial changes in average quality over time. For example, Kelly et al. (2018) create a text-based measure of patent quality, identifying "breakthrough" patents as those patents where the patent's text differs from the text of past patents but is similar to the text of future patents. This measure has the advantage of covering a longer time series (1860-present) than citation based measures (1940-present). Using this measure, Kelly et al. (2018) find that periods with high average patent quality coincide with the discovery of new general purpose technologies, including the ICT revolution in the 1990s, consistent with Bresnahan and Trajtenberg (1995)'s theory of "innovational complementarities" between general purpose technologies and inventions in other sectors of the economy.[7] The most recent wave of high patent quality driven by ICT began to subside in the late 1990s according to this measure (see Appendix A.2).

To explore heterogeneity in the decline in patent quality across firms, I use a measure of patent value from Kogan et al. (2017) that estimates the market value of all

[7] See Helpman (1998) and Aghion, Akcigit, and Howitt (2014) for reviews of the study of GPTs.

patents issued in the U.S. and assigned to public firms from 1926-2010 using firms' excess stock returns in a window around patent approval dates to infer the market value of the patent.[8,9] This measure has the advantage of capturing the private value of the patent to the firm, which will determine firms' investment decisions in the model.

In the model presented in section 3, firms make innovations that grow the quality of their product variety by a random amount. I use the dollar value estimates of Kogan et al. (2017) to construct a measure of each public firm's "patent stock" as the cumulative value of all past patents, intuitively corresponding to the current knowledge or quality embodied in the firm's product(s).[10,11] From 1980 to 2017 this measure covers 1,339,541 patents issued to 4,360 different U.S. public firms. With this measure of the patent stock in hand, I define *patent quality* as the marginal contribution of a new patent to the total value of the firm's existing patent stock. Figure 2 plots the average of this measure over time, splitting the sample into market leaders (largest firms by sales in 4-digit SIC industries) and followers (all other firms).

Figure 2 illustrates the two key facts for the subsequent analysis:

1. Smaller firms have higher patent quality than market leaders on average.
2. Smaller firms' patent quality rose from 1990 to 2000, but has declined significantly since 2000.

Fact 1 is related to a large debate on the relative innovativeness of large versus small firms (see Akcigit and Kerr (2018)). Typically this debate centers on small startups versus large companies with more than 500 employees (more than 72% of observations in the sample of patenting firms in Compustat have more than 500 employ-

[8]I use updated data through 2017 from Noah Stoffman's webpage:
https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data.

[9]Kelly et al. (2018) document the strong correlation between the market- and text-based measures at the patent level as well as the correlation of these measures with forward citation-weighted measures. All three measures show a sharp uptick in average patent quality and in the right tail during the 1990s and a subsequent decline beginning in the late 1990s.

[10]Construction details in appendix A.1.

[11]Some depreciation can be applied to the patent stock measure. For example Peters and Taylor (2017) use the Bureau of Economic Analysis' R&D expenditure depreciation rates by sector, ranging from 5-20% per year to construct a measure of firms' intangible capital stock. Applying depreciation rates in this range increases the level of the estimated quality improvements but does not affect the magnitude of the slowdown or the differential decline between leaders and laggards.

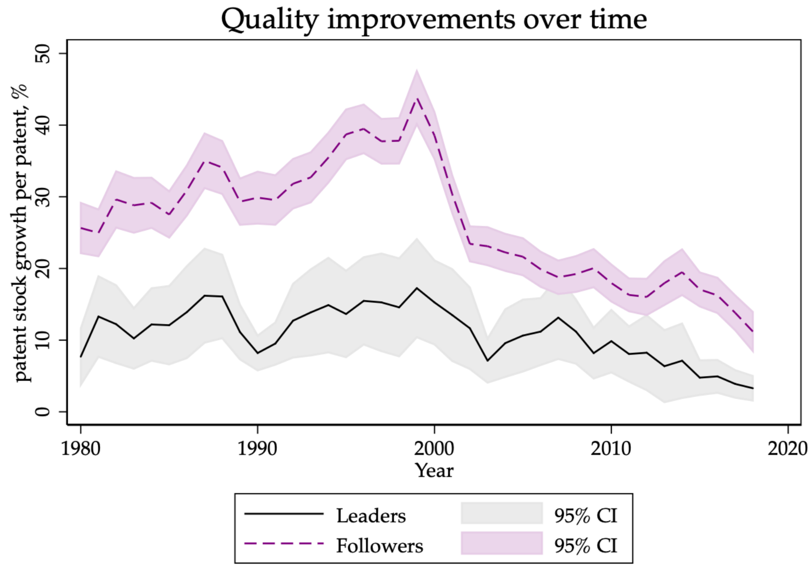


Figure 2: Contribution of average new patent to value of filing firm's existing stock of patents, using estimated patent values from Kogan et al. (2017). Leader indicates sales leaders in 4-digit SIC industries and followers are all other firms.

ees). My finding is that even among firms that are large relative to the entire firm size distribution, there are differences in patent quality by size (measured by sales) within industries. Arguments supporting the greater innovativeness of smaller firms should still be relevant even among public firms: managers at smaller firms tend to be more flexible and to be closer both to customers and to researchers within the firm, enhancing their ability to allocate spending to more productive projects (Knott and Vieregger (2016)). Rosen (1991) develops a model to explain why smaller firms within sectors make a disproportionate share of major innovations. In the model, large firms find it optimal to focus on innovations that are complementary to their existing products and processes to avoid Arrow (1962)'s replacement effect. Small firms instead choose to allocate funds toward developing revolutionary technologies, having more to gain in post-innovation rents from doing so.

Fact 2 is new. The patented innovations of relatively smaller firms seem to be more incremental recently than they were in the 1990s. One candidate explanation is that, because this is a market-based measure, investors are internalizing the fact that it is harder for non-leading firms to make a dent in the advantage of their leading competitor than before, perhaps because of anti-competitive practices of market leaders or because of the rise of platform-based technologies. It's not possible to fully rule this out, but constructing a similar measure of patent quality based on forward citation counts instead of dollar values shows a similar decline beginning in 2000 (see Appendix A.2). If this was the case and nothing else changed we might expect to see a wedge opening up between the private and social value of laggards' patents, but in fact both declined, perhaps pointing towards technological explanations.

Another possible explanation for fact (2) is that general purpose technologies, or at least ICT, have greater complementarities with some types of firms than others (Jovanovic and Rousseau (2005) find that initial public offerings surge during GPT waves, for example). Smaller firms, with greater flexibility and more incentive to invest in riskier, disruptive ideas, may be better positioned to take advantage of the gains associated with disruptive technologies. After the general purpose technology has diffused through the economy, opportunities for disruption may lessen and laggards' improvements may become more incremental.[12] Consistent with this idea, the pattern of boom and bust in patent quality is more pronounced in high tech sectors than in manufacturing, healthcare, or consumer good sectors, though the trend is present to some extent across all four categories (see Appendix A.2).

The very sharp decline in laggards' patent quality between 1999 to 2001 is worth exploring. The only significant change to U.S. patent law in the late 1990s was the American Inventor's Protection Act of 1999 to publish most patent applications 18 months after filing.[13] Previously, only approved patents were published. This change might deter inventors who thought their patent was unlikely to be approved from applying for a patent for fear that their idea would be published but they would not get the exclusive rights to it. In that case one would expect the patent application approval rate to rise. In fact, according to Carley, Hegde, and Marco (2015), the approval rate declined from about 70% in 1996 to 40% in 2005. Moreover, Graham and Hegde (2015) find that firms given the option to opt out of this pre-grant disclosure (U.S. firms that did not file any foreign versions of the same patent) chose to do so less than 10% of the time.

The decline is also not likely driven by the dot-com bubble since the same pattern appears in the citation-based measure of patent quality. It also does not appear to be driven by the increasing age of public firms: this pattern appears even among

[12] Aum, Lee, and Shin (2018) find that the productivity boom from computerization had normalized by 2004.

[13] https://www.uspto.gov/about-us/news-updates/uspto-will-begin-publishing-patent-applications

firms that have been public at least 20 years when the patent is issued (Appendix A.2). Nor is it driven by ideas being embodied in multiple patents in recent years: the same pattern is present in the annual patent stock growth rather than the marginal contribution of each individual patent (Appendix A.2).

2.3 Declining Dynamism and Leadership Turnover

In the model presented in section 3, innovations drive growth in market share at the expense of the firm's competitor. Turning to this outcome, Figure 3 plots the fraction of U.S. industries with a new sales leader each year to measure the frequency with which smaller firms overtake the largest firm. This fraction has fallen from around 15% per year in the late 1990s to around 9% in recent years (see Bessen et al. (2020) for a detailed empirical analysis of this phenomenon in the U.S. and Andrews, Criscuolo, and Gal (2016) for a cross-country analysis).

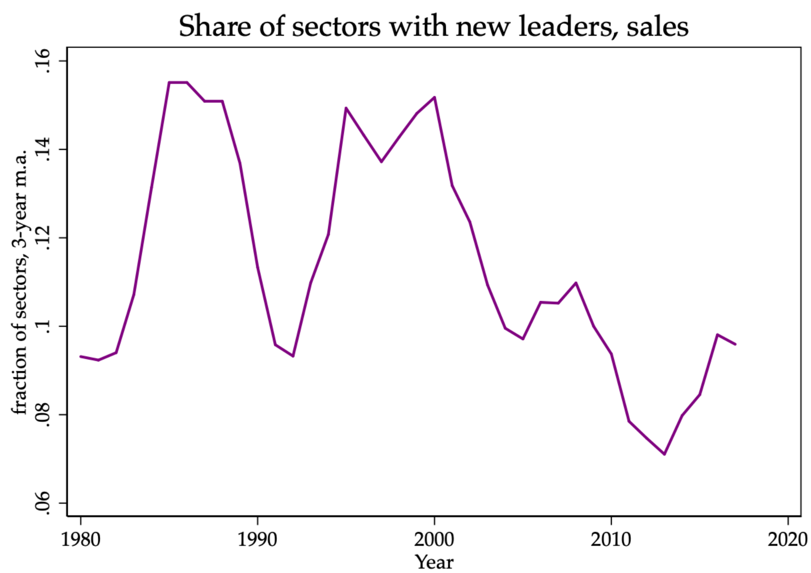


Figure 3: Share of 4-digit SIC sectors in Compustat with a new sales leader each year.

Firm-level productivity data also shows that the "advantage of backwardness" has fallen relative to the 1990s, consistent with the idea that it is now harder to catch up through innovation than it was in the 1990s. Andrews, Criscuolo, and Gal (2016) show that in a regression of firm-level productivity growth on a variety of explanatory variables, the coefficient on the lagged productivity gap to the most productive competitor has been declining over the 2000s, suggesting that distance to the productivity frontier is becoming a less important predictor of future productivity growth.[14] Decker et al. (2016) also find that the right skewness of the firm-level productivity growth distribution in the U.S. has declined over this period.

3 Model

To capture the effect of declining innovativeness of laggards firms, I develop a model along the lines of Aghion et al. (2001) but building on models with heterogeneous patent quality rather than step by step innovations (Akcigit, Ates, and Impullitti (2018), Acemoglu et al. (2018), Akcigit and Kerr (2018)). Relative to Aghion et al. (2001), I also introduce a competitive fringe of firms in each sector that constrains the pricing behavior of the incumbents in order to match observed levels of concentration in the data. The model features endogenous markups and each sector's level of sales concentration evolves over time as the result of innovation. The model will be used to infer changes to the nature of the economy between the 1990s and the 2000s.

The model is of a closed economy in continuous time. There are three types agents: a representative household, a representative final good firm, and firms producing intermediate goods. This section presents the model going through the problem of each type of agent in the economy, then analyzes the equilibrium of the model.

3.1 Households

A representative household consumes, saves, and supplies labor inelastically to maximize:

$$U_t = \int_t^\infty \exp(-\rho(s-t))\frac{C_s^{1-\psi}}{1-\psi}ds,$$

subject to:

$$r_t A_t + W_t L = P_t C_t + \dot{A}_t,$$

where ρ is the discount rate, ψ is the inverse intertemporal elasticity of substitution, C_t is consumption at time t, W_t is the nominal wage rate, and P_t is the price of the

[14]This empirical observation is endogenous according to the model, because it may be a result of both structural change to catchup speeds and to the endogenously lower innovation effort by laggard firms since their regression does not control for innovation effort (R&D investment).

consumption good C_t. Households' labor supply L will be normalized to 1 and there is no population growth. Households own all the firms, and the total assets in the economy A_t are:

$$A_t = \int_0^1 \sum_{i=1}^{2} \left(V_{ijt} + V_{ijt}^e\right) dj,$$

where V_{ijt} is the value of an incumbent intermediate good firm i in sector j at time t and V_{ijt}^e is the value of an entrant that can displace firm i in sector j at time t. These value functions are explained in greater detail in section 3.3. r_t is the rate of return on the portfolio of firms. On a balanced growth path with constant growth rate of output g this yields the standard Euler equation $r = g\psi + \rho$.

3.2 Final Good Producers

The competitive final goods sector combines intermediate goods and labor to create the final output good which is used in consumption, research, and intermediate good production. The final good firm operates a constant return to scale technology:

$$Y_t = \frac{1}{1-\beta}\left(\int_0^1 K_{jt}^{1-\beta} dj\right) L^{\beta}, \tag{1}$$

where K_{jt} is a composite of two intermediate good firms' products within sector j described below. β determines both the elasticity of substitution across sectors ($\frac{1}{\beta}$) and the labor share. The final good firm's problem of hiring sector composite goods K_{jt} for $j \in [0,1]$ and labor is:

$$\max_{K_{jt},L} P_t \frac{1}{1-\beta}\left(\int_0^1 K_{jt}^{1-\beta} dj\right) L^{\beta} - \int_0^1 P_{jt} K_{jt} dj - W_t L.$$

The first order condition for sector j's composite good given sector j's composite price index P_{jt} yields the following demand for sector j's good:

$$K_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\frac{1}{\beta}} L,$$

and the real wage is equal to the marginal product of labor:

$$\beta \frac{Y_t}{L} = \frac{W_t}{P_t}.$$

To derive the demand curve for each intermediate good producer i within sector j we need to define the sector composite goods K_{jt} explicitly:

$$K_{jt} = \left((q_{1jt} k_{1jt})^{\frac{\epsilon-1}{\epsilon}} + (q_{2jt} k_{2jt})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where q_{ijt} is the quality of firm i's product at time t (equivalently as firm i's productivity) and k_{ijt} is the output of firm i purchased by the final good producer.[15] The elasticity of substitution between product varieties in the same sector is ϵ.

The first order condition for the final goods firm's problem yields the following demand curve for firm i in sector j's output:

$$k_{ijt} = q_{ijt}^{\epsilon-1} \left(\frac{p_{ijt}}{P_{jt}} \right)^{-\epsilon} \left(\frac{P_{jt}}{P_t} \right)^{-\frac{1}{\beta}} L. \tag{3}$$

That is, demand is increasing in the firm's quality, decreasing in its price relative to the sector j price index, and decreasing in the sector's price index relative to the price index in the economy as a whole.

3.3 Intermediate Goods Producers

Each intermediate good sector features competition between two large incumbent firms with differentiated products and access to an R&D technology, plus a competitive fringe that constrains the price-setting of the incumbents. Incumbents are periodically hit with exit shocks that cause them to be replaced by a new firm. This section covers the static pricing game played by intermediate good firms and their dynamic R&D investment decision.

3.3.1 Production and Price Setting

Production Intermediate goods producers purchase final goods to transform them into differentiated intermediate goods. Each unit of intermediate output requires $\eta < 1$ units of the final good to produce. There are no other inputs to intermediate good production.

Competitive fringe Each industry contains a competitive fringe of firms that is able to produce a perfect substitute to the lower quality variety at marginal cost η. I call the incumbent firm with lower quality the *follower*, or laggard, and the incumbent firm with higher quality the *leader*. When $q_{1jt} = q_{2jt}$, the fringe can produce

[15] I use quality and productivity interchangeably because final output is homogeneous of degree one in either the qualities or the quantities of the intermediate goods firms' products.

perfect substitutes to both incumbents' varieties. One way to micro-found this assumption is by introducing a cost to filing and maintaining a patent that is sufficiently high that only the leader, who exercises some additional market power by possessing the higher quality and thus earns higher profits in duopoly competition without the fringe, would be willing to pay. The follower then allows its patent to expire and faces imitation by the fringe. Intuitively, this means that sectors in the model feature a high quality variety like a brand name product and competition among other firms to produce a generic version of that sector's product. The competitive fringe firms do not have access to an innovation technology.

This assumption of the presence of a competitive fringe is not necessary to solve the model, but makes it possible to match the average level of sales concentration across sectors in the data and generates plausible predictions for profit shares as a function of market shares (see Appendix A.4.) I solve a version of the model without the competitive fringe in Appendix B.5 and replicate the main exercise in this setting. The main results in section 4 are qualitatively unchanged.

Price setting Firms set prices a la Bertrand at each instant t. The presence of the competitive fringe implies the follower must set its price $p_{ijt} = \eta$.[16] Understanding this, the leader chooses its price as a best response to the price set by the follower.

Dropping the subscript t, the pricing problem of technology leader i in sector j is:

$$\max_{p_{ij}} p_{ij}k_{ij} - \eta k_{ij},$$

subject to the demand:

$$k_{ij} = q_{ij}^{\epsilon-1}\left(\frac{p_{ij}}{P_j}\right)^{-\epsilon}\left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L,$$

where

$$P_j = \left(\sum_{i=1}^{2} q_{ij}^{\epsilon-1} p_{ij}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

is sector j's price index.

Let $s_{ij} = \frac{p_{ij}k_{ij}}{\sum_{i=1}^{2} p_{ij}k_{ij}}$ denote firm i's market share in sector j. Then the optimal pricing policy for the market leader is:

$$p_{ij} = \frac{\epsilon - (\epsilon - \frac{1}{\beta})s_{ij}}{\epsilon - (\epsilon - \frac{1}{\beta})s_{ij} - 1}\eta. \tag{4}$$

[16] I resolve the indeterminacy of which firm(s) produces the lower quality variety in equilibrium by having the incumbent capture all sales of the lower quality variety so the fringe is not active in equilibrium.

The optimal price is the standard one for two-layered constant elasticity of demand structures (nested CES): a variable markup that rises in market share. This is easiest to see for the two extreme cases where market share is 0 or 1. When market share is 0, the firm is atomistic with respect to the sector and charges a markup $\frac{\epsilon}{\epsilon-1}$, the CES solution for an elasticity of substitution equal to ϵ. On the other hand, if the market share is 1, the firm only weighs the elasticity of substitution across sectors and sets a markup $\frac{1}{1-\beta} > \frac{\epsilon}{\epsilon-1}$ since products are less substitutable across sectors than within sectors.

3.3.2 Innovation

Incumbent intermediate goods producers have access to a research and development technology that allows them to choose an amount of research spending R_{ijt} of the final good to maximize the discounted sum of expected future profits. The decision to model R&D as a process of own-product quality improvement by incumbents is consistent with the evidence in Garcia-Macia, Hsieh, and Klenow (2019) that: (i) incumbents are responsible for most employment growth in the U.S., and this share has increased in recent years; (ii) growth mainly occurs through quality improvements rather than new varieties; (iii) creative destruction by entrants and incumbents over other firms' varieties accounted for less than 25% of employment growth from 2003-2013, consistent with earlier evidence from Bartelsman and Doms (2000).

Innovations arrive randomly at Poisson rate x_{ijt} which depends on research spending according to the function:

$$x_{ijt} = \left(\frac{\gamma R_{ijt}}{\alpha}\right)^{\frac{1}{\gamma}} q_{ijt}^{\frac{1-\frac{1}{\beta}}{\gamma}}.$$

That is, since $\beta < 1$, at higher quality levels more research spending is needed to achieve the same arrival rate of innovations x. γ and α are R&D technology parameters.

Innovations improve the quality of the incumbent firm's variety.[17] Conditional on innovating the size of the quality improvement is random. Formally, conditional on innovating,

$$q_{ij(t+\Delta t)} = \lambda^{n_{ijt}} q_{ijt},$$

[17] See Griliches (2001) for a survey of the relationship between R&D and productivity at the firm level and Zachariadis (2003) for a leading empirical test.

where $\lambda > 1$ is some minimum quality improvement and $n_{ijt} \in \mathbb{N}$ is a random variable. Note that each competitor improves over their own quality when they innovate, rather than over the quality frontier.[18] Initial qualities of all firms at $t = 0$ are normalized to 1. Let $N_{ijt} = \int_0^t n_{ijs} ds$ denote the total number of λ step improvements over a product line i since the beginning of time. The *technology gap* m_{ijt} from firm 1 in sector j's perspective at time t is defined as:

$$\frac{q_{1jt}}{q_{2jt}} = \frac{\lambda^{N_{1jt}}}{\lambda^{N_{2jt}}} \equiv \lambda^{m_{1jt}}.$$

Given λ, m_{ijt} parameterizes the relative qualities of the two firms within sector j from firm $i \in \{1, 2\}$'s perspective, representing the number of λ steps ahead or behind its competitor firm i is. m_{ijt} turns out to be the only payoff relevant state variable for the incumbent firms. For tractability I will impose a maximal technology gap $\bar{m}$, but in calibrating the model I will set the parameters so that this maximal gap rarely occurs in steady state. I assume that the only knowledge spillover between incumbents in the model occurs when a firm at the maximal gap innovates. In that case, both the innovating firm and its competitor's quality increase by the factor λ, keeping the technology gap unchanged but raising the absolute quality of the sector composite good.

The probability distribution of possible quality improvements depends on the firm's current technology gap, consistent with the evidence in section 2 that patent quality varies between market leaders and laggards. It is useful to instead imagine firms draw a new position in technology gap space $n \in \{-\bar{m}, \ldots, \bar{m}\}$ when they innovate, rather than an absolute number of λ steps, though given n and m the number of steps can be easily derived as $n - m$. As in Akcigit, Ates, and Impullitti (2018), I assume there exists a fixed distribution $\mathbb{F}(n) \equiv c_0(n + \bar{m})^{-\phi}$ for all $n \in \{-\bar{m} + 1, \ldots, \bar{m}\}$ that applies to firms that are the furthest possible distance behind their competitor and describes the probability that they move to each position in technology gap space. An example is shown in the left panel of 4. The curvature parameter ϕ is critical in the model and determines the speed of catchup by increasing or decreasing the relative probability of larger innovations. A higher ϕ means a lower probability of these "radical" improvements.[19,20] c_0 is simply a shifter to ensure $\sum_n \mathbb{F}(n) = 1$.

Given this fixed distribution for the most laggard firm, the new position distribu-

[18]Luttmer (2007) provides an additional rationale for this type of assumption: entrants are typically small and enter far from the productivity frontier, implying that imitation of other firms' technologies is difficult.

[19]As noted by Akcigit, Ates, and Impullitti (2018), this formulation converges to the less general step-by-

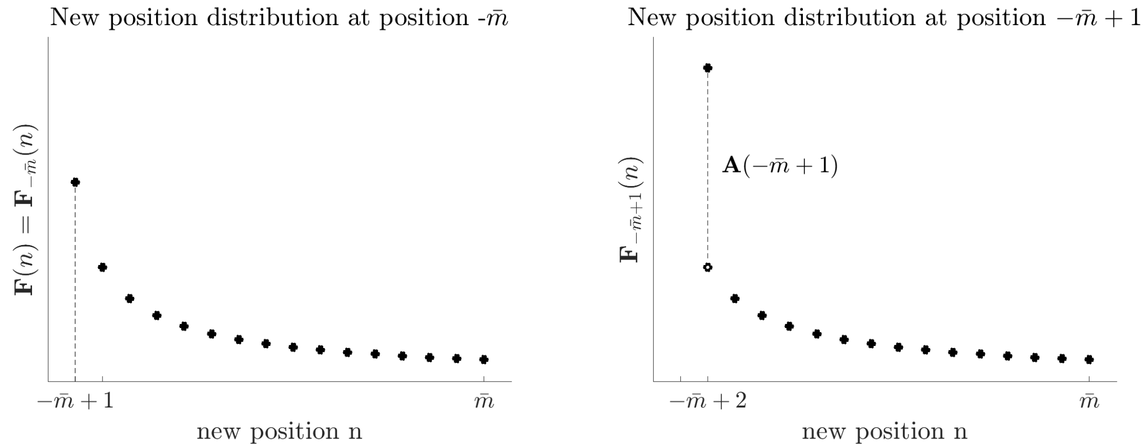


Figure 4: Examples of new position distributions for positions $-\bar{m}$ and $-\bar{m}+1$.

tion specific to each technology gap $m > -\bar{m}$ is given by:

$$\mathbb{F}_m(n) = \begin{cases} \mathbb{F}(m+1) + \mathbb{A}(m) & \text{for } n = m+1 \\ \mathbb{F}(s) & \text{for } n \in \{m+2, \ldots, \bar{m}\} \end{cases},$$

where $\mathbb{A}(m) \equiv \sum_{-\bar{m}+1}^{m} \mathbb{F}(n)$. This distribution is shown in the right panel of Figure 4 for a firm at gap $-\bar{m}+1$. Simply put, all the mass of the fixed distribution on positions lower than the current position m is put on one-step ahead improvements. This formulation can capture the feature that laggard firms make larger improvements than leaders on average.

3.3.3 Entry and Exit

Incumbent firms face a constant exit risk δ_e. If an incumbent is hit with this shock the incumbent is replaced by an entrant that takes over the product line with the same quality level (and thus technology gap to the other incumbent in the sector) as the incumbent it replaces. This shock captures many reasons why incumbents may exit or be displaced by entrants that are not directly related to the incumbent firms' innovations such as adverse financial shocks, negative taste shocks for the incumbent's brand, expiration of the incumbent's patent or knowledge diffusion as in Akcigit and Ates (2020), or cost shocks to specific inputs used by the incumbent.

step model as $\phi \to \infty$.

[20]The use of "radical innovation" in this paper to describe a relatively large quality improvement differs from some other papers in the literature such as Acemoglu and Cao (2015) who use "radical innovation" to refer to an entrant replacing an incumbent.

3.3.4 Intermediate Goods Firms Value Functions

Turning to the firm value functions, I will show that the technology gap $m \in \{-\bar{m}, \ldots, \bar{m}\}$ is sufficient to describe the firms' pricing and innovation strategies, and that firm values scale in some function of their current product quality q_{ijt}.

The proof that pricing decisions and market shares depend only on m_{ijt} (and not on the level of quality q_{ijt}) is in Appendix B.1 which shows that we can define $p(m)$ as the price set by a firm at gap m. Next consider the flow profits of an incumbent, denoting the optimal price of the leader at technology gap m as $p(m)$, and dropping subscripts t and j for now:

$$\pi(m, q_i) = \begin{cases} 0 & \text{if } m \leq 0 \\ (p(m) - \eta)k_i & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}.$$

Plugging in equation 3 for k_i and using the definition of the sector price index yields:

$$\pi(m, q_i) = \begin{cases} 0 & \text{if } m \leq 0 \\ q_i^{\frac{1}{\beta}-1}(p(m) - \eta)p(m)^{-\epsilon}(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1}p(-m)^{1-\epsilon})^{\frac{\epsilon-\frac{1}{\beta}}{1-\epsilon}} & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}.$$

For the dynamic problem, I will use a guess and verify method to verify that firms' strategies depend only on m and that firm values scale in some function of q_{ijt}. Dropping the subscript ij and given an interest rate r_t, the value function of a firm with technology gap m_t to its competitor and quality level q_t can be written:

$$\begin{aligned} r_t V_{mt}(q_t) - \dot{V}_{mt}(q_t) = \max_{x_{mt}} \{ & \pi(m, q_t) - \alpha \frac{(x_{mt})^\gamma}{\gamma} q_t^{\frac{1}{\beta}-1} \\ & + x_{mt} \sum_{n_t=m+1}^{\bar{m}} \mathbb{F}_m(n_t)[V_{nt}(\lambda^{n_t-m} q_t) - V_{mt}(q_t)] \\ & + x_{(-m)t} \sum_{n_t=-m+1}^{\bar{m}} \mathbb{F}_{-m}(n_t)[V_{(-n)t}(q_t) - V_{mt}(q_t)] \\ & + \delta_e(0 - V_{mt}(q_t) \}. \end{aligned} \tag{5}$$

The firm chooses the arrival rate of innovations x_{mt}. The first line denotes the flow profits and the research cost R_{ijt} given the choice of x_{mt}. The second line denotes the probability the firm innovates and sums over the possible states the firm could move to using the new position distribution and the firm's new value function with higher quality and a larger quality advantage over its rival. The third line denotes the chance the firm's rival innovates and the change in the firm's value because its relative quality falls when the rival innovates. The final line denotes the chance the entrant displaces the incumbent. The slightly altered equations for firms at the minimum and maximum gaps because of knowledge spillovers are given in Appendix B.2.

A guess and verify approach verifies that $V_{mt}(q_t) = v_{mt} q_t^{\frac{1}{\beta}-1}$. Thus one can focus on a Markov Perfect equilibrium where firms' strategies depend only on the payoff-relevant state variable m, which characterizes the technology gap between incumbents.

The firm's optimal innovation rate x_{mt} is the solution to the first order condition of equation (5), which gives:

$$x_{mt} = \begin{cases} \left(\frac{\sum_{n=m+1}^{\bar{m}} \mathbb{F}_m(n_t)[(\lambda^{n_t-m})^{\frac{1}{\beta}-1} v_{nt} - v_{mt}]}{\alpha} \right)^{\frac{1}{\gamma-1}} & \text{for } m < \bar{m} \\ \left[\frac{1}{\alpha}(\lambda^{\frac{1}{\beta}-1} - 1) v_{\bar{m}t} \right]^{\frac{1}{\gamma-1}} & \text{for } m = \bar{m} \end{cases}.$$

Intuitively, firms choose a higher arrival rate of innovations when the cost of R&D α is low, and when the expected gain from innovating is high, captured by the probability of moving to different positions in technology gap space upon innovating $\mathbb{F}_m(n)$, the value v_n of being at gap n, and the minimum size of quality improvements λ. All else equal, greater expected innovativeness of laggards (more weight on states where they catch up to or overtake the leader), should encourage more innovation by laggard firms. However, the v_n terms also capture the probability of being displaced in the future, so these values are endogenously determined along with the chance of displacement by rivals due to innovation or the chance of being hit with an exit shock δ_e. At t, the value of a potential entrant in product line i in sector j is simply $V_{ijt}^e = \delta_e V_{ijt}$.

3.4 Equilibrium Output

Plugging in the intermediate goods firms' pricing decisions yields the following expression for final output Y_t, derived in Appendix B.3:

$$Y_t = \frac{1}{2} \frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \sum_{m=-\bar{m}}^{\bar{m}} Q_{mt}, \tag{6}$$

where Q_{mt} is defined as:

$$Q_{m,t} = \int_0^1 \left(q_{it}^{\epsilon-1} p(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i\in\mu_{mt}\}} di$$
$$= \left(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{\frac{1-\beta}{\beta(\epsilon-1)}} \int_0^1 q_{i,t}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i\in\mu_{mt}\}} di. \tag{7}$$

Here, μ_{mt} is the measure of firms at each technology gap m at time t (normalizing measure of firms to one) and Q_{mt} is a particular index of the qualities of all firms at gap m. The change in output between t and $t+dt$ will therefore depend on the changes $\dot{Q}_{mt}$ for each technology gap m which in turn depend on the innovation arrival rates x_{mt} chosen by firms and the exogenous distribution of quality improvement sizes $\mathbb{F}(n)$. The term $(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon})^{\frac{1-\beta}{\beta(\epsilon-1)}}$ weights the change in qualities of firms at gap m depending on the prices set by firms at gap m and $-m$, capturing static distortions from firms' markups. Note that entry and exit are not a source of growth in the model because they have no impact on the qualities of the intermediate goods in the economy or on markups. The final component determining output will be the measure of firms at each technology gap μ_{mt} that is itself an endogenous object. The next section describes how to solve for the measures μ_{mt}.

3.5 Distribution Over Technology Gaps

Firms move to technology gap n through innovation from a lower technology gap, or because their competitor innovates to gap $-n$. The distributions $\mathbb{F}_m(n)$ and $\mathbb{F}_{-m}(-n)$ respectively determine these probabilities, combined with the innovation efforts of firms at m and $-m$, for all $m < n$ and $-m < -n$. The outflows from gap n are due to the firm at n or its competitor at $-n$ innovating. Putting this together into the Kolmogorov forward equations for the evolution of the mass of firms at each gap:

$$\dot{\mu}_{nt} = \sum_{m=-\bar{m}}^{n-1} x_m \mathbb{F}_m(n)\mu_{mt} + \sum_{m=n+1}^{\bar{m}} x_{-m}\mathbb{F}_{-m}(-n)\mu_{mt} - (x_n + x_{-n})\mu_{nt}. \tag{8}$$

The highest and lowest technology gaps are special cases because of spillovers: if the firm at the highest gap innovates both firms remain at the same gap in the next instant:

$$\dot{\mu}_{-\bar{m}t} = \sum_{m=-\bar{m}+1}^{\bar{m}} x_{-m}\mathbb{F}_{-m}(\bar{m})\mu_{mt} - x_{-\bar{m}}\mu_{-\bar{m}t} \tag{9}$$

$$\dot{\mu}_{\bar{m}t} = \sum_{m=-\bar{m}}^{\bar{m}-1} x_m \mathbb{F}_m(\bar{m}) \mu_{mt} - x_{-\bar{m}} \mu_{\bar{m}t}. \tag{10}$$

On a balanced growth path, $\mu_{mt} = \mu_m$ for all m, t. Replacing the left hand side of the above equations with zero change in equilibrium and the measures on the right hand side with the constants μ_n, μ_m defines a system of $2\bar{m}+1$ equations in $2\bar{m}+1$ unknowns that determine the steady state distribution of firms over possible technology gaps. There are several additional restrictions on the solution to this system. First, for each firm at m there is a firm at $-m$ (that is, the stationary distribution is symmetric). Second, I impose the restriction that the measure of all incumbent firms sums to one.

3.6 Output Growth

Differentiating equation 6 with respect to time yields the following expression for the growth rate:

$$\frac{\dot{Y}_t}{Y_t} = g_{Yt} = \frac{1}{2} \frac{1}{1-\beta} \sum_{m=-\bar{m}}^{\bar{m}} \frac{\dot{Q}_{mt}}{Y_t}.$$

It's useful to define:

$$\tilde{Q}_{mt} = \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_{mt}\}} di \tag{11}$$

So that:

$$g_{Yt} = \frac{1}{2} \frac{1}{1-\beta} \sum_{m=-\bar{m}}^{\bar{m}} (p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon})^{\frac{1-\beta}{\beta(\epsilon-1)}} \frac{\dot{\tilde{Q}}_{mt}}{Y_t}.$$

The subsequent analysis focuses on a balanced growth path where $\frac{\dot{\tilde{Q}}_{mt}}{Y_t}$ is constant for all m. On this balanced growth path consumption and output grow at a constant growth rate g and the mass of firms at each technology gap μ_m is constant. In general it is not possible to solve for this growth rate in closed form, but for a given set of model parameters it is possible to check the existence and uniqueness of such a balanced growth path and find the value of g as the solution to a system of equations. A more detailed derivation of these results is provided in Appendix B.4.

3.7 Equilibrium Definition

Let $R_t = \int_0^1 \sum_{i=1}^2 R_{ijt} dj$ denote total research and development spending by incumbents, C_t total consumption, and $K_t = \int_0^1 \sum_{i=1}^2 \eta k_{ijt} dj$ total purchases of final goods for production of intermediate goods.

A Markov-Perfect equilibrium is an allocation $\{k_{ijt}, K_t, x_{ijt}, R_t, Y_t, C_t, L, \mu_{mt}, Q_{mt}, A_t\}_{i\in\{1,2\},j\in[0,1],m\in[-\bar{m},\bar{m}]}^{t\in(0,\infty)}$ and prices $\{r_t, W_t, p_{ijt}\}_{i\in\{1,2\},j\in[0,1]}^{t\in(0,\infty)}$ such that for all t:

1. Households choose C_t and A_t to solve the problem described in section 3.1.
2. Final goods firms solve their problem to hire labor L and purchase intermediate goods k_{ijt} optimally according to the problem in section 3.2.
3. Intermediate good firms choose p_{ijt} and x_{ijt} to solve their innovation and price-setting problems described in section 3.3.
4. The final goods market clears: $Y_t = C_t + R_t + K_t$.
5. The asset market clears, pinning down r_t via the household's Euler equation.
6. Labor market clears, pinning down the wage rate from the final good producer's problem.
7. μ_{mt} and Q_{mt} are consistent with firms' choices of x_{ijt}.

This completes the description of the model. The next section develops further intuition about the model under reasonable model parameter values for the U.S. economy.

4 Model Estimation

The quantitative analysis precedes as follows. I estimate an initial steady state for the model by matching various moments for the U.S. economy in the period of high patent quality between 1994 and 2003 ("1990s") using data on U.S. public firms from Compustat as well as aggregate moments. Using this initial calibration I describe firms' pricing and innovation strategies to develop intuition about the model. I then re-estimate the model parameters for 2004-2017 ("2000s") in order to infer changes to the economy between these two periods.

4.1 Baseline Calibration for the 1990s

Four parameters are calibrated outside the model. The inverse intertemporal elasticity of substitution ψ is set to 1. The labor share, β, is set to 0.6. This implies an elasticity of substitution across sectors of $\frac{1}{\beta} = \frac{5}{3}$, within the range of upper-level elasticities of substitution estimated in Hobjin and Nechio (2019). The curvature of the R&D cost function, γ, is calibrated outside to match the empirical evidence on the elasticity of patenting to R&D expenditures, discussed in Acemoglu et al. (2018). The maximum technology gap, $\bar{m}$, is set to 16.

The rest of the parameters for the baseline model, shown in Table 1, are estimated using a simulated method of moments approach described in appendix C.2 to match targets for the 1990s equilibrium (1994-2003). These targets are given in Table 2. The data sources and computation methods for the data moments are given in appendix A.1. Appendix C.1 describes the solution method for finding the model steady state.

Parameter	Value	Meaning/source
ψ	1	Inverse intertemporal elasticity of substitution
ρ	**0.026**	Rate of time preference (annual)
β	0.6	Labor share/Nechio & Hobijn (2017)
ϵ	**4.21**	Elasticity of substitution within sectors
η	**0.64**	Marginal cost of intermediate producers
δ_e	**0.089**	Exogenous entry/exit rate (annual)
λ	**1.059**	Min. qual. improvement
γ	2	Curvature of R&D function
α	**4.18**	R&D cost parameter
$\bar{m}$	16	Maximum number of steps ahead
ϕ	**0.88**	Curvature of patent quality distribution

Table 1: Model parameters (estimated parameters in bold), 1990s.

The moments include the main phenomena of interest: aggregate productivity growth, average market leader's share of industry sales, the profit share of total output, average patent quality, and the rate of leadership turnover from either entry or being overtaken by an incumbent rival. In addition to average patent quality across all firms, I include the average patent quality of market leaders to help identify λ and ϕ separately. The other two moments, R&D as a share of output and R&D as a share of sales at the firm level, are included to help discipline the R&D cost parameter and the discount rate.

The model performs relatively well in fitting the data, particularly for productivity growth and concentration. Intuitively, the minimum step size λ and ϕ govern the average patent quality, with λ acting as a level shift in patent quality for all types of firms and ϕ shifting the probability that laggards make drastic or incremental improvements, holding patent quality of leaders fixed. The R&D cost parameter α influences the amount all firms spend on R&D and helps match aggregate expenditures as a share of output and R&D as a share of firms' sales. The entry/exit shock δ_e helps match leadership turnover. One problem with the model fit is for the R&D as a share of sales at the firm level. This can be attributed to the fact that productivity growth is purely due to R&D in the model, whereas in the reality productivity may improve for other reasons, such as management practices or improved human capital.

The estimated parameters are reasonable: a discount rate ρ of 2.6% annually implies a real interest rate in the model of 4.4%. An elasticity of substitution ϵ of 4.21 results in an average markup of 1.24, in line with the evidence summarized in Mongey (2017), particularly de Loecker, Eeckhout, and Unger (2020), that suggests markups for U.S. public firms in the 1990s ranged from 1.2 to 1.3. The entry/exit rate of about 9% per year is in line with entry and exit rates for the U.S. reported by Decker et al. (2016). The model also matches non-targeted heterogeneity in R&D intensity (R&D as a share of sales) well, as shown in Table 3. Section 4.4 describes the model's fit for additional non-targeted moments.

Targeted moments, 1994-2003	Data	Model
Avg. TFP growth, %	1.74	1.75
Avg. leader market share, %	43.34	44.62
R&D share of GDP, %	1.8	1.91
Profit share of GDP, %	5.24	6.02
Avg. R&D/sales, %	2.56	5.18
Avg. patent stock growth per patent, %	23.52	22.26
Avg. patent stock growth per patent, leaders, %	9.08	10.52
Avg. leadership turnover, %	13.74	13.26

Table 2: Model fit for targeted moments from estimation of 7 parameters for 1990s.

Non-targeted moments, 1994-2003	Data	Model
Avg. R&D/sales, followers, %	4.80	6.98
Avg. R&D/sales, leaders, %	1.66	2.51

Table 3: Model fit for R&D heterogeneity, 1990s equilibrium.

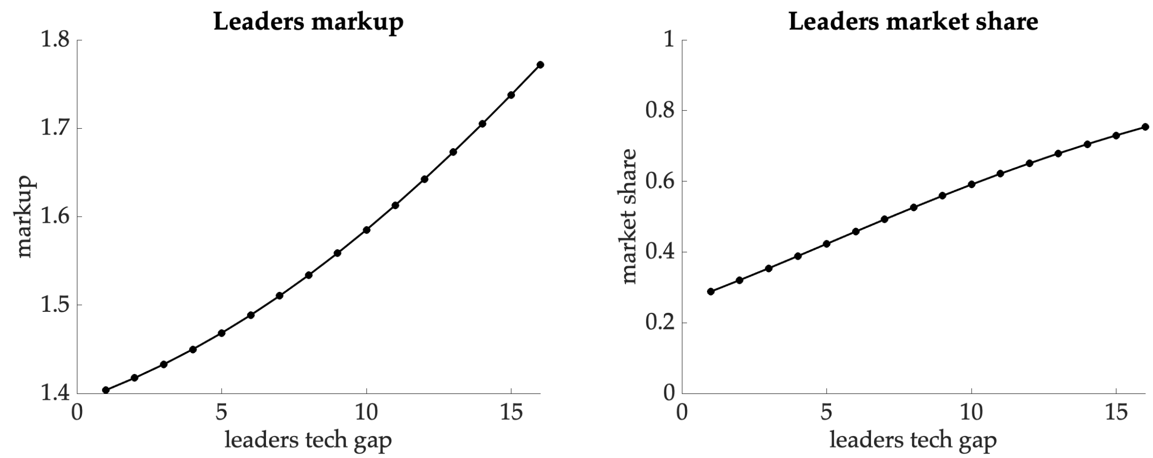


Figure 5: Markups and resulting market shares as a function of the leader's technology gap m.

4.2 Properties of the Baseline Model

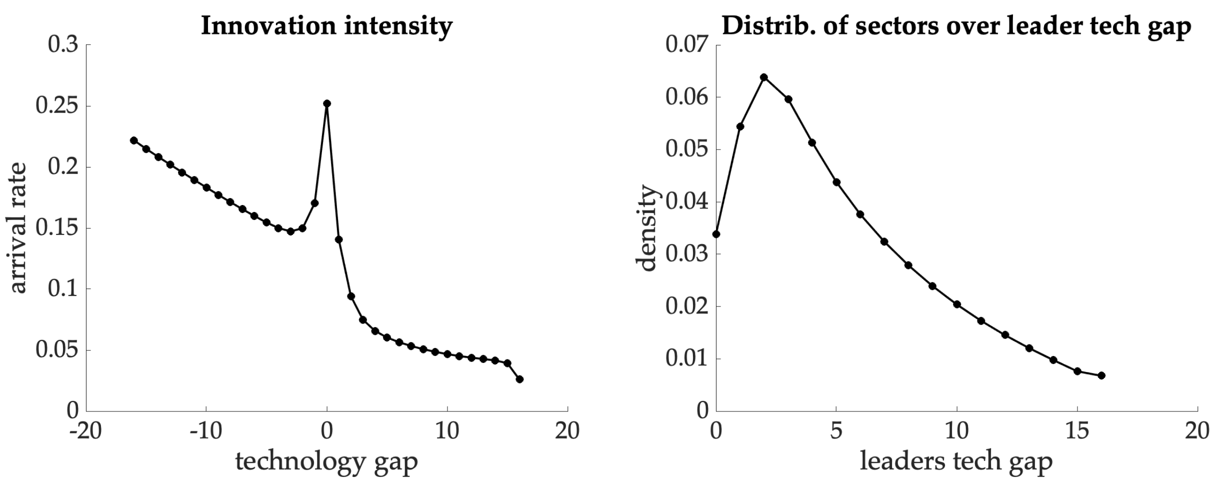
Before turning to how the estimated parameters change when targeting the same moments for the 2000s, I use the 1990s steady state to develop intuition about the quantitative model, particularly the incumbent firms' strategies.

Using the parameters from Table 1, the market shares and prices for market leaders as a function of the leader's technology gap are plotted in Figure 5. The leader's optimal price $p(m)$ rises as the technology gap widens (that is, as the leader's relatively quality improves). The leader's market share rises from around 30% of sales when the leader is one step ahead (that is, when the leader's quality is 5.9% higher than the laggard's) to 80% of the market at the maximum 16 steps ahead. The follower, which must set price equal to marginal cost because of the presence of the competitive fringe, has a large market share due to its relatively low price, and its market share is increasing in its relative quality.

The competitive fringe assumption also plays an important role in determining the shape of the innovation policy as a function of technology differences depicted in

Figure 6a, specifically the hump shape. This shape has been suggested theoretically in the work of Harris and Vickers (1987), Aghion et al. (2001), and Akcigit, Ates, and Impullitti (2018), and found in a variety of studies including Aghion et al. (2005), Aghion et al. (2014), Aghion et al. (2019b), and Zhang (2018). The hump shape appears in this model because the competitive fringe assumption means that the greatest incremental gain in flow profits comes from obtaining quality leadership (and thus escaping competition with the fringe), so the arrival rate of innovations will be highest when firms have equal quality.[21]

Figure 6: Baseline Innovation Policies and Stationary Distribution



(a) Innovation policies

(b) Stationary distribution

Finally, Figure 6b shows the stationary distribution of sectors over the market leader's technology gap to its rival. Because followers innovate more frequently than leaders and have a high chance of catching their competitor when they do, there is a high rate of turnover in market leadership and technology gaps do not grow very large on average. Most sectors feature a leader that is just a few steps ahead of its rival, but there is a right tail of sectors with a large and dominant "superstar" that has a much higher quality product than its rival and captures a large share of industry sales.

[21]See Appendix B.5 for the version without the competitive fringe. The mechanism and main results are qualitatively unchanged.

4.3 Re-estimation for the 2000s

Re-estimating the model for the 2000s uses the model to infer the role of different channels suggested in the literature to explain changes in concentration and productivity growth and compare the strength of these other channels to the strength of declining laggard patent quality to explain these trends.

Changing the discount rate ρ captures the interest rate channel proposed by Liu, Mian, and Sufi (2019).[22] A decrease in entry and exit shocks δ_e can capture declining knowledge diffusion from incumbent firms to new entrants or rising entry costs (Akcigit and Ates (2020), Corhay, Kung, and Schmid (2020)). An increase in the research cost parameter α implies that more R&D spending is needed to achieve the same arrival rate of innovations, capturing the cost side of the hypothesis of Bloom et al. (2020) that ideas are getting harder to find. A decrease in the elasticity of substitution within sectors ϵ captures increased market power over the leader's variety, in line with Jones and Philippon (2016). On the other hand, an increase in ϵ captures the superstar firm hypothesis of Autor et al. (2020) that competitive pressures within industries have risen, causing the most productive firms to capture a larger share of total industry sales.[23] Finally, changes in ϕ govern the expected patent quality for different types of firms by changing the distributions $\mathbb{F}_m(n)$. Changing ϕ represents the research *output* side of Bloom et al. (2020)'s hypothesis, capturing the possibility that the quality of new ideas, particularly for laggard firms, is falling.

Table 4 shows the targeted moments and model fit for the 2000s estimation. Productivity growth slowed substantially compared to 1994-2003, while the average market leader's sales share grew by about 4 percentage points. Both aggregate and firm level research and development expenditures grew, as noted by Bloom et al. (2020). As discussed in detail in section 2, patent quality and leadership turnover declined. The estimation has some trouble matching the decline in the growth rate alongside an increase in R&D expenditure, but otherwise performs well.

Table 5 compares the estimated parameters to fit the two steady states. The households' discount rate declines slightly in the 2000s. Consistent with Decker et al. (2016), the entry rate of new firms declines (alternately, incumbents are less likely to be dis-

[22] However, the model economy is not close to the very low interest rate environment discussed in Liu, Mian, and Sufi (2019) where strategic effects can dominate. See Goldberg, Lopez-Salido, and Chikis (2020) for further discussion.

[23] Autor et al. (2017) speculate that such pressures may have risen because of increasing competition from foreign firms or greater price sensitivity due to better search technology such as online retail.

Targeted moments, 2004-2017	Data	Model
Avg. TFP growth, %	0.49	0.74
Avg. leader market share, %	48.12	48.89
R&D share of GDP, %	1.89	1.32
Profit share of GDP, %	6.61	6.71
Avg. R&D/sales, %	3.8	3.54
Avg. patent stock growth per patent, %	11.71	11.88
Avg. patent stock growth per patent, leaders, %	5.38	7.3
Avg. leadership turnover, %	9.27	9.37

Table 4: Targeted moments from estimation of 7 parameters for 2000s.

placed, consistent with the hypothesis of Akcigit and Ates (2019) that the rate of knowledge diffusion is slowing down). To match the fact that R&D expenditures as a share of GDP rose between the 1990s and the 2000s, the cost α of performing R&D declines. However, the expected *output* of R&D (patent stock growth per patent) conditional on innovating declines substantially due to the decrease in the probability of radical innovations, driven by the substantial increase in ϕ. The elasticity of substitution within sectors ϵ rises slightly. The marginal cost of the intermediate goods firms rises modestly. I explore these results in more detail in Section 5.

Parameter	1990s	2000s	Meaning/source
ρ	**0.026**	**0.025**	Rate of time preference (annual)
ϵ	**4.21**	**4.32**	Elasticity of substitution within sectors
η	**0.64**	**0.70**	Marginal cost of intermediate producers
δ_e	**0.089**	**0.081**	Exog. entry/exit rate
λ	**1.059**	**1.063**	Min. qual. improvement
α	**4.18**	**3.33**	R&D cost parameter
ϕ	**0.88**	**1.52**	Curvature of patent quality distribution

Table 5: Comparison of estimated parameters, 1990s vs. 2000s model equilibria.

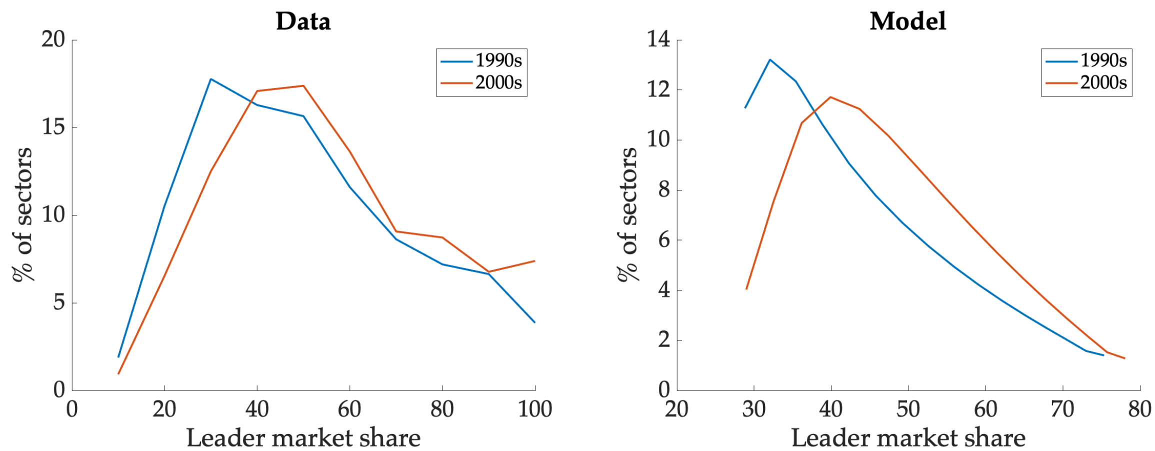


Figure 7: Shift in market concentration across sectors, data vs. model. Leader market share in the data is for 4-digit SIC industries in Compustat.

4.4 Model Validation

The model performs well in matching not just the average level of concentration but the entire distribution of market leaders' market shares across sectors of the economy in both periods. Figure 7 compares the empirical distribution of leader market shares in the two study periods (1994-2003 and 2004-2017) in the data and in the model. This shift is mainly due to increased average quality differences between leaders and followers in steady state, consistent with the findings of Andrews, Criscuolo, and Gal (2015) and Andrews, Criscuolo, and Gal (2016) that productivity differences within industries have grown over this period. They also find that this divergence is particularly pronounced in ICT intensive sectors, and that sectors with wider productivity gaps have experienced deeper productivity slowdowns. In Figure 20 of Appendix A.5 I also show that rising market concentration and the productivity slowdown are correlated at the sector level.

The model also predicts that the average leader's share of total industry R&D expenditures rises from 20% to 53%, I find that among Compustat firms, market leaders now perform a larger share of total R&D expenditures in their industries than during the 1990s (Figure 21 in Appendix A.5), though the increase in the data is less dramatic than in the model, from around 38% in 1999 to a peak of 50% in 2010. Anderson and Kindlon (2019) also find a decline in R&D intensity among companies with fewer than 250 employees and an increase among larger firms in the National Science Foundation's Business R&D and Innovation Survey covering both public and private firms over this period. Akcigit and Ates (2019) also document increasing concentration of patents among the top 1% of patenting firms and increasing flows of R&D employees from small to large firms.

5 Results

This section obtains the main results of the paper by decomposing the role of different parameters changes estimated in Section 4.3 in explaining rising concentration and the productivity slowdown. The decomposition suggests the greatest scope for declining patent quality of laggards, compared with other explanations such as declining real interest rates, slowing knowledge diffusion from incumbents to entrants, and declining entry rates, to explain the observed trends in productivity growth and concentration. Holding the other parameters fixed at their initial levels, the model-implied change to the patent quality distribution explains just over 100 percent of the productivity slowdown and about 40 percent of the rise in concentration observed in the data, and is consistent with the decline in patent quality documented in section 2. Two different decomposition exercises suggest that between 25 and 60 percent of the productivity slowdown generated by an exogenous change in the patent quality distribution is due to firms' responses to this change, in particular a relocation of innovation effort from laggard firms towards market leaders.

Only one other estimated parameter change moves both market concentration and productivity growth in the directions suggested by the data. This is an increase in the elasticity of substitution between product varieties within sectors. I discuss how such a change represents a dynamic version of the exercise in Autor et al. (2017) modelling the rise of superstar firms. This experiment in the quantitative model suggests that on impact this change raises measured TFP, but has a negative effect on growth dynamically through a standard Schumpeterian channel that lower markups reduce incentives for innovation. A change in patent quality, on the other hand, generates a modest rise in markups and the profit share that is consistent with the data.

5.1 Decomposition

To understand the contribution of each estimated parameter change to matching the trends in the data, Table 13 in Appendix C.3 reports the effect of changing each parameter from its 1990s value to its 2000s value, holding the other model parameters fixed at their 1990s estimated values. Note that these are not the marginal effects of each parameter on each moment since the moments are endogenously determined in steady state.

The decline in the discount rate ρ and the exit rate δ_e play a similar role in increasing incumbents' R&D expenditures in order to match the rise in the R&D as a share of GDP, since all incumbent firms discount expected future profits less which increases incentives for innovation. A decrease in the cost of R&D α also helps match the rise in R&D expenditures in the data. However, because they result in more R&D, these changes all have the additional effect of *raising* the TFP growth rate absent the other parameter changes. They do not substantially change the average level of concentration. Only the estimated changes in ϕ, governing relative patent quality of leaders and laggards, and ϵ, governing product substitutability, push both concentration and productivity growth in the same direction as in the data. I next explore these two parameter changes in more detail.

5.2 Role of Changing Patent Quality

Table 6 summarizes the role of the model-implied change in ϕ compared to changes in all the other parameters at once to match the moments of interest. To decompose the effect of a change in the patent quality distribution, I use the values in Table 13 to compute the share of the changes in the data that are explained by a change in ϕ as follows:

$$\frac{M_j(\boldsymbol{\theta}_{1990s}, \phi_{2000s}) - M_j(\boldsymbol{\theta}_{1990s}, \phi_{1990s})}{D_{j,2000s} - D_{j,1990s}} \times 100$$

Where M_j is moment j in the model steady state with the other parameters $\boldsymbol{\theta}$ held fixed at their estimated 1990s values and $D_{j,t}$ denotes the moment's value in the data at time $t \in \{1990s, 2000s\}$.

A change in the patent quality distribution alone, consistent with lower probability that the followers catch up to leaders through innovation, can explain 102% of the productivity slowdown and about 46% of the rise in concentration in the data.[24] It explains about a quarter of the rise in the profit share and more than three quarters of the decline in turnover in market leadership. As can be seen in Table 6, the model-implied

[24]Transition dynamic analysis in Appendix C.4 suggests the productivity slowdown occurs within a few years, while concentration takes a long time to reach its new steady state level, consistent with Figure 1.

Moment	Data change (pp)	Decomposition Innov. (%)	All others (%)
TFP	-1.28	102.4	-60.0
Concent.	4.78	45.6	31.0
R&D/GDP	0.09	-1322.2	1056.6
Profits/GDP	1.37	23.4	16.1
R&D/Sales	1.24	-262.1	221.0
Pat. qual.	-11.81	95.5	-15.7
New leader	-4.47	76.3	-7.6

Table 6: Share of changes in moments between 1990s and 2000s explained by estimated parameter changes in Table 5. Column labelled "Innov." holds other parameters fixed at 1990s values while ϕ changes to its estimated 2000s value. "All others" column holds ϕ fixed and allows the six other estimated parameters to change to 2000s values. Positive sign in the second and third columns indicates same direction of change as in the data.

change in ϕ from the re-estimation of the model also closely matches the observed decline in average patent quality documented in section 2.2. Figure 8 shows the expected quality improvement from innovation for a firm at each technology gap for the two different model-implied values of ϕ, holding the minimum quality improvement λ fixed at its 1990s value.

Turning to the mechanisms behind these changes, firms with lower quality than their rival ($m < 0$) respond strongly to a decline in the expected return from innovating by choosing a lower arrival rate of innovations (Figure 9a). Facing a lower probability of being overtaken, market leaders discount future gains to innovation less and choose a slightly higher rate of innovations.

Taken together, this relocation of innovative activity from followers to leaders causes the stationary distribution of sectors over the leader's technology gap to shift right: more sectors now feature a leader that is further ahead (Figure 9b). The rise in concentration, average markups (from 23.8% to 25%), and the profit share in the model equilibrium with lower patent quality is driven purely by this composition effect. Note that fixing innovation effort at its 1990s level in the model but increasing ϕ (decreasing the average innovation size) would result in a *tighter* distribution

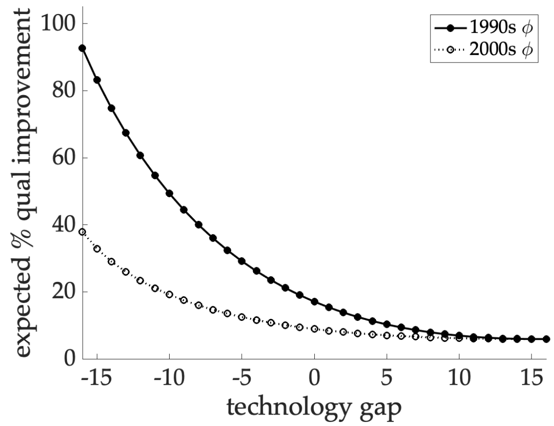
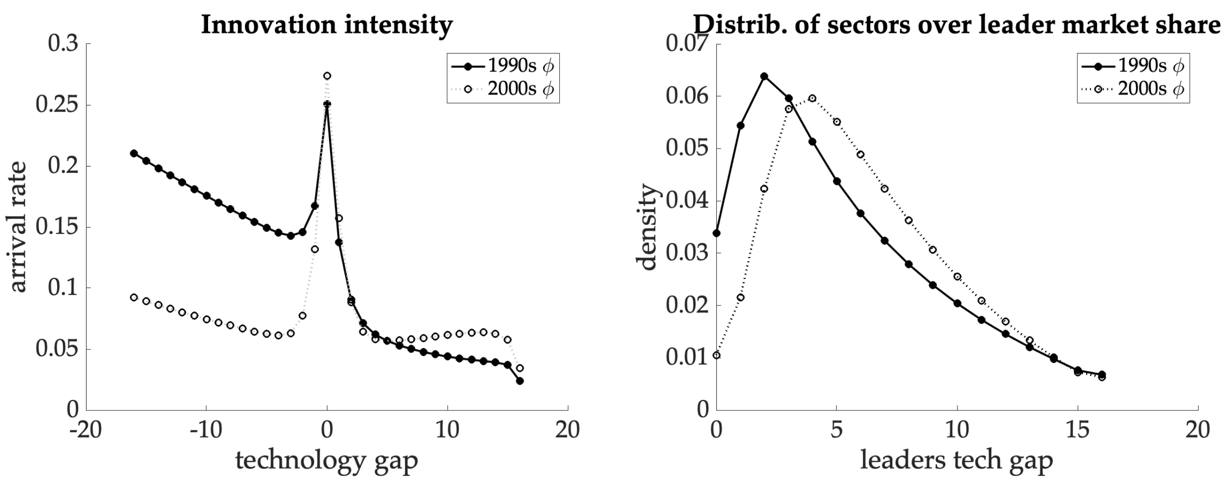


Figure 8: Expected quality improvement from innovation as a function of firm's current technology gap, two different values of patent quality parameter ϕ.

Figure 9: Innovation Policies and Stationary Distribution, Role of Patent Quality



(a) Innovation policies

(b) Stationary distribution

around the neck-and-neck state, because competitors pull away from each other less frequently under a higher ϕ regime. This counterfactual is plotted in Figure 10).

The growth rate declines when laggards' patent quality is lower for two reasons. First, there is an endogenous effect that comes from changes in firms' innovation policies x (Figure 9a). R&D expenditures as a share of output decline from 1.9% to 0.7% (Table 6). The decline in R&D expenditures is concentrated among industry laggards, whose average R&D intensity declines from 7% in the 1990s to 1.5% in the 2000s. Leaders' average R&D intensity declines just slightly from 2.5% to 2%. As a result,

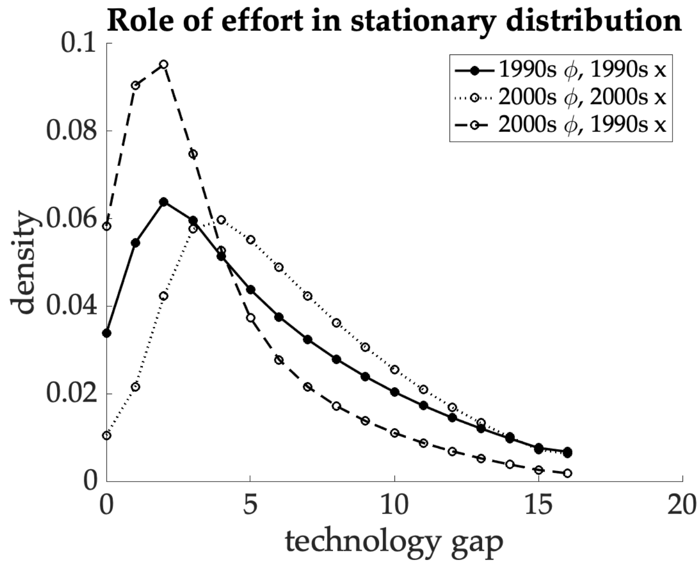


Figure 10: Stationary distribution of firms over technology gaps in 1990s and 2000s equilibria, plus distribution assuming firms innovation policies x_m are fixed at their 1990s values while ϕ changes to its estimated value in the 2000s equilibrium (counterfactual).

the average leader's share of total industry R&D rises from 20% in the equilibrium corresponding to the 1990s to 51% in the equilibrium with lower patent quality. Leaders' quality improvements are more incremental on average, so both the level effect of reduced R&D expenditures and the reallocation effect contribution to slower productivity growth.

Second, even if firms' innovation policies were unchanged, lowering the average patent quality exogenously lowers the growth rate. To decompose the importance of these two channels, I conduct two different counterfactual exercises (Table 7). First, I solve the model holding ϕ fixed at its 1990s estimated value but allowing firms' innovation policies x to change to their 2000s values. The growth rate under this counterfactual is 0.9% per year, which accounts for 60% of the decline in productivity growth due to changing patent quality in the model. The other decomposition fixes firm innovation policies at their 1990s values and reduces patent quality exogenously. The growth rate in this counterfactual is 0.8%, accounting for about 74% of the productivity slowdown in the model.

Decomposition	% of slowdown explained
Role of effort (ϕ fixed, x changes)	60.2
First order effect (x fixed, ϕ changes)	74.4

Table 7: Growth decompositions. Details in the text.

5.3 Role of Elasticity of Substitution

The elasticity of substitution within sectors ϵ plays an important role in the determination of the level of concentration and the growth rate.[25] Though the re-estimation exercise suggests only a small change in ϵ, I next explore larger changes in both directions in ϵ because each can capture (in a very reduced form) different structural changes in the U.S. economy that have been suggested in the literature recently to explain rising concentration or rising markups. These exercises illustrate how the model can be used to unify the neo-Schumpeterian endogenous growth literature with the literature on superstar firms and rising market power. Neither change matches the direction of all the moments of interest that declining patent quality of laggard firms does, though the superstar firm experiment gets closer to the data than increased market power. This is because the calibrated model has the standard Schumpeterian feature that increased market power gives a greater incentive for innovation.

5.3.1 Increasing Market Power?

Recent research has focused on the potential costs of rising market power and markups (see de Loecker, Eeckhout, and Unger (2020), Eggertsson, Robbins, and Wold (2018) and Edmond, Midrigan, and Xu (2018) for example) for growth and welfare. Can an increase in market power generate the same predictions for the macroeconomic changes experienced in the U.S. in recent years as a change in the probability of radical innovations in the model? I model an increase in market power as a decrease in the substitutability of products in the same sector, ϵ, making the incumbents' varieties more differentiated and increasing the markup the leader charges for the same level of quality differences.

The calibration remains the same as in Table 1. I decrease ϵ from 4.2 in the baseline to 3 as an illustration. The model-generated moments for this exercise are compared

[25] Following the same decomposition as in the previous section, the estimated change in ϵ explains 1.6% of the productivity slowdown and 8% of the rise in concentration.

to the 1990s baseline results in Table 8. Average markups rise by about 10 percentage points, about a third of the total rise estimated by de Loecker, Eeckhout, and Unger (2020). With the exception of markups, R&D, and the profit share, the results from this exercise are the opposite of what has happened in the data. Because of greater market power, the leader's markups and profits are higher for the same level of the technology gap (see Figure 12b) and this induces more innovation effort by laggard firms as they try harder to overtake the market leader (R&D/GDP rises from 1.9% to 2.8%, see Table 8 and Figure 11a). This results in a higher growth rate. There is also greater turnover in market leadership and average quality differences between leaders and followers go down (Figure 11b), contrary to the data.

Moment	Model	
	$\epsilon = 4.2$	$\epsilon = 3$
TFP growth, %	1.75	2.12
Leader market share, avg, %	44.62	40.6
R&D share of GDP, %	1.91	2.77
Profit share of GDP, %	6.02	6.54
Pat stock growth/patent, avg, %	22.26	22.08
R&D intensity, avg, %	5.18	6.76
Leadership turnover, %	13.26	14.18
Markup, avg, %	23.84	33.25

Table 8: Model comparison, market power experiment.

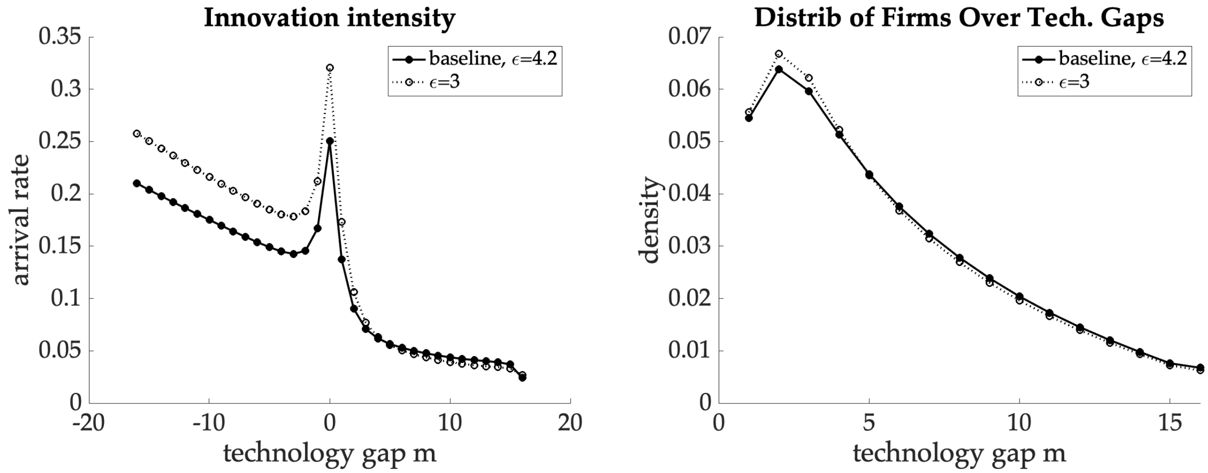
5.3.2 Superstar Firms?

Seminal work on the macroeconomic effects of superstar firms is Autor et al. (2020), who show that the rise of superstar firms can explain the declining labor share of GDP. In their static industry model (Autor et al. (2017)), firms draw labor productivities from an exogenous distribution and then make an entry decision.[26] Firms that decide to enter produce differentiated varieties of the sector good and set prices a la Bertrand.

The force for reallocation to more productive firms in Autor et al. (2017)'s model is an increase in product substitutability between varieties. The authors argue that

[26]The model in Autor et al. (2020) generalizes this formulation. The exercise presented here studies the specific shock to market toughness suggested in Autor et al. (2017).

Figure 11: Innovation Policies and Stationary Distribution, Market Power



(a) Innovation policies

(b) Stationary distribution

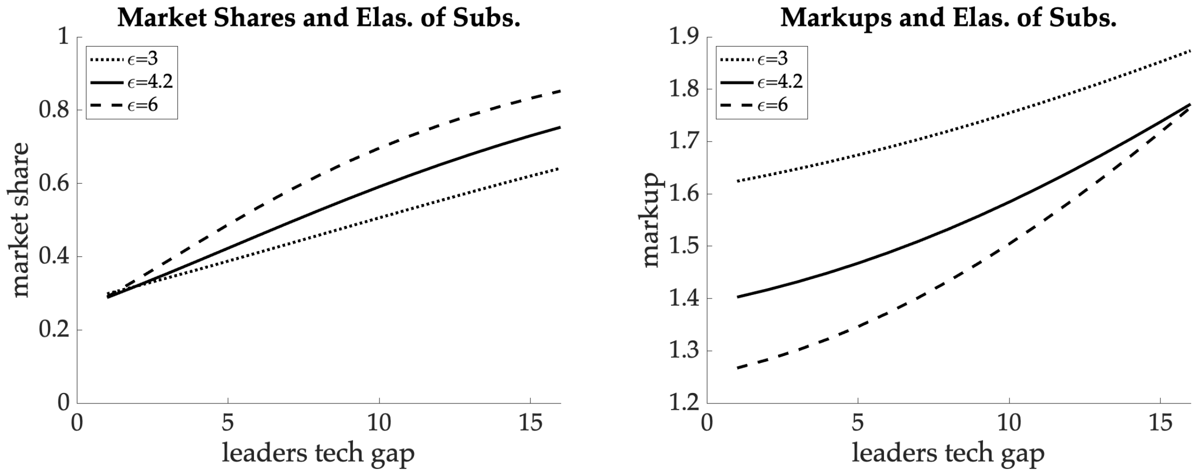
this increase could represent more fierce import competition from abroad, particularly China, in recent years or increased price sensitivity due to better search technology such as online retail. Keeping the exogenous productivity distribution fixed, an ancillary result of their analysis is that a sector's measured TFP will rise unambiguously when substitutability increases because of two forces: first, the minimum productivity threshold for entrants rises, and second, more productive firms increase their sales share.

I first show that this static reallocation result is present in my two-firm industry model with a competitive fringe for the estimated parameter values. Figure 12a plots market shares as a function of the technology gap for different values of ϵ for the baseline parameterization of the model given in Table 1. For most values of the technology gap, increasing the substitutability of the incumbents' varieties statically increases the leader's market share which raises sale-weighted sector TFP.[27]

On impact, therefore, increasing the substitutability of the firms' products tends to raise measured TFP as in Autor et al. (2017). To analyze the dynamic effect of this change I compare the steady state of the model with higher ϵ to the 1990s equilibrium of the model in Table 9. Under this parameterization, raising ϵ lowers the growth rate

[27]Without the competitive fringe this is always true. But it is not necessarily true under the assumption that the follower sets price equal to marginal cost: if relative quality differences are small, increasing ϵ can cause a drop in the leader's market share.

Figure 12: Markups and Market Shares, Role of Elasticity of Substitution



(a) Market shares (b) Markups

while dramatically increasing concentration. The rise in concentration comes from two forces. First, the static reallocation force operates: even if technology gaps were unchanged from one steady state to another, these same gaps would generate a higher average leader market share according to Figure 12a. Second, changes in patenting frequency across different types of firms (Figure 13a) cause the average technology gap to grow (Figure 13b).

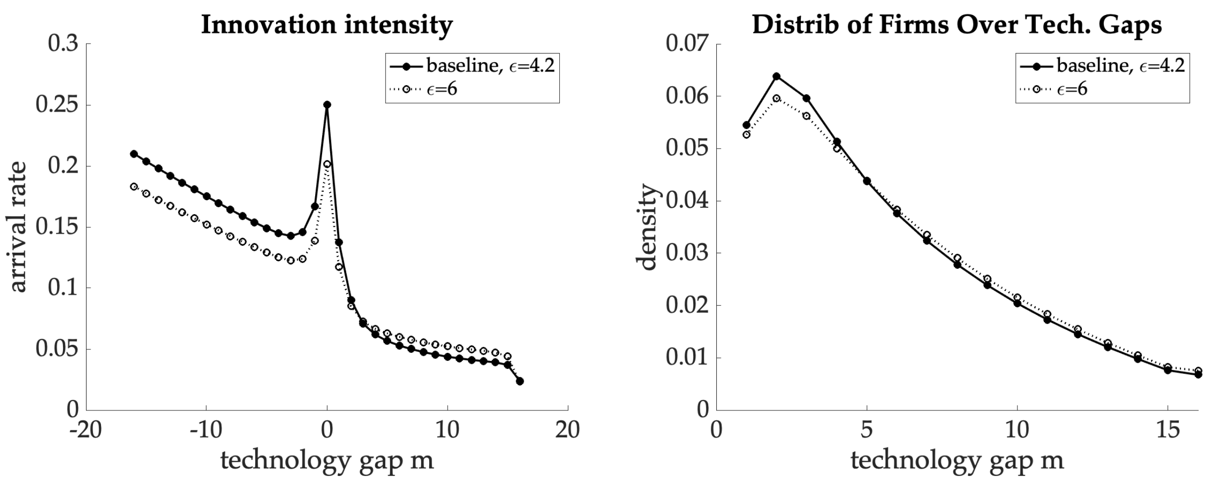
Moment	Model	
	$\epsilon = 4.2$	$\epsilon = 6$
TFP growth, %	1.75	1.56
Leader market share, avg, %	44.62	51.12
R&D share of GDP, %	1.91	1.52
Profit share of GDP, %	6.02	6.05
Pat stock growth/patent, avg, %	22.26	22.54
R&D intensity, avg, %	5.18	4.75
Leadership turnover, %	13.26	12.72
Markup, avg, %	23.84	18.79

Table 9: Model comparison, superstar firm experiment.

This latter effect is due to the fact that, for the given parameter values, the markup the leader charges as a function of its technology gap is lower at all possible values of the technology gap when ϵ changes from 4.2 to 6 (Figure 12b). This reduces the post-innovation gains to attaining market leadership, reducing the innovation effort of laggard and neck and neck firms (Figure 11a). On the other hand, markups and profits become more elastic in the technology gap when ϵ is higher, and the likelihood of being overtaken falls, so leaders choose a higher arrival rate of innovations than before.

Similar to the experiment reducing laggards' patent quality, there is both a reduction in the level and a shift in the location of R&D expenditures that generates a productivity slowdown, though this slowdown (from 1.75% annual productivity growth to 1.56%) is much smaller than the slowdown driven by changing patent quality. Moreover, the average markup falls, contradicting the findings of de Loecker, Eeckhout, and Unger (2020). Compared to laggards' declining patent quality, the main exercise suggests only a very modest scope for the superstar firm channel.

Figure 13: Innovation Policies and Stationary Distribution, Superstar Firms



(a) Innovation policies (b) Stationary distribution

6 Conclusion

This paper documents a striking decline in the contribution of new patents to firms' existing portfolios of patents since 2000, raising the possibility that the quality of new ideas has declined relative to the 1990s. Smaller firms within industries drove the boom in innovation quality that has been attributed to the arrival of information and communication technology as a general purpose technology, as well as the bust that began in the early 2000s. This finding contributes to the debate on whether ideas are getting harder to find, emphasizing heterogeneity in this phenomenon across firms. Further empirical work should investigate heterogeneity in the complementarities of general purpose technologies with different types of firms' R&D investments, and determine whether similar patterns of firm dynamics and productivity growth were present in the wake of previous general purpose technology waves.

To understand the consequences of this empirical fact I develop a general equilibrium model of innovations and growth where multiple firms are active in each sector in each period and goods within sectors are imperfect substitutes. A quantitative version of the model estimated for the U.S. in two different periods, the 1990s and the 2000s, points to declining patent quality as the main driver of rising concentration and the productivity slowdown over this period. Rising concentration in the 2000s is driven by a decline in the research effort of laggard firms and an increase in research effort by large firms, which causes average quality differences between competitors to grow, consistent with the rise of superstar firms. Because leaders make more incremental improvements on average, the economy grows more slowly as a result.

Through the lens of the model I unify the Schumpeterian endogenous growth literature with the growing literature on the rise of superstar firms. The estimated model points to only a modest rise in the elasticity of substitution within sectors, though I show that larger changes also have the potential to explain rising concentration and the productivity slowdown, providing a dynamic complement to the experiment in Autor et al. (2017), one that rationalizes the emergence of superstar firms alongside a productivity slowdown for a standard Schumpeterian reason: in this environment, laggard firms' incentives to innovate fall because varieties are less differentiated and the value of market leadership is lower. However, this is inconsistent with patterns of markups and the profit share, which have risen rather than fallen over this period.

Analyzing welfare and optimal policy is left to future research, though the preceding discussion offers some insight into the relevant trade-offs between reducing static markup distortions and providing dynamic incentives to innovate. Such analysis should ensure that knowledge spillovers between firms are properly accounted for. Another area for future research is how policy can incentivize the development of new general purpose technologies. Past research suggests this is difficult because there are significant positive externalities for other sectors that the inventor of the general purpose technology does not internalize. This paper suggests there may also be winners and losers within other industries, further complicating this problem.

References

Acemoglu, Daron, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr. 2018. "Innovation, Reallocation, and Growth." *American Economic Review* 108 (11): 3450–3491.

Acemoglu, Daron, and Dan Cao. 2015. "Innovation by Entrants and Incumbents." *Journal of Economic Theory* 157:255–294.

Aghion, Philippe, Ufuk Akcigit, and Peter Howitt. 2014. "What Do We Learn From Schumpeterian Growth Theory?" Chapter 1 of *Handbook of Economic Growth*, Volume 2, 515–563. Elsevier B.V.

Aghion, Philippe, Stefan Bechtold, Lea Cassar, and Holger Herz. 2014. "The Causal Effects of Competition on Innovation : Experimental Evidence." pp. 1–31.

Aghion, Philippe, Antonin Bergeaud, Timo Boppart, Peter J. Klenow, and Huiyu Li. 2019a. "A Theory of Falling Growth and Rising Rents." *Federal Reserve Bank of San Francisco, Working Paper Series*, pp. 01–43.

Aghion, Philippe, Antonin Bergeaud, Mattheiu Lequien, and Marc J. Melitz. 2019b. "The Heterogenous Impact of Market Size on Innovation: Evidence from French Firm-Level Exports." *NBER Working Paper*.

Aghion, Philippe, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. 2005. "Competition and Innovation: An Inverted-U Relationship." *Quarterly Journal of Economics* 120 (2): 701–728.

Aghion, Philippe, Christopher Harris, Peter Howitt, and John Vickers. 2001. "Competition, Imitation and Growth with Step-by-Step Innovation." *Review of Economic Studies* 68 (3): 467–492.

Akcigit, Ufuk, and Sina Ates. 2019. "What Happened to U.S. Business Dynamism?"

———. 2020. "Ten Facts on Declining Business Dynamism and Lessons from Endogenous Growth Theory." *AEJ: Macroeconomics, Forthcoming*.

Akcigit, Ufuk, Sina Ates, and Giammario Impullitti. 2018. "Innovation and Trade Policy in a Globalized World." *NBER Working Paper*.

Akcigit, Ufuk, and William R. Kerr. 2018. "Growth Through Heterogeneous Innovations." *Journal of Political Economy* 126, no. 4.

Anderson, Gary, and Audrey Kindlon. 2019. "Indicators of R&D Performance by Size of Company." *National Science Foundation*, pp. 1–9.

Andrews, Dan, Chiara Criscuolo, and Peter Gal. 2016. "The Global Productivity

Slowdown, Technology Divergence and Public Policy: A Firm Level Perspective." *The Future of Productivity: Main Background Papers*, pp. 1–50.

Andrews, Dan, Chiara Criscuolo, and Peter N Gal. 2015. "Frontier Firms, Technology Diffusion and Public Policy: Micro Evidence From OECD Countries." *The Future of Productivity: Main Background Papers*, pp. 1–38.

Anzoategui, Diego, Diego Comin, Mark Gertler, and Joseba Martinez. 2019. "Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence." *AEJ: Macroeconomics* 11 (3): 67–110.

Arrow, Kenneth J. 1962. "The Economic Implications of Learning by Doing." *The Review of Economic Studies* 29 (3): 155–173.

Aum, Sangmin, Sang Yoon (Tim) Lee, and Yongseok Shin. 2018. "Computerizing industries and routinizing jobs: Explaining trends in aggregate productivity." *Journal of Monetary Economics*.

Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen. 2020. "The Fall of the Labor Share and the Rise of Superstar Firms." *Quarterly Journal of Economics* 135 (2): 645–709.

Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John van Reenen. 2017. "The Fall of the Labor Share and the Rise of Superstar Firms." *NBER Working Paper Series*, no. 23396.

Baqaee, David Rezza, and Emmanuel Farhi. 2020. "Productivity and Misallocation in General Equilibrium." *Quarterly Journal of Economics* 135 (1): 105–163.

Barkai, Simcha, and Seth G Benzell. 2018. "70 Years of US Corporate Profits 70 Years of US Corporate Profits *." no. 22.

Bartelsman, Eric J., and Mark E. Doms. 2000. "Understanding Productivity: Lessons from Longitudinal Microdata." *Journal of Economic Literature* XXXVIII (September): 569–594.

Berger, David W., Kyle F. Herkenhoff, and Simon Mongey. 2019. "Labor Market Power." *NBER Working Paper Series* 25719:1–80.

Bessen, James. 2017. "Information Technology and Industry Concentration." *Boston University School of Law, Law & Economics Paper* 17, no. 41.

Bessen, James, Erich Denk, Joowon Kim, and Cesare Righi. 2020. "Declining Industrial Disruption." *Boston University School of Law, Law & Economics Paper* 20, no. 28.

Bloom, Nicholas, Charles Jones, John Van Reenen, and Michael Webb. 2020. "Are Ideas Getting Harder to Find?" *American Economic Review* 110 (4): 1104–1144.

Bornstein, Gideon. 2018. "Entry and Profits in an Aging Economy: The Role of Consumer Inertia." pp. 909–927.

Bresnahan, Timothy F., and M. Trajtenberg. 1995. "General Purpose Technologies 'Engines of growth'?" *Journal of Econometrics* 65 (1): 83–108.

Brynjolfsson, Erik, Daniel Rock, and Chad Syverson. 2018. "The Productivity J-Curve: How Intangibles Complement General Purpose Technologies." *NBER Working Paper*, no. 25148.

Carley, Michael, Deepak Hegde, and Alan Marco. 2015. "What is the Probability of Receiving a U.S. Patent?" *Yale Journal of Law and Techonology*, vol. 203.

Cavenaile, Laurent, Murat Alp Celik, and Xu Tian. 2020. "Are Markups Too High? Competition, Strategic Innovation, and Industry Dynamics."

Chatterjee, Satyajit, and Burcu Eyigungor. 2019. "The Firm Size and Leverage Relationship and Its Implications for Entry and Concentration in a Low Interest Rate World." *Federal Reserve Bank of Philadelphia, Working Papers, Research Department* 19, no. 18.

Corhay, Alexandre, Howard Kung, and Lukas Schmid. 2020. "Q: Risk, Rents, or Growth ?"

Council of Economic Advisers. 2016. "Benefits of Competition and Indicators." *Issue Brief*, no. May.

Covarrubias, Matias, German Gutierrez, and Thomas Philippon. 2019. "From Good to Bad Concentration? U.S. Industries over the Past 30 Years." *NBER Macro Annual*.

Crouzet, By Nicolas, and Janice Eberly. 2018. "Intangibles, Investment, and Efficiency." *AEA Papers and Proceedings* 108:426–431.

Decker, Ryan A., John Haltiwanger, Ron S. Jarmin, and Javier Miranda. 2016. "Where Has All the Skewness Gone? The Decline in High-Growth (Young) Firms in the U.S." *European Economic Review* 86:4–23.

de Loecker, Jan, Jan Eeckhout, and Gabriel Unger. 2020. "The Rise of Market Power and the Macroeconomic Implications." *Quarterly Journal of Economics* 135 (2): 561–644.

de Loecker, Jan, and Frederic Warzynski. 2012. "Markups and Firm-Level Export Status: Appendix." *American Economic Review* 102 (6): 2437–2471.

de Ridder, Maarten. 2020. "Market Power and Innovation in the Intangible Economy."

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu. 2018. "How Costly Are Markups?"

Eggertsson, Gauti, Neil Mehrotra, and Jacob Robbins. 2019. "A Model of Secular Stagnation." *American Economic Journal: Macroeconomics* 11, no. 1.

Eggertsson, Gauti, Jacob Robbins, and Ella Getz Wold. 2018. "Kaldor and Piketty's Facts: The Rise of Monopoly Power in the United States." pp. 1–69.

Engbom, Niklas. 2019. "Firm and Worker Dynamics in an Aging Labor Market." *Working Paper*, pp. 1–115.

Fernald, John. 2014. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." *Federal Reserve Bank of San Francisco*.

Garcia-Macia, Daniel, Chang-Tai Hsieh, and Peter J. Klenow. 2019. "How Destructive Is Innovation?" *Econometrica* 87 (5): 1507–1541.

Goldberg, Jonathan, David Lopez-Salido, and Craig A Chikis. 2020. "Interest Rates and the Innovation Multiplier."

Graham, Stuart, and Deepak Hegde. 2015. "U.S. Patents and Their Disclosure Status." *Science* 347, no. 6219.

Griliches, Zvi. 1998. "Patent Statistics as Economic Indicators: A Survey." Chapter 13 of *R&D and Productivity: The Econometric Evidence*, 287–343. University of Chicago Press.

———. 2001. *R&D, Education, and Productivity: A Retrospective*. Cambridge, Mass.: Harvard University Press.

Grullon, Gustavo, Yelena Larkin, and Roni Michaely. 2019. "Are US Industries Becoming More Concentrated?" *Review of Finance* 23 (4): 697–743.

Hall, Robert E. 2015. "Quantifying the Lasting Harm to the US Economy from the Financial Crisis." *NBER Macroeconomics Annual* 29 (1): 71–128.

Harris, Christopher, and John Vickers. 1987. "Racing with Uncertainty." *Review of Economic Studies* 54 (1): 1–21.

Helpman, Elhanan. 1998. *General Purpose Technologies and Economic Growth*. Edited by Elhanan Helpman. Cambridge: MIT Press.

Hoberg, Gerard, and Gordon Phillips. 2010. "Product Market Synergies and Competition in Mergers and Acquisitions: A Text-Based Analysis." *Review of Financial Studies* 23 (10): 3773–3811.

Hobjin, Bart, and Fernanda Nechio. 2019. "Sticker Shocks: Using VAT Changes to Estimate Upper-Level Elasticities of Substitution." *Journal of the European Economic Association* 17 (3): 799–833.

Hopenhayn, Hugo, Julian Neira, and Rish Singhania. 2018. "From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share." *National Bureau of Economic Research Working Paper Series*, pp. 1–55.

Hsieh, Chang-Tai, and Esteban Rossi-Hansberg. 2020. "The Industrial Revolution in Services." pp. 139–158.

Jones, Callum, and Thomas Philippon. 2016. "The Secular Stagnation of Investment?" *Working Paper*, no. December:1–25.

Jovanovic, Boyan, and Peter L Rousseau. 2005. *General Purpose Technologies*. Volume 1. Edited by Philippe Aghion and Steven N. Durlauf. Elsevier B.V.

Karahan, Fatih, Benjamin W. Pugsley, and Aysegul Sahin. 2019. "Demographic Origins of the Startup Deficit." *NBER Working Paper*, no. 25874.

Kehrig, Matthias, and Nicolas Vincent. 2020. "The Micro-Level Anatomy of the Decline of the Labor Share." *NBER Working Paper*, no. 25275:1689–1699.

Kelly, Bryan T., Dimitris Papanikolaou, Amit Seru, and Matt Taddy. 2018. "Measuring Technological Innovation over the Long Run." *SSRN Electronic Journal*.

Klette, Tor Jakob, and Samuel Kortum. 2004. "Innovating Firms and Aggregate Innovation." *Journal of Political Economy* 112 (5): 986–1018.

Knott, Anne Marie, and Carl Vieregger. 2016. "Reconciling the Firm Size and Innovation Puzzle." *CES Working Paper*.

Kogan, Leonid, Dimitris Papanikolaou, Amit Seru, and Noah Stoffman. 2017. "Technological Innovation, Resource Allocation, and Growth." *Quarterly Journal of Economics*, pp. 665–712.

Lentz, Rasmus, and Dale Mortensen. 2008. "An Empircal Model of Growth Through Product Innovation." *Econometrica* 76 (6): 1317–1373.

Liu, Ernest, Atif R. Mian, and Amir Sufi. 2019. "Low Interest Rates, Market Power, and Productivity Growth."

Luttmer, Erzo G.J. 2007. "Selection, Growth, and the Size Distribution of Firms." *Quarterly Journal of Economics*, no. August:1103–1145.

Mongey, Simon. 2017. "Market Structure and Monetary Non-neutrality."

Olley, G. Steven, and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica* 64 (6): 1263–1297.

Pellegrino, Bruno. 2020. "Product Differentiation, Oligopoly, and Resource Allocation."

Peters, Michael, and Conor Walsh. 2019. "Declining Dynamism, Increasing Markups and Missing Growth: The Role of the Labor Force." *SSRN Electronic Journal*, pp. 1–49.

Peters, Ryan H., and Lucian A. Taylor. 2017. "Intangible Capital and the Investment-q Relation." *Journal of Financial Economics* 123 (2): 251–272.

Pugsley, Benjamin W., Petr Sedlacek, and Vincent Sterk. 2020. "The Nature of Firm Growth."

Rosen, Richard J. 1991. "Research and Development with Asymmetric Firm Sizes." *RAND Journal of Economics* 22 (3): 411–429.

Rossi-Hansberg, Esteban, Pierre-Daniel Sarte, and Nicholas Trachter. 2020. "Diverging Trends in National and Local Concentration." *NBER Macroeconomics Annual* 35:1–39.

Syverson, Chad. 2017. "Challenges to Mismeasurement Explanations for the U.S. Productivity Slowdown." *Journal of Economic Perspectives*.

van Reenen, John. 2018. "Increasing Differences Between Firms: Market Power and the Macro-Economy." *CEP Discussion Paper*, no. 1576.

Zachariadis, Marios. 2003. "R&D, Innovation, and Technological Progress: A Test of the Schumpeterian Framework Without Scale Effects." *Canadian Journal of Economics/Revue Canadienne d'Economique* 36 (3): 566–586.

Zhang, Linyi. 2018. "Escaping Chinese Import Competition? Evidence from U.S. Firm Innovation." *Working Paper*, pp. 1–69.

A Data Appendix

The main source of data for the paper is the Compustat Fundamentals Annual database, 1962-2017 (though most analysis focuses on the post-1980 period). I restrict attention to firms incorporated in the U.S. reporting in U.S. dollars. I further restrict attention to non-financial, non-agricultural, non-utilities firms.

A.1 Data Sources and Moment Computations

Table 10 lists the source and, where necessary, computation method for each target moment from the data.

A.2 Additional Patent Quality Figures

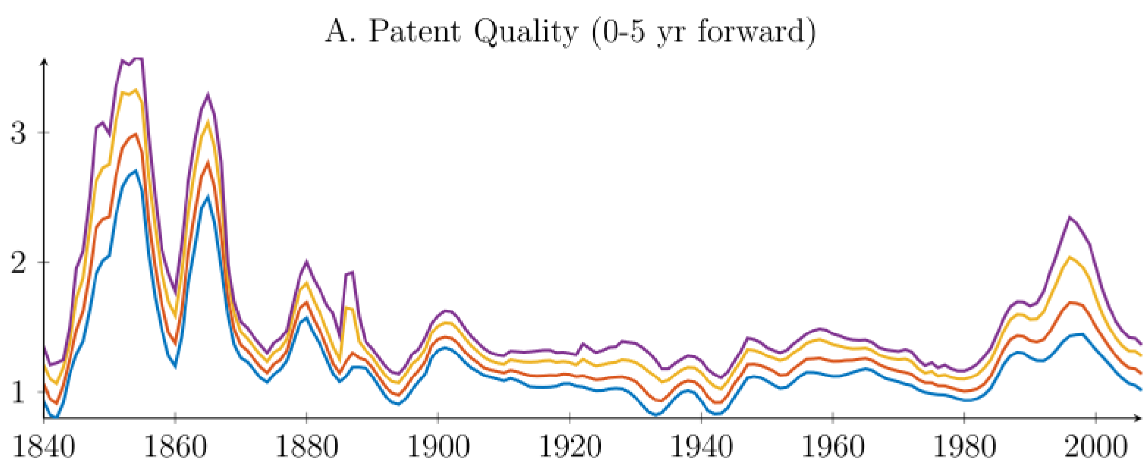


Figure 14: Percentiles of text-based patent quality distribution over time. Blue = P50, Red = P75, Yellow = P90, Purple = P95. Source: Kelly et al. (2018) Figure 3a.

Moment	Source	Computation/Series Name
TFP growth	Fernald (2014)	Utilization-adjusted annual total factor productivity growth
Leader market share	Compustat	Average of sales share (SALE) of largest firm in each 4-digit SIC industry (weighted by industry size)
Patent quality $\equiv$ patent stock growth per patent (psgpp)	Kogan et al. (2017)	$rTsm_{it} = \frac{Tsm_{it}}{GDPdefl_t}$ is the real value of firm i's patents issued in year t. $psgpp_{it} = \frac{\frac{rTsm_{it}}{fNpats_{it}}}{\sum_{s=1}^{t-1} rTsm_{is}}$; s =first year in Compustat. Citation-based version substitutes Tcw (not deflated).
R&D share of GDP	OECD Main Science and Technology Indicators	Business Expense R&D (private)/GDP
R&D intensity	Compustat	XRD/SALE, mean across all firms with real sales over 1 million in 2012 USD, assuming 0 if XRD missing.
Profit share of GDP	Bureau of Economic Analysis/FRED	Profits after tax with inventory valuation and capital consumption adjustments/Gross domestic income
Leader's share of R&D	Compustat	Average sales leader share of total R&D in 4-digit sector (weighted by industry size)
Leadership turnover	Compustat	Share of 4-digit SIC industries with new sales leader per year

Table 10: Data sources and computation method for each moment used in the text.

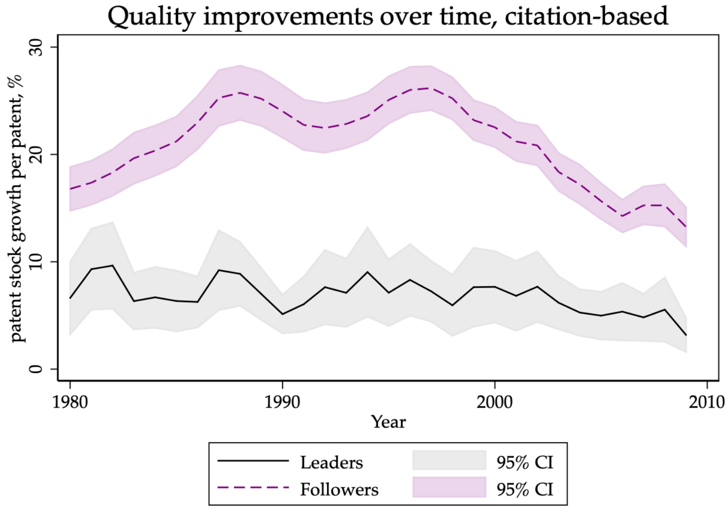


Figure 15: Contribution of average new patent to firm's existing stock of patents, substituting forward citations counts for dollar value, from Kogan et al. (2017). Leader indicates sales leaders in 4-digit SIC industries and followers are all other firms.

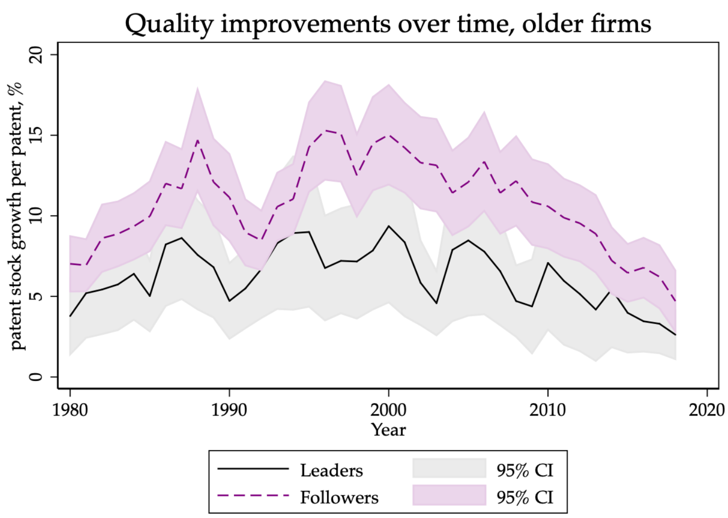


Figure 16: Contribution of average new patent to value of firm's existing stock of patents, using estimated patent values from Kogan et al. (2017). Leader indicates sales leaders in 4-digit SIC industries and followers are all other firms, restricting attention to firms that have been public at least 20 years in the year patent is issued.

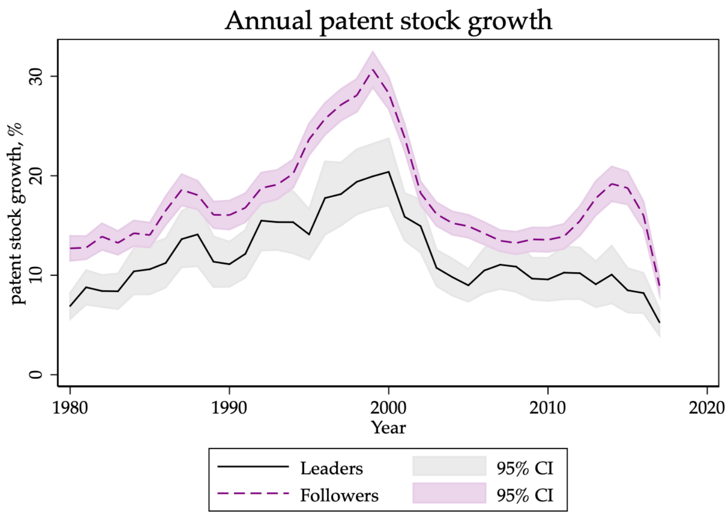


Figure 17: Average annual growth of firm's patent stock conditional on patenting at least once in that year, using estimated patent values from Kogan et al. (2017). Leader indicates sales leaders in 4-digit SIC industries and followers are all other firms.

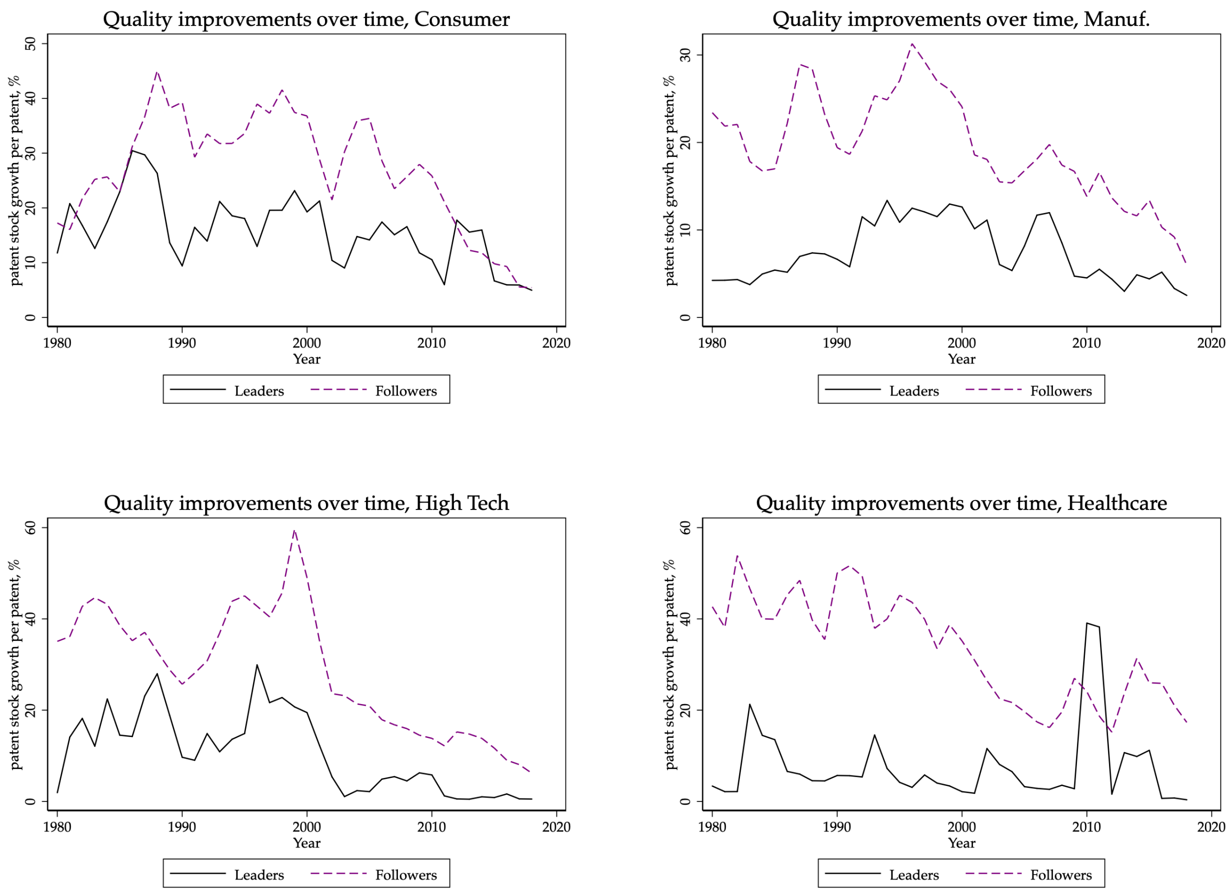


Figure 18: Average patent quality differences between leaders and followers in Fama-French 5 broad industry categories (excluding "Other" category), using estimated patent values from Kogan et al. (2017). Leader indicates sales leaders in 4-digit SIC industries and followers are all other firms.

A.3 TFP and Markup Estimation

I use Compustat data on U.S. public firms from 1962-2017 to estimate revenue-based total factor productivity (TFPR) and markups at the firm level. I focus on the non-farm, non-financial sector and exclude utilities and firms without an industry classification. I keep only those companies that are incorporated in the U.S. The sample includes around 3,000 firms per year, though this number varies over time.

I construct each firm's capital stock $K_{i,t}$ by initializing the capital stock as PPEGT (total gross property, plant, and equipment) for the first year the firm appears. I then construct $K_{i,t+1}$ recursively:

$$K_{i,t+1} = K_{i,t} + I_{i,t+1} - \delta K_{i,t}$$

where PPENT (total net property, plant, and equipment) is used to capture the last two terms (net investment). I deflate the nominal capital stock using the Bureau of Economic Analysis (BEA) deflator for non-residential fixed investment.

In de Loecker and Warzynski (2012) the authors show that under a variety of pricing models firm i's markup at time t, μ_{it}, can be computed as a function of the output elasticity θ^V_{it} of any variable input and the variable input's cost share of revenue[28] :

$$\mu_{it} = \theta^V_{it} \frac{P_{it} Q_{it}}{P^V_t V_{it}} \tag{12}$$

where P_{it} is the output price of firm i's good at time t, Q_{it} its output, P^V_t the price of the variable input and V_{it} the amount of the input used.

Following de Loecker, Eeckhout, and Unger (2020) I use COGS (cost of goods sold) deflated by the BEA's GDP deflator series as the real variable input cost $M_{i,t}$ of the firm. While the number of employees is well measured in Compustat and would be sufficient to estimate productivity, the wage bill is usually not available and would be needed to compute the labor cost share needed to compute the markup simultaneously with productivity.

For the results presented in this paper, I assume a Cobb-Douglas production function[29] for firm i in 2-digit SIC sector s in year t so that factor shares may vary across sectors but not over time:

[28] This approach requires several assumptions. First, the production technology must be continuous and twice differentiable in its arguments. Second, firms must minimize costs. Third, prices are set period by period. Fourth, the variable input has no adjustment costs. No particular form of competition among firms need be assumed.

[29] Alternative estimation of a translog production function yielded similar estimates.

$$Y_{i,s,t} = A_{i,s,t} M_{i,s,t}^{\beta_{M,s}} K_{i,s,t}^{\beta_{K,s}}$$

I use the variable SALE to measure firm output $Y_{i,s,t}$. I deflate SALE using the GDP deflator series to obtain real revenue at the firm level. I include firm and time fixed effects and obtain revenue-based TFP in logs (lower case variables denote variables in logs) by computing the residual (including fixed effects) of the following regressions for each 2-digit sector:

$$y_{i,t} = \alpha + \eta_t + \delta_i + \beta_{M,s} m_{i,t} + \beta_{K,s} k_{i,t-1} + \varepsilon_{i,t}.$$

In the above equation, $\beta_{M,s}$ captures the sector specific variable output elasticity, so I use equation 12 to obtain the markup from the estimated $\hat{\beta}_{M,s}$ and the inverse cost share $\frac{SALE}{COGS}$.

A.4 Industry Profit Shares

The competitive fringe assumption generates empirically plausible predictions about profit shares: the largest U.S. public firms (by sales) capture by far the largest share of industry profits (see Figure 19).[30]

A.5 Additional Model Validation Figures

An empirical exploration of the causal relationships among productivity growth, productivity gaps, and concentration is beyond the scope of this paper. However, especially given the sectoral heterogeneity in the decline in laggards patent quality in Figure 18 which suggests that the extent of this phenomena differs across industries, we might expect rising concentration and the productivity slowdown to be correlated at the sector level. I use data from Bureau of Economic Analysis estimates of multifactor productivity[31] at the 3-digit NAICS level and data from Compustat to check the association between the change in the leader's market share in Compustat and the change in the sector's average productivity growth rate from 1994-2003 to 2004-2017 at the sector level. Sectors experiencing greater slowdowns in average productivity growth rates between 1994-2003 and 2004-2017 also saw greater increases in concentration, measured as the market leader's share of total industry sales, on average (Figure 20).

[30] TFP and sales share are correlated, and the figure looks similar if one uses a productivity ranking instead of sales-share based ranks.

[31] https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems

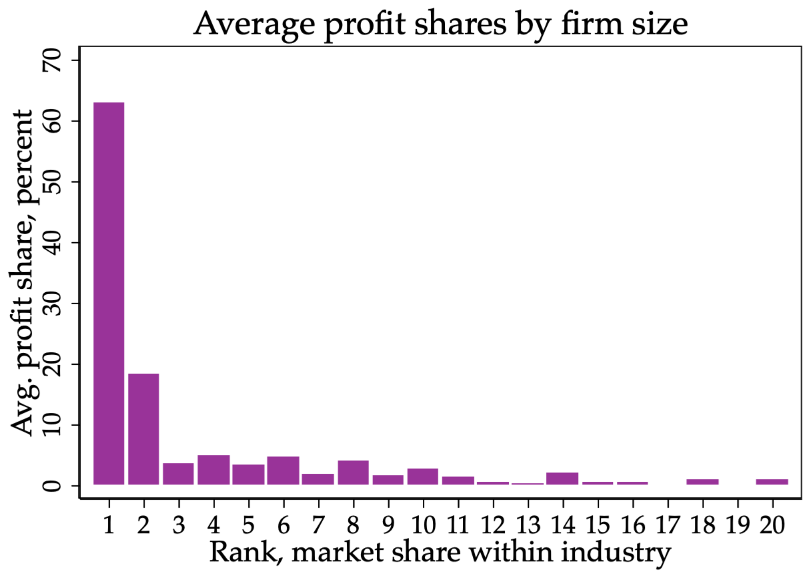


Figure 19: Source: Compustat, 1975-2015. Firms are ranked by market share (sales) within 4-digit SIC industries, and these ranks are compared to profit shares (firm's own operating income (OIDBP) as a share of industry-total operating income). The Figure averages across 4-digit sectors.

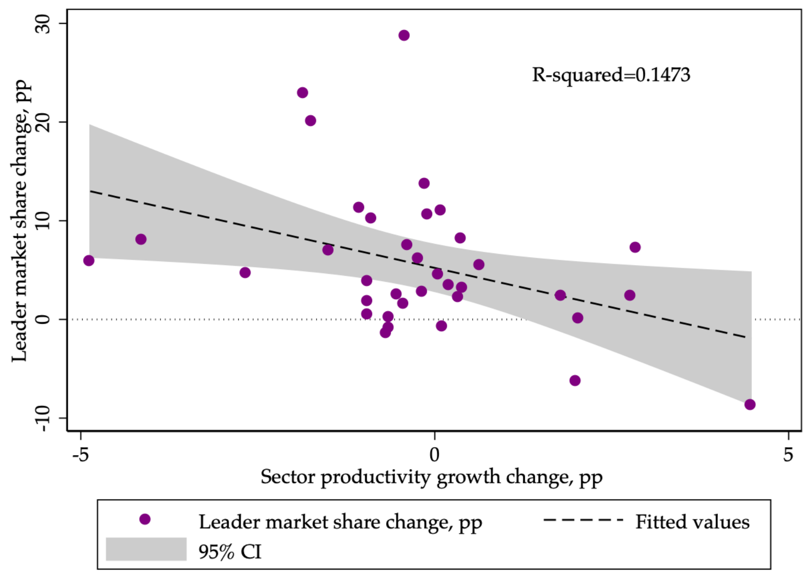


Figure 20: Author's calculations from Compustat and BEA Integrated Industry-Level Production Accounts. 3-digit NAICS sectors, comparing 1994-2003 average to 2004-2017 average.

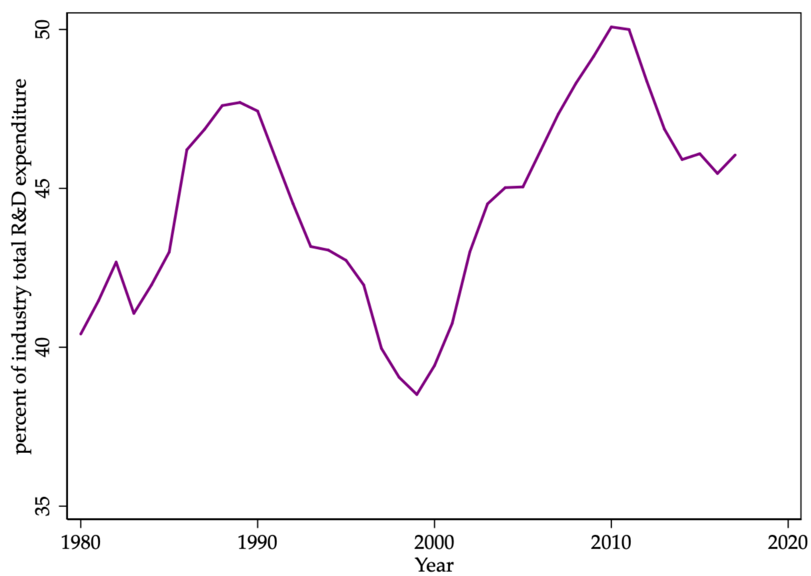


Figure 21: Research and development expenditures (XRD) of sales leaders in 4-digit SIC industries in Compustat as a share of total R&D expenditures of all firms in that sector. Average across industries, sale-weighted by industry size.

B Model Appendix

B.1 Proof Prices Depend on Relative Quality

Relative quality refers to the ratio of qualities of the two incumbent firms in a sector (dropping the sector notation j) $\frac{q_1}{q_2}$ for firm 1 and $\frac{q_2}{q_1}$ for firm 2. Below I show that the firms' pricing strategies depend only on relative quality, not the level of their own or their rival's quality.

First, this is clearly satisfied for the technology follower ($m_i < 0$) who sets price equal to marginal cost η regardless of absolute quality, and for sectors where $m_1 = m_2 = 0$, that is, when firms are neck-and-neck, because of the presence of the competitive fringe.

For the leader ($m_i > 0$), plugging the final good firm's demand for good i into the definition of the market share and using the definition of the price index yields:

$$\begin{aligned} s_i &= q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{1-\epsilon} \\ &= \frac{q_i^{\epsilon-1}p_i^{1-\epsilon}}{q_i^{\epsilon-1}p_i^{1-\epsilon}+q_{-i}^{\epsilon-1}\eta^{1-\epsilon}} \\ &= \frac{1}{1+\left(\frac{q_{-i}}{q_i}\right)^{\epsilon-1}\left(\frac{p_i}{\eta}\right)^{\epsilon-1}}, \end{aligned}$$

where $-i$ denotes the follower. Now using the pricing decision of the leader:

$$s_i = \frac{1}{1+\left(\frac{q_{-i}}{q_i}\right)^{\epsilon-1}\left(\frac{\epsilon-(\epsilon-\frac{1}{\beta})s_i}{\epsilon-(\epsilon-\frac{1}{\beta})s_i-1}\right)^{\epsilon-1}}.$$

Thus there is a mapping from technology gaps to market shares and prices that is independent of quality levels. ■

B.2 Value Function Boundary Equations

For the firm that's furthest behind (at gap $-\bar{m}$ with quality q_t):

$$
\begin{aligned}
r_t V_{-\bar{m},t}(q_t) - \dot{V}_{-\bar{m},t}(q_t) = \max_{x_{-\bar{m},t}} \{ & 0 - \alpha \frac{(x_{-\bar{m},t})^{\gamma}}{\gamma} q_t^{\frac{1}{\beta}-1} \\
& + x_{-\bar{m},t} \sum_{n_t=-\bar{m}+1}^{\bar{m}} \mathbb{F}_m(n_t)[V_{nt}(\lambda^{n_t-(-\bar{m})} q_t) - V_{-\bar{m},t}(q_t)] \\
& + x_{\bar{m},t}(V_{-\bar{m},t}(\lambda q_t) - V_{-\bar{m},t}(q_t)) \\
& + \delta_e(0 - V_{-\bar{m},t}(q_t)\}.
\end{aligned}
$$

The difference between this and equation 5 is in the third line, where if the firm's competitor innovates, there is a spillover that causes the firm at gap $-\bar{m}$ to improve its quality by λ.

For a firm at gap $\bar{m}$ the value function is:

$$
\begin{aligned}
r_t V_{\bar{m},t}(q_t) - \dot{V}_{\bar{m},t}(q_t) = \max_{x_{\bar{m},t}} \{ & \pi(\bar{m}, q_t) - \alpha \frac{(x_{\bar{m},t})^{\gamma}}{\gamma} q_t^{\frac{1}{\beta}-1} \\
& + x_{\bar{m},t}(V_{\bar{m},t}(\lambda q_t) - V_{\bar{m},t}(q_t)) \\
& + x_{-\bar{m},t} \sum_{n_t=-\bar{m}+1}^{\bar{m}} \mathbb{F}_{-\bar{m}}(n_t)[V_{nt}(q_t) - V_{\bar{m},t}(q_t)] \\
& + \delta_e(0 - V_{\bar{m},t}(q_t)\},
\end{aligned}
$$

where:

$$
\pi(m, q_t) = \begin{cases} 0 & \text{if } m \leq 0 \\ q_t^{\frac{1}{\beta}-1}(p(m) - \eta) p(m)^{-\epsilon} (p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} \eta^{1-\epsilon})^{\frac{\epsilon - \frac{1}{\beta}}{1-\epsilon}} & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}.
$$

B.3 Derivation of Final Output

Dropping the time subscript t, plugging the pricing strategies in equation 4 and $p_i = \eta$ for firms with $m \leq 0$ into the demand curve 3 to obtain the output of each incumbent and plugging these outputs into equation 2 and equation 2 into equation 1 simplifies as:

$$
\begin{aligned}
Y &= \frac{1}{1-\beta}\left(\int_0^1 K_j^{1-\beta} dj\right) L^\beta \\
&= \frac{1}{1-\beta}\left(\int_0^1 \left(\sum_{i=1}^{2} q_i^{\frac{\epsilon-1}{\epsilon}} (q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{-\epsilon}\left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}1-\beta} dj\right) L^\beta \\
&= \frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}}\left(\int_0^1 P_j^{\epsilon(1-\beta)-\frac{1-\beta}{\beta}}\left(\sum_{i=1}^{2} q_i^{\epsilon-1} p_i^{1-\epsilon}\right)^{\frac{\epsilon(1-\beta)}{\epsilon-1}} dj\right) \\
&= \frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}}\left(\int_0^1 P_j^{-\frac{1-\beta}{\beta}} dj\right).
\end{aligned}
$$

The demand shifter $P^{\frac{1}{\beta}}L$ index is common to all firms and can be taken out entirely (and normalized to one since I assume zero population growth). The quality-adjusted price index P_j of each sector falls as the qualities of the two firms in the sector grow, and the exponent is negative for all $\beta \in (0,1)$ so Y is increasing in firms' qualities.

Common to all firms with a particular technology gap m are the prices $p(m)$ of the firm at gap m and its competitor at $-m$, $p(-m)$. At time t, therefore, Y can be expressed as:

$$
Y_t = \frac{1}{2}\frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \sum_{m=-\bar{m}}^{\bar{m}}\left(\int_0^1 \left(q_{it}^{\epsilon-1} p_i(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p_{-i}(-m)^{1-\epsilon}\right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i\in\mu_{mt}\}} di\right)
$$

where μ_{mt} is the measure of firms at technology gap m at time t and the above integration is taken over firms rather than sectors. More simply:

$$
Y_t \equiv \frac{1}{2}\frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \sum_{m=-\bar{m}}^{\bar{m}} Q_{mt},
$$

where Q_{mt} is defined as:

$$
\begin{aligned}
Q_{m,t} &= \int_0^1 \left(q_{it}^{\epsilon-1} p(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i\in\mu_{mt}\}} di \\
&= (p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon})^{\frac{1-\beta}{\beta(\epsilon-1)}} \int_0^1 q_{i,t}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i\in\mu_{mt}\}} di.
\end{aligned}
$$

B.4 Output Growth on Balanced Growth Path

To understand how aggregate output evolves, this section studies the evolution of $\tilde{Q}_{m,t}$ (defined in equation 11) between t and $t+dt$ for all m. These expressions are similar to those for the stationary distribution (equations 8-10) because they are based on the movement of firms to different technology gaps from their rival, but account for the quality improvements that occur because of innovation.

Assuming fixed distribution $\mu_{mt} = \mu_m$ for all m, t:

$$\dot{\tilde{Q}}_{mt} = \int_0^1 q_{m,t+dt,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_m\}} di - \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_m\}} di.$$

that is, quality growth at gap m is due to the change an index of the qualities of all the firms with technology gap m. Consider an arbitrary $m \in (-\bar{m}, \bar{m})$ ($-\bar{m}$ and $\bar{m}$ are special cases because of spillovers). A portion of firms at m at t innovate to a different gap, and another portion leave gap m because their competitor innovates. Because all firms at gap m choose the same arrival rate x_m, these are a random sample of the firms at gap m at time t. The outflows from $\dot{\tilde{Q}}_m$ are:

$$-(x_m + x_{-m}) \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i \in \mu_m\}} di = -(x_m + x_{-m}) \tilde{Q}_m.$$

The inflows to m's quality index come from two sources. First, some firms innovate into position m from a lower position n, improving their quality by λ^{m-n}. The probability they innovate and reach gap m is given by $x_n F_n(m)$. Some firms fall back to m from a higher gap n because their competitor innovates to $-m$. The probability their competitor reaches $-m$ is given by $x_{-n} F_{-n}(-m)$. So cumulative inflows are:

$$\sum_{n=-\bar{m}}^{m-1} x_n F_n(m) (\lambda^{(m-n)})^{\frac{1-\beta}{\beta}} \tilde{Q}_n + \sum_{n=m+1}^{\bar{m}} x_{-n} F_{-n}(-m) \tilde{Q}_n.$$

Putting it together:

$$\dot{\tilde{Q}}_{mt} = \sum_{n=-\bar{m}}^{m-1} x_n F_n(m) (\lambda^{(m-n)})^{\frac{1-\beta}{\beta}} \tilde{Q}_n + \sum_{n=m+1}^{\bar{m}} x_{-n} F_{-n}(-m) \tilde{Q}_n - (x_m + x_{-m}) \tilde{Q}_m. \quad (13)$$

For lowest gap there are spillovers when competitor innovates:

$$\dot{\tilde{Q}}_{-\bar{m}t} = \sum_{n=-\bar{m}+1}^{\bar{m}} x_{-n} F_{-n}(\bar{m}) \tilde{Q}_n + x_{\bar{m}} (\lambda^{\frac{1-\beta}{\beta}} - 1) \tilde{Q}_{-\bar{m}} - x_{-\bar{m}} \tilde{Q}_{-\bar{m}}. \quad (14)$$

For highest gap the firm does not exit that gap when they innovate:

$$\dot{\tilde{Q}}_{\bar{m}t} = \sum_{n=-\bar{m}}^{\bar{m}-1} x_n F_n(\bar{m}) (\lambda^{(m-n)})^{\frac{1-\beta}{\beta}} \tilde{Q}_n + x_{\bar{m}} (\lambda^{\frac{1-\beta}{\beta}} - 1) \tilde{Q}_{\bar{m}} - x_{-\bar{m}} \tilde{Q}_{\bar{m}}. \quad (15)$$

Given equations 13, 14, and 15, on a balanced growth path where $\frac{\dot{\tilde{Q}}_{mt}}{Y_t}$ is constant, it's sufficient to assume $\frac{\tilde{Q}_{mt}}{Y_t}$ is constant over time for all $m \in [-\bar{m}, \bar{m}]$. Differentiating $\frac{\tilde{Q}_{mt}}{Y_t}$ with respect to time yields:

$$
\begin{aligned}
\dot{\left(\frac{\tilde{Q}_m}{Y}\right)} &= \frac{\dot{\tilde{Q}}_m}{Y} - \frac{\tilde{Q}_m}{Y}\frac{\dot{Y}}{Y} \\
&= \frac{\dot{\tilde{Q}}_m}{Y} - g\frac{\tilde{Q}_m}{Y}.
\end{aligned}
$$

Imposing that the left hand side is zero implies:

$$
\frac{\dot{\tilde{Q}}_m}{Y} = g\frac{\tilde{Q}_m}{Y}.
$$

The vector on the left hand side is defined above by the flow equations (13), (14), and (15) divided by GDP. Use those equations to form a matrix A that captures the flow equations:

$$
\frac{\dot{\tilde{Q}}_m}{Y} = A\frac{\tilde{Q}_m}{Y} = g\frac{\tilde{Q}_m}{Y}.
$$

The values in A depend on λ, ϕ, and x_m. The above equation means that the growth rate g is an eigenvalue of the matrix A and $\frac{\tilde{Q}_m}{Y}$ is the corresponding eigenvector of A. If there is only one positive, real eigenvalue there is only one such balanced growth path where the contribution of the growth of the quality index of each technology gap to the total growth rate is constant and the growth rate of the economy is constant.

B.5 Alternate Model With No Competitive Fringe

It is also possible to solve the full dynamic model without the presence of the competitive fringe imitating the follower's variety so that both firms exercise market power over their variety i of sector j's good. In this alternative model, only the incumbent firms compete a la Bertrand. There is still the possibility of exogenous entry/exit, though this assumption can be relaxed as well. The analogy from the model to the data becomes less obvious under this assumption, since the laggard firm can no longer be thought of representing many firms producing generic products that are perfectly substitutable with other generic products but imperfectly substitutable with the brand produced by the leader. In this setup the quality leader always has at least 50% market share, unlike in the data. This assumption also gives empirically counterfactual predictions that the profit shares of total industry profits of the market leader and the other firm in the industry are relatively similar, contradicting the pattern shown in Figure 19.

Nonetheless, many of the main results carry through under this alternate assumption. Before describing these alternate results, I return to the pricing problem of the firms assuming the follower can now choose its optimal markup. Using the same derivation as in section 3.3.1 it can be shown that both firms follow the pricing policy the leader follows in the baseline model:

$$p_i = \frac{\epsilon - (\epsilon - \frac{1}{\beta})s_i}{\epsilon - (\epsilon - \frac{1}{\beta})s_i - 1}\eta,$$

where

$$s_i = q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{1-\epsilon}.$$

I look for a Markov perfect equilibrium with balanced growth where each firm's price is the best response to its competitor's price at time t. The algorithm for finding the steady state remains the same, plugging in the pricing functions of the firms, illustrated in Figure 22.

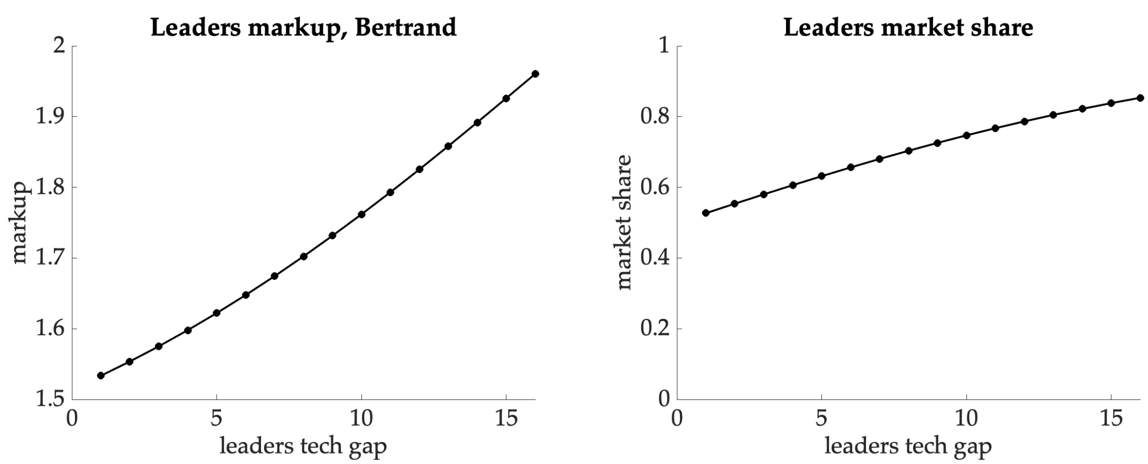


Figure 22: Markups and resulting market shares as a function of the technology gap (ratio of firm qualities), Bertrand pricing.

Table 11 gives the results of the same experiment as in section 4.3 under the alternate pricing strategies with the same parameters as in table 1 and Figure 23 shows the policy functions and stationary distributions. Note that the escape competition motive around the neck and neck state disappears in the version without the competitive fringe. As before, changing ϕ has a level effect on total innovation effort but also changes the location of R&D from laggard firms to leading firms.

The level of the growth rates and the change in the growth rate from one steady state to the other under Bertrand pricing due to a change in ϕ are very similar to the baseline model with marginal cost pricing of the follower. The increase in concentration is smaller since the change in technology gaps is not as dramatic as in the main case (Figure 23), though technology gaps do increase modestly. As for the growth decomposition, the effects of the firms' innovation responses is smaller, and the first order effect of lowering the probability of radical innovations is a bit larger than in the baseline model with the competitive fringe.

Moment	Data			Model		
	1990s	2000s	Chg. (pp)	1990s	2000s	Chg. (pp)
TFP growth, %	1.74	0.49	-1.25	1.82	0.27	-1.55
Leader market share, avg, %	43.34	48.12	4.78	62.44	63.52	1.08
R&D share of GDP, %	1.8	1.89	0.09	2.29	1.72	-0.57
Profit share of GDP, %	5.24	6.61	1.37	14.58	14.45	-0.13
R&D intensity, avg, %	2.56	3.8	1.24	8.13	5.33	-2.8
Pat stock growth/patent, avg, %	23.52	11.81	-11.71	21.44	11.10	-10.44
Leadership turnover, %	13.74	9.27	-4.47	13.49	10.43	-3.06

Table 11: Model and data comparison, role of ϕ, model with no competitive fringe.

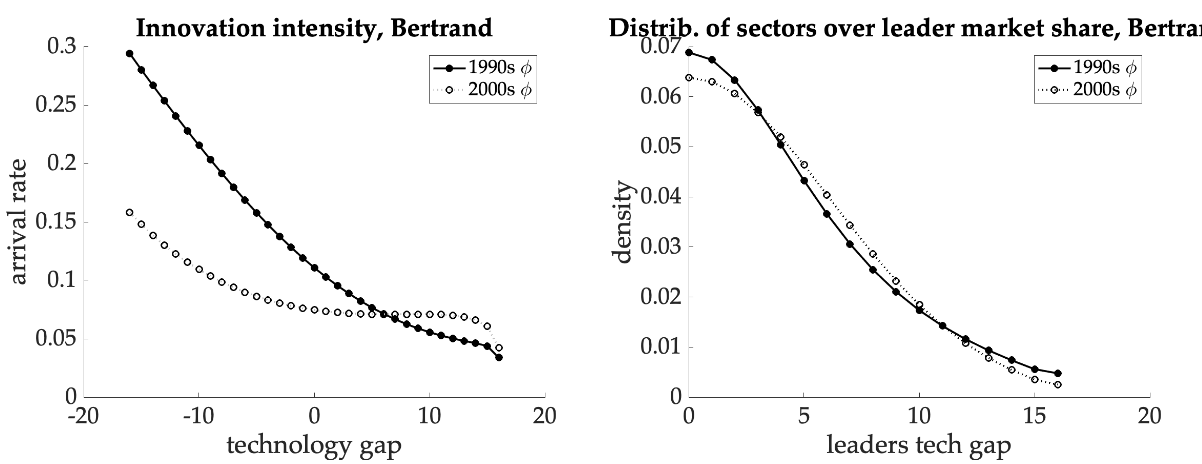


Figure 23: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), model with no competitive fringe.

Decomposition	% of slowdown explained
Role of effort (ϕ fixed, x changes)	49.9
First order effect (x fixed, ϕ changes)	82.37

Table 12: Growth decompositions, model without competitive fringe.

C Numerical Appendix

C.1 Solution Algorithm

For a given set of parameter values, the solution algorithm involves first guessing a steady state interest rate. Given this interest rate, solve the value functions for each technology gap by policy function iteration using the fact that $\dot{v}_{mt} = 0$ on a balanced growth path. This process yields the optimal innovation policies of firms at each technology gap. Given the policy functions the stationary distribution of firms over technology gaps can be obtained by solving the system of equations described in section 3.5. To obtain the growth rate of GDP, solve the system described in appendix B.4. Check whether this growth rate is consistent with the interest rate guess using the household's Euler equation: $r = g\psi + \rho$. Update the guess of the interest rate and repeat until the interest rate guess and the interest rate implied by the resulting growth rate and the Euler equation are consistent.

To obtain micro-level moments, I simulate a discrete time version of the model with ten subperiods per year for a panel of 3000 firms for 400 years after the model reaches the steady state distribution over technology gaps.

C.2 Simulated Method of Moments Estimation

Let $M_j(\boldsymbol{\theta})$ denote the steady state value of moment j in the model as a function of the model parameters in vector $\boldsymbol{\theta}$. Let D_j denote the same moment in the data. The simulated method of moments estimation procedure seeks to find the vector of parameters $\boldsymbol{\theta}^*$ that solves:

$$\min_{\boldsymbol{\theta}} \sum_{j=1}^{J} \left| \frac{M_j(\boldsymbol{\theta}) - D_j}{\frac{1}{2}M_j(\boldsymbol{\theta}) + \frac{1}{2}D_j} \right|$$

for J moments. The moments are weighted equally.

C.3 Decomposition Table

Moment	Model		Data		Effect of each parameter						
	1990s	2000s	1990s	2000s	ρ	ϵ	η	δ_e	λ	α	ϕ
TFP	1.75	0.74	1.74	0.49	1.76	1.73	1.87	1.81	1.94	2.06	0.47
Concent.	44.62	48.89	43.34	48.12	44.66	45.01	44.67	44.68	45.61	44.7	46.8
R&D/GDP	1.91	1.32	1.8	1.89	1.95	1.88	2.33	2.06	2.06	2.13	0.72
Profits/GDP	6.02	6.71	5.24	6.61	6.02	6.01	6.01	6.0	6.23	6.02	6.34
R&D/Sales	5.18	3.54	2.56	3.8	5.22	5.09	6.23	5.5	5.61	5.7	1.93
Pat. qual.	22.26	11.88	23.52	11.71	22.26	22.28	22.27	22.27	24.06	22.28	10.98
New leader	13.26	9.27	13.74	9.27	13.28	13.21	13.53	12.6	13.4	13.91	9.85

Table 13: Effect of each estimated parameter change in Table 5 on the model steady state, holding other parameters fixed at estimated 1990s values.

C.4 Transition Dynamics

This appendix details the computational approach to solving the model's transition dynamics from one steady state to another and then presents the results. I assume the economy begins in the initial (1990s) steady state in period $t = 1$ and arrives at the new steady state by T, where T is large. I consider a discrete version of the model with small time increments dt. Given a conjecture for the interest rate path over the transition, I solve the value and policy functions backwards from T. Then, given the sequence of innovation policies, I use the flow equations 13, 14, 15 and 8, 9, 10 to solve for aggregate output and the distribution of sectors over technology gaps on the transition path respectively. Then I use the household's Euler equation to check the consistency of the growth rate over the transition with the interest rate guess, and update the guess until the conjectured and implied interest rate paths are within some minimum distance from one another. Formally:

1. Guess an interest rate path $\mathbf{r} = \{r_1, r_{1+dt}, r_{1+2*dt}, \ldots, r_T\}$.
2. Given the steady state values $v_{m,T}$ assumed at T, solve for innovation policies at $T - dt$ as:

$$x_{m,T-dt} = \begin{cases} \left(e^{-r_T dt} \frac{\sum_{n=m+1}^{\bar{m}} \mathbb{F}_{m,T-dt}(n)[(\lambda^{n-m})^{\frac{1}{\beta}-1} v_{n,T} - v_{m,T}]}{\alpha} \right)^{\frac{1}{\gamma-1}} & \text{for } m < \bar{m} \\ \left[e^{-r_T dt} \frac{1}{\alpha} (\lambda^{\frac{1}{\beta}-1} - 1) v_{\bar{m},T} \right]^{\frac{1}{\gamma-1}} & \text{for } m = \bar{m} \end{cases}.$$

3. Given the policy functions at $T - dt$ and the interest rate guess, solve for the value functions $v_{m,T-dt}$:

$$\begin{aligned} v_{m,T-dt} = & \left(\pi(m) - \alpha \frac{x_{m,T-dt}^{\gamma}}{\gamma} \right) dt \\ & + e^{-r_T dt} (x_{m,T-dt} dt \sum_{n=m+1}^{\bar{m}} \mathbb{F}_{m,T-dt}(n) [v_{n,T} (\lambda^{n-m})^{\frac{1}{\beta}-1} - v_{m,T}] \\ & + x_{-m,T-dt} dt \sum_{-m+1}^{\bar{m}} \mathbb{F}_{-m,T-dt}(n) [v_{-n,T} - v_{m,T}] \\ & + \delta_e dt (0 - v_{m,t}) + v_{m,T}). \end{aligned}$$

4. Repeat this backwards iteration for $x_{m,t}$ and $v_{m,t}$ until $t = 1$.

5. Beginning at $t = 1$, initialize $Y = 1$ and use the flow equations 13-15 and 8-10 and the sequence of innovation policies $\{x_{m,t}\}_{t=1}^{T-dt}$ to obtain the distribution of firms over technology gaps and the sequence of growth rates $\mathbf{g}$ over the transition.
6. Check if the interest rate sequence $\mathbf{r}$ is consistent with the resulting sequence of growth rates $\mathbf{g}$ using the households' Euler equation.
7. Update the guess of $\mathbf{r}$ to $\mathbf{r}_{new}$ using the implied sequence of interest rates from the Euler equation.
8. Repeat until $|\mathbf{r}_{new} - \mathbf{r}| < \epsilon_{tol}$ for some small tolerance value ϵ_{tol}.

C.5 Transition Results

The experiment is to consider a surprise, permanent decrease in laggards' patent quality consistent with the increase in ϕ estimated in section 5 and with the pattern of declining patent quality in Figure 2 in section 2.2.

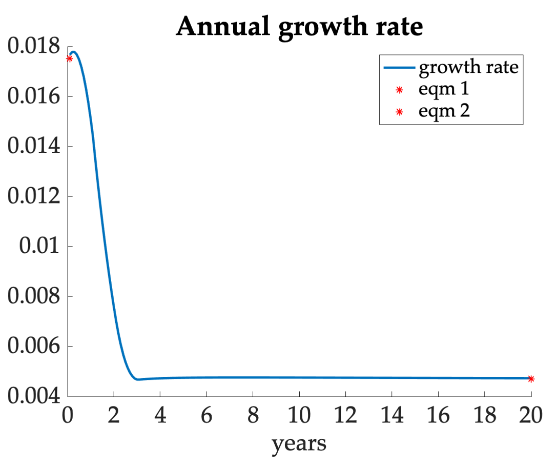


Figure 24: Productivity growth and concentration over the transition to a lower level of patent quality.

For illustrative purposes, I assume the transition takes 20 years, with ϕ increasing smoothly for the first two years and then remaining permanently higher (that is, innovations becoming more incremental). Figure 24 illustrates the evolution of productivity growth and concentration over the transition. Productivity growth closely tracks the decline in patent quality, while concentration rises more slowly and after 20 years has not fully reached its new steady state value. Because innovations are somewhat infrequent, it takes a long time for market leaders to pull ahead and laggards to fall as far behind as they are in the new steady state on average. These patterns are fairly consistent with Figure 1 which shows the TFP growth rate declining quickly in the early 2000s and then remaining roughly flat while concentration has continued to rise through 2017. As discussed in section 5.3, an increase in product substitutability immediately increases concentration, and fitting the model over the transition would help inform the relative roles of innovation technology and the superstar firm hypothesis, and might also imply an even sharper decline in laggards patent quality in order to arrive at the high level of concentration observed recently by 2017.