Capturing Value of Reliability through Road Pricing in Congested Traffic under Uncertainty

Shanjiang Zhu, Ph.D.,

Dept. of Civil, Environmental and Infrastructure Engineering
George Mason University

Gege Jiang
Hong K. Lo, Ph.D
Dept. of Civil and Environmental Engineering
Hong Kong University of Science and Technology

Acknowledgements

- General Research Fund #615712 and #616113 of the Research Grants Council of the HKSAR Government and the Hong Kong PhD Fellowship
- Junior faculty study leave program of George Mason University

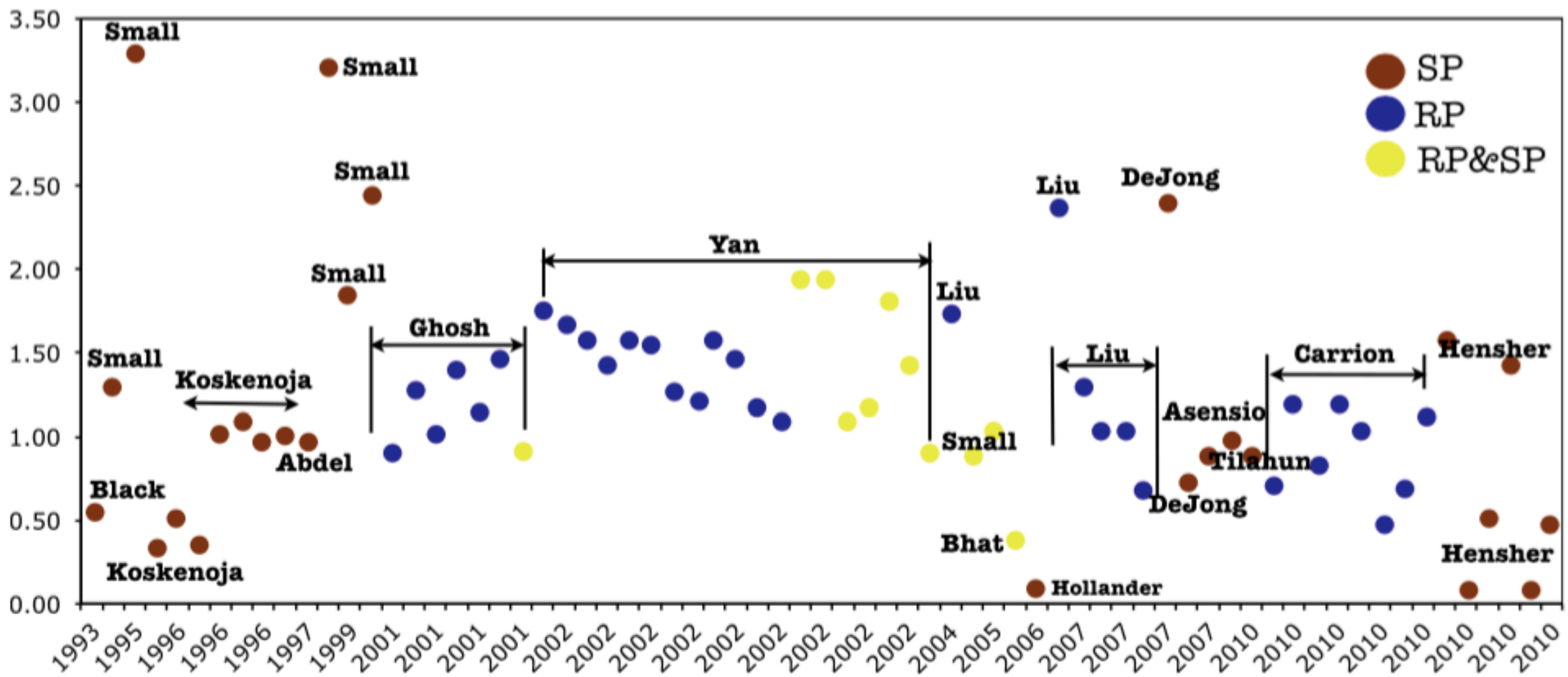
Motivations

- Importance of travel time reliability in travel decision making
  - Small 1982, Noland and Small 1995, Bates et al. 2001, and Fosgerau and Karlström 2010
  - Travel time reliability (or unreliability) is often measured using its standard deviation, although measures such as travel time variance (Jackson and Jucker 1982) and inter-quantile ranges (e.g. Small et al. 2005) have also been used.

$$VOT = \frac{\partial U / \partial \mu_T}{\partial U / \partial C} \qquad VOR = \frac{\partial U / \partial \sigma_T}{\partial U / \partial C} \qquad RR = \frac{\partial U / \partial \sigma_T}{\partial U / \partial \mu_T} = \frac{VOR}{VOT}$$

Motivations

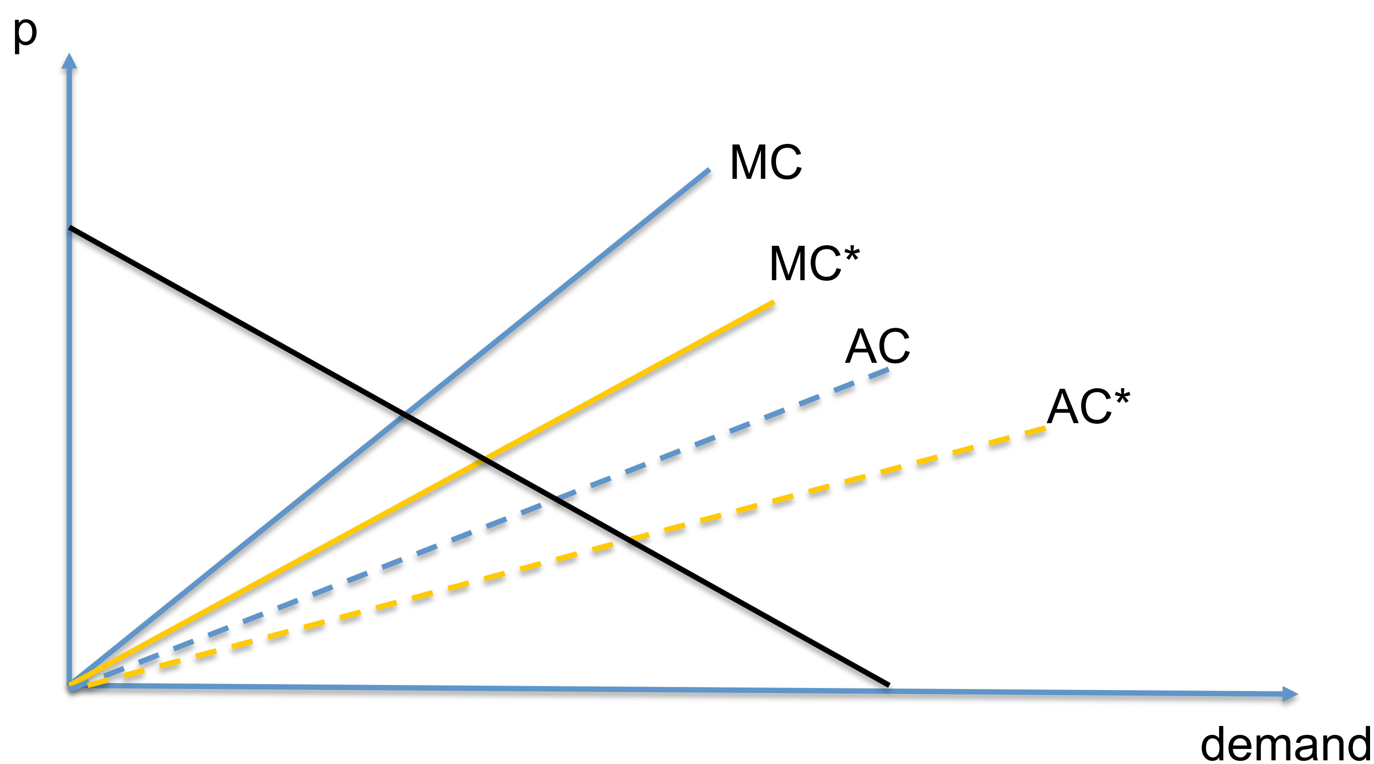
- Empirical evidences
  - Review of empirical work by Carrion and Levinson 2012, showing a Reliability Ratio ranging from 0.10 to 2.51



Source: Carrion and Levinson 2012

Motivations

- Road pricing
  - Button and Verhoef 1998, Yang and Huang 2005, etc…



Motivations

- Correlation between travel time and travel time variance
  - Chen et al (2003) reported a correlation coefficient of 0.85 using data collected from I-5 in Los Angles, CA
- Opportunities to improve welfare through not only travel time reduction, but also reliability improvement
- The relative importance of travel time reliability when compared with travel time itself could be become more pronounced when automatous vehicles hit the market.

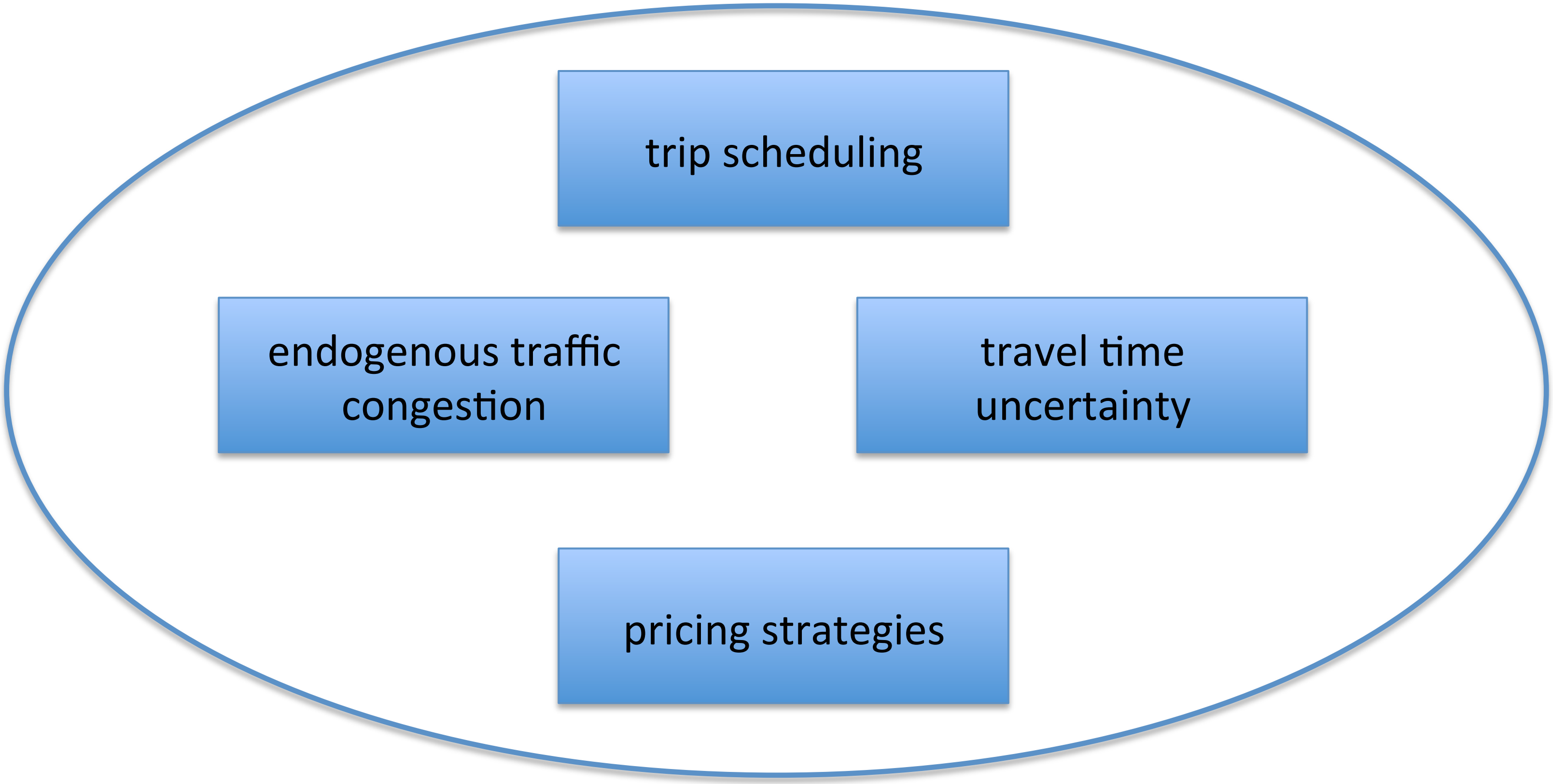
Modeling Time Variance Consideration

- Mean-variance approach
  - Jackson and Jucker 1982, Small et al. 2005

- Scheduling approach
  - Bates et al (2001), Fosgerau and Karlström (2010)

Objectives

- Capture the value of travel time reliability by road pricing



Problem Formulation: Trip Scheduling

- Following Siu and Lo (2009)

$$t_{in} = 0 - b(X) = -b(X)$$

$$t_{out} = -b(X) + Q(X)/s + R$$

R is a random delay

- Scheduling Cost

$$SDE(X) = max[-t_{out}(X), 0]$$

$$SDL(X) = max[0, t_{out}(X)]$$

- Separating fix and random time

$$t_{out}(X) = t_{out}(X) + R$$

$$t_{out}(X) = -b(X) + Q(X)/s$$

Problem Formulation: Trip Scheduling

- Chance of not being late

$$P(t_{out}(X) \leq 0) = P(t_{out}(X) + R \leq 0) = P(R \leq -t_{out}(X)) = F(-t_{out}(X)) = \rho_X$$

Within Budget Time Reliability (WBTR) following Lo et al. (2006), depends on the CDF of the random delay R

$$E(SDE(X)) = -\int_0^{\max\{-t_{out}(X),0\}} (t_{out}(X) + r) f(r) dr, \forall t_{out}(X) \leq 0$$

$$E(SDL(X)) = \int_{\max\{-t_{out}(X),0\}}^{t} (t_{out}(X) + r) f(r) dr, \forall t_{out}(X) \geq -t$$

Equilibrium Trip Scheduling under Travel Time Uncertainty

- Expected Cost

$$EC(X)=Q(X)/s+E(R)+\beta_X E(SDE(X))+\gamma_X E(SDL(X))$$

$$EC^e=EC(X)=Q(X)/s+E(R)+SC(X),\ \forall X$$

- Under FIFO condition and homogeneous R

$$t_{out}(X)=t_{out}(0)+X/s$$

$$Q(X)/s=EC^e-E(R)-SC(X)$$

$$-b(X)=t_{out}(X)-Q(X)/s=-b_0+X/s-Q(X)/s$$

Time-varying Tolls with Exogenous and Identical Random Delay

- Expected Cost

$$EC\,(X)=\tau\,(X)/\xi+Q\,(X)/s+E(R)+\beta_X\,E(SDE(X))+\gamma_X\,E(SDL(X))$$

- Efficient toll scheme should not add additional costs when there is no queue.

$EC\,(0)=EC(0)=E(R)+SC(0)$ and $EC\,(X)=EC(X)$

- Toll Rate

$$\tau\,(X)=\xi(Q(X)/s)$$

- Queue can be eliminated

$$EC\,(X)=EC(X)\Longrightarrow\tau\,(X)/\xi+Q\,(X)/s=Q(X)/s\Longrightarrow Q\,(X)=0$$

Endogenous Random Delay

- Correlation between travel time and travel time reliability
  - Empirical work by Chen et al. 2003; Ng et al, 2011; Rakha et al. 2010 etc...
  - Random capacity degradation approach by Lo and Tung, 2003; Lo et al. 2006; Siu and Lo, 2008 etc...
  - Travel time unreliability is positivley correlated with the level of congestion.

$$R_N = \omega(N/s)R \qquad R \sim U[0,1]$$

$$P(t_{out}(X) \le 0) = P(t_{out}(X) + \omega N/s\, R \le 0) = P(R \le -s/\omega N\, t_{out}(X)) = F(-s/\omega N\, t_{out}(X)) = \rho_X$$

Endogenous Random Delay

- Scheduling Costs due to Random Delay

$$E(SDE(X)) = -\int_0^{\max\{-\frac{s}{\omega N} t_{out}(X),0\}} \left(t_{out}(X) + \frac{\omega N}{s} r\right) f(r)\, dr, \quad \forall t_{out}(X) \le 0$$

$$E(SDL(X)) = \int_{\max\{-\frac{s}{\omega N} t_{out}(X),0\}}^{1} \left(t_{out}(X) + \frac{\omega N}{s} r\right) f(r)\, dr, \quad \forall t_{out}(X) \ge -\frac{\omega N}{s}$$

$$EC(X) = \frac{Q(X)}{s} + \frac{\omega N}{s} E(R) + \beta_X E(SDE(X)) + \gamma_X E(SDL(X))$$

- Change of departure profile

$$b(X) = t_{out}(X) - \frac{Q(X)}{s} = -b_0 + \frac{X}{s} - \frac{Q(X)}{s}$$

Although the random delay $\frac{\omega N}{s} R$ does not appear in the expression of departure time $-b(X)$, it does affect the departure time choice through the queueing process.

Problem Formulation: Endogenous Congestion and Pricing Strategies

- Adopt a uniform toll scheme
- Choose a linear demand function to keep the discussions succinct
  - Many functional forms have been used in literature.
  - Most of them assume a monotonically decreasing function to maintain certain desireable properties.
  - Linear function has been used in many studies for its simplicity.

$$c = \kappa - \psi * N$$

κ is the maximum price and ψ is the downward slope

Value of Reliability

- Following Fosgerau and Karlström (2010) and Carrion and Levinson (2012), we define the value of reliability as:

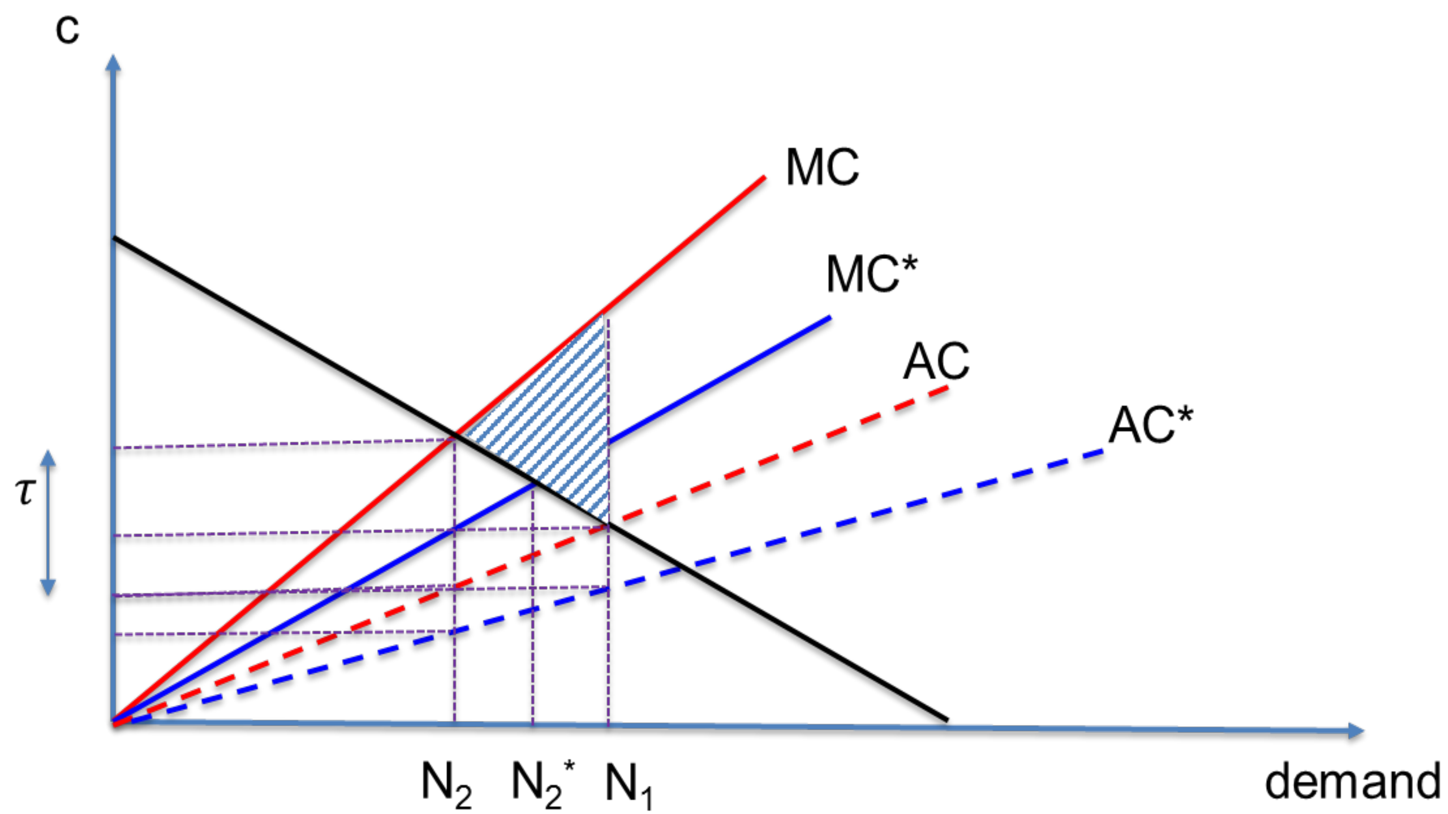
$$VOR = dEC\uparrow e\,/d\sigma$$

- In our model, the reliability measure is:

$$\sigma = \sqrt{\omega N/12s}$$

- The travel time reliability can be improved through:
  - Reducing the level of congestion (smaller N/s ratio)
  - Better operation and management (thus smaller ω)

Marginal Pricing



- Marginal pricing:

$$MC(N) = EC \uparrow e\,(N) + N * \partial EC \uparrow e\,(N) / \partial N$$

$$\tau = \partial EC \uparrow e / \partial N\ N$$

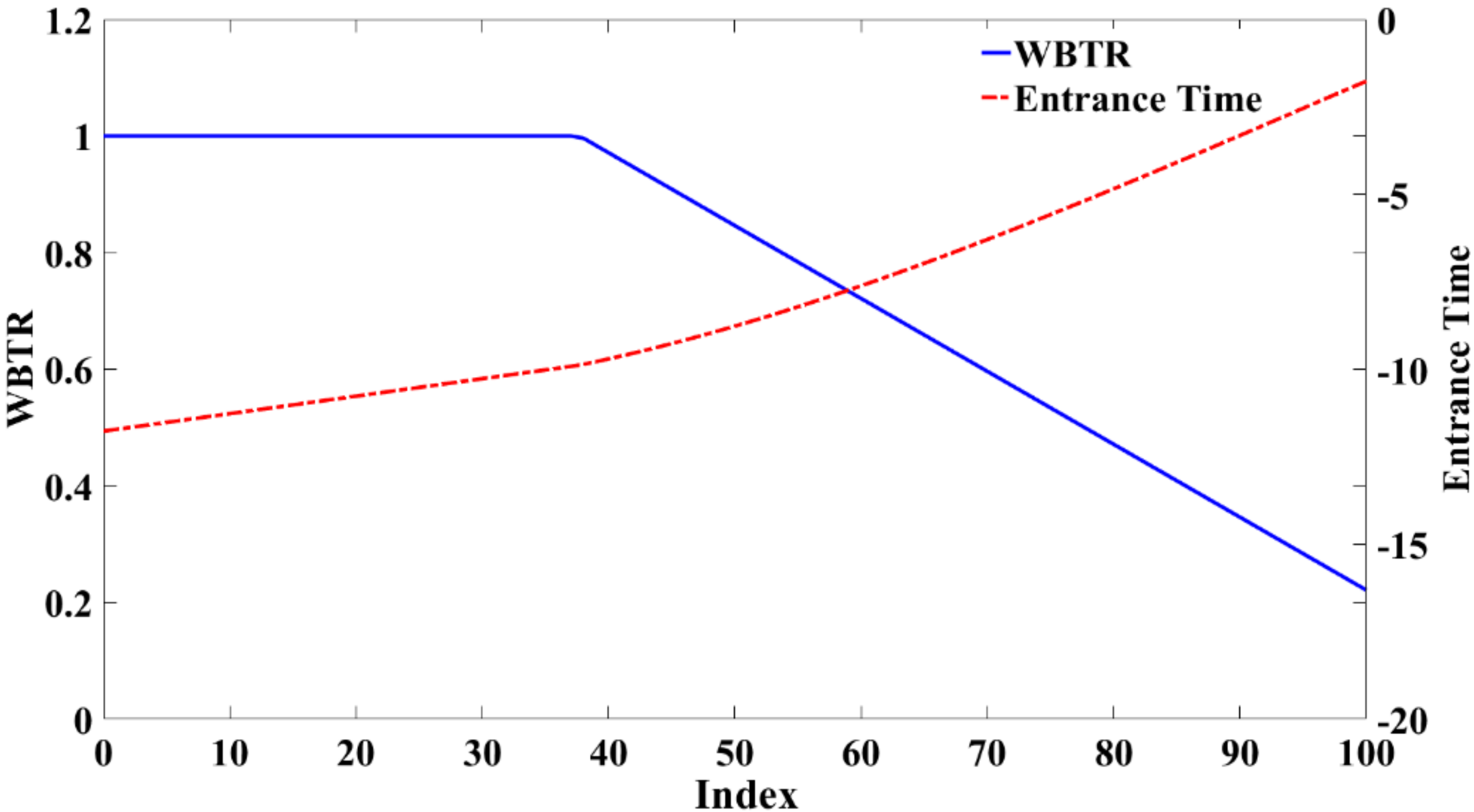
Numerical Example

$$\beta_X = \beta^0 - \beta^1 \rho_X$$

$$\gamma_X = \gamma^0 - \gamma^1 \rho_X$$

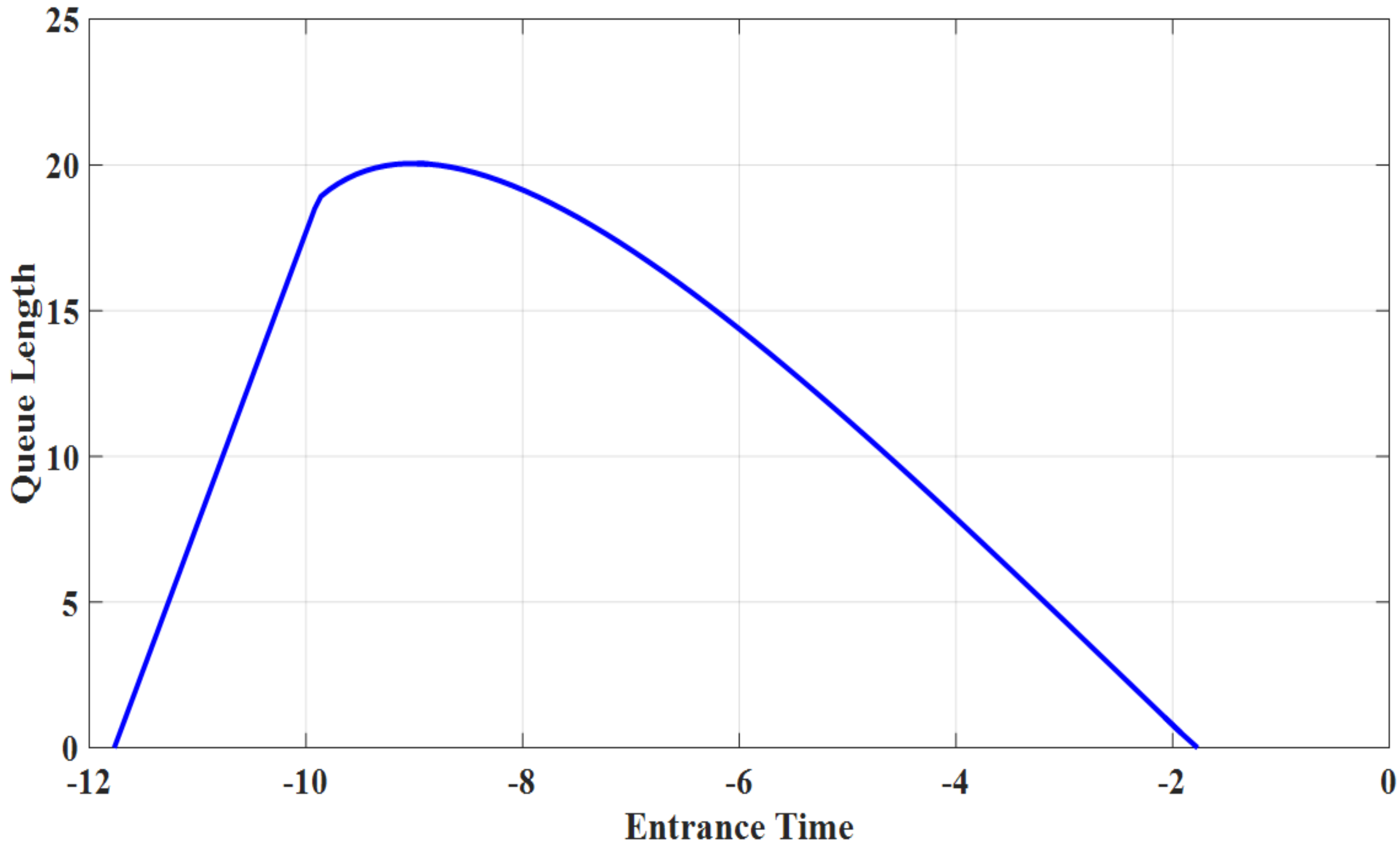
Variables	Symbol	Value
Free-flow travel time	t_f	0
Values in the taste functions	β^0	1
	β^1	0.5
	γ^0	1.2
	γ^1	1.5
Total demand	κ	20
	ψ	0.133
Bottleneck capacity	c	10
Upper bound of random delay	ω	0.8

Mapping of the Traveler, Departure Schedule and WBTR under Random Delay



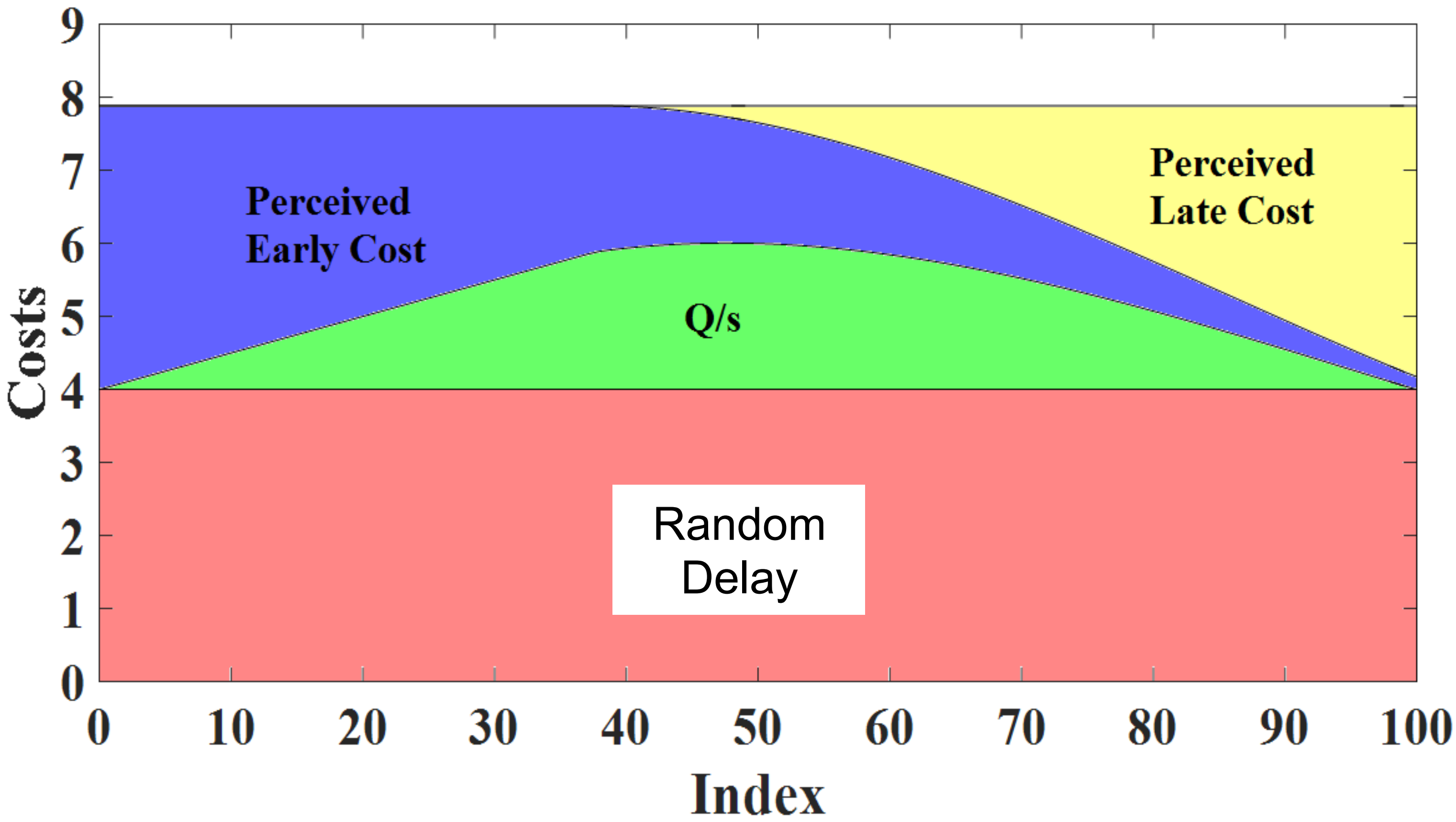
- Travelers with a strong preference for punctual arrival (high WBTR) chose to leave home early.

Queue Length Dynamics



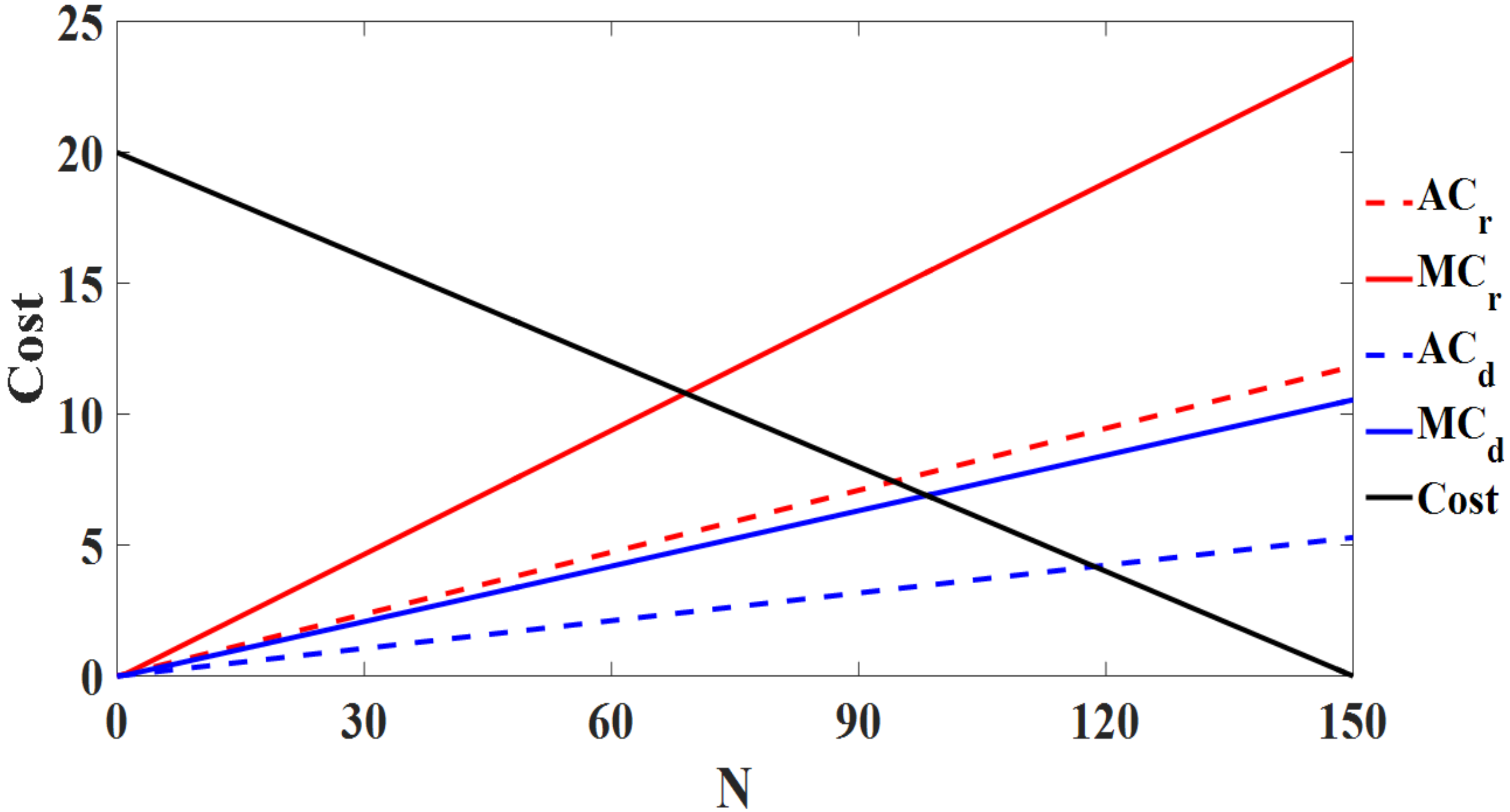
- Queue length dynamics under random delay

Cost Decomposition at Equilibrium with a Given Demand



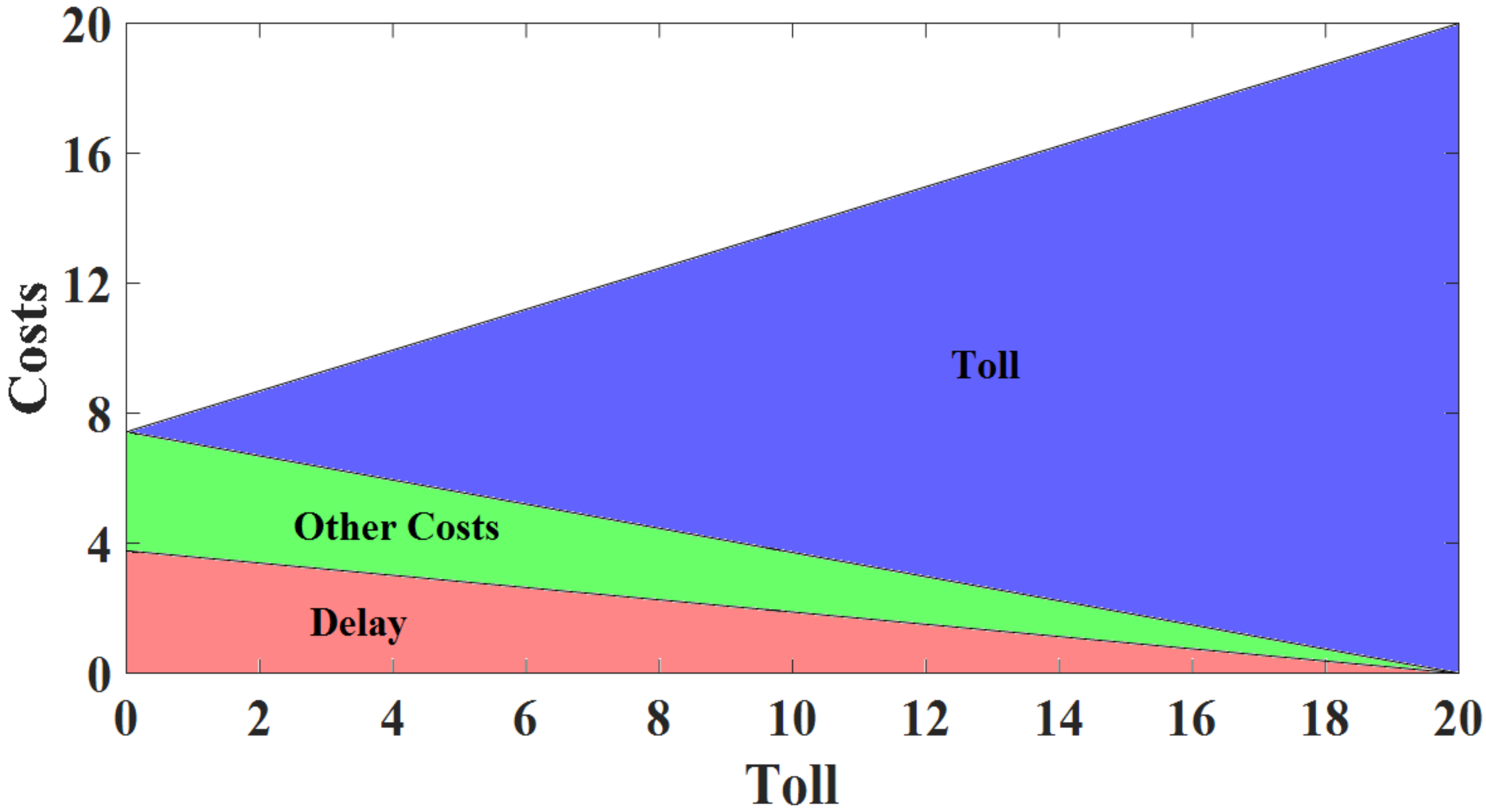
- Cost structure is different for travelers with different WBTR.

Supply and Demand



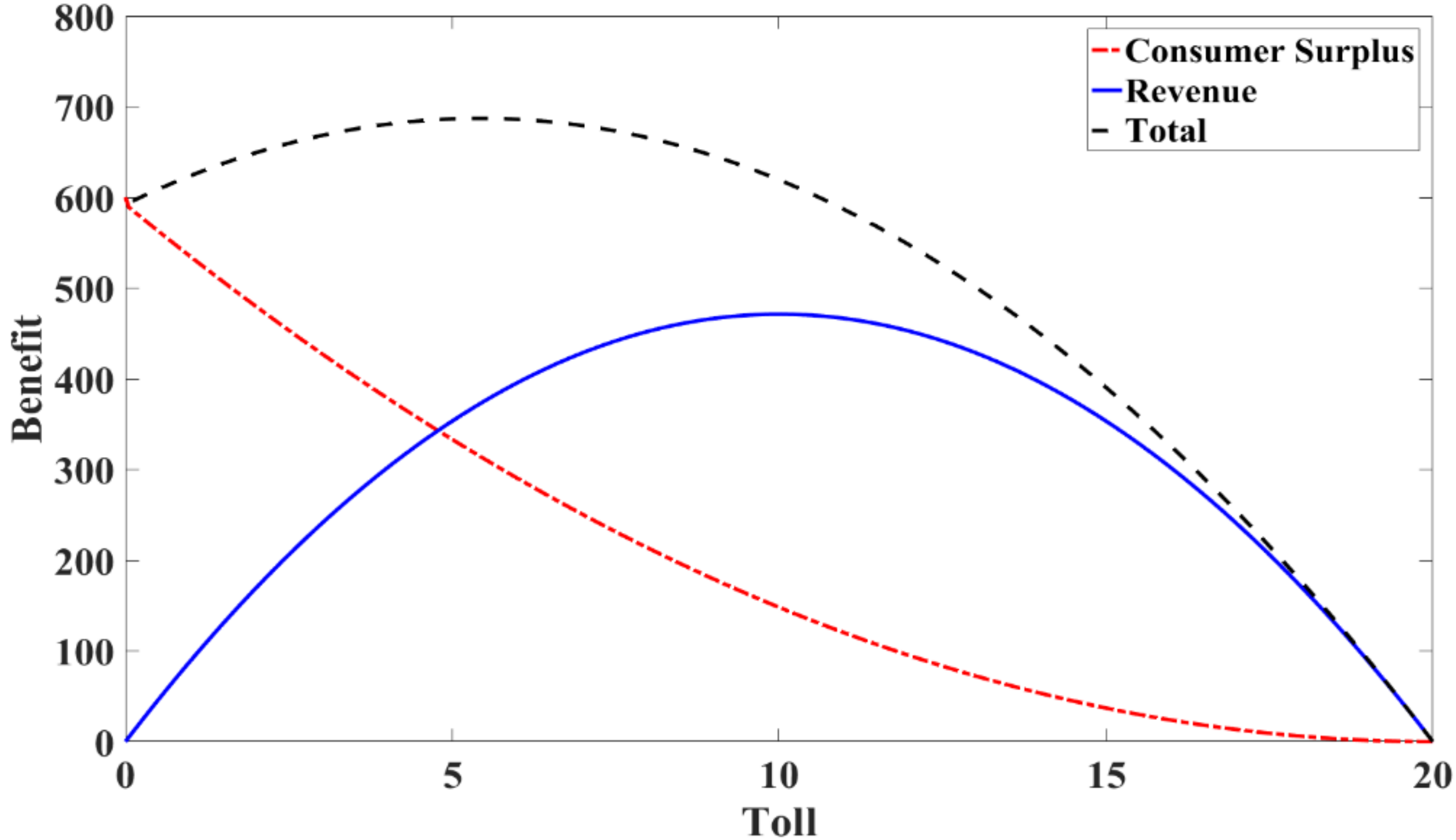
- Marginal and average costs with a full understanding of the utility structure (in red) and partial understanding of the utility structure (in blue)

Impact of Tolling on Costs



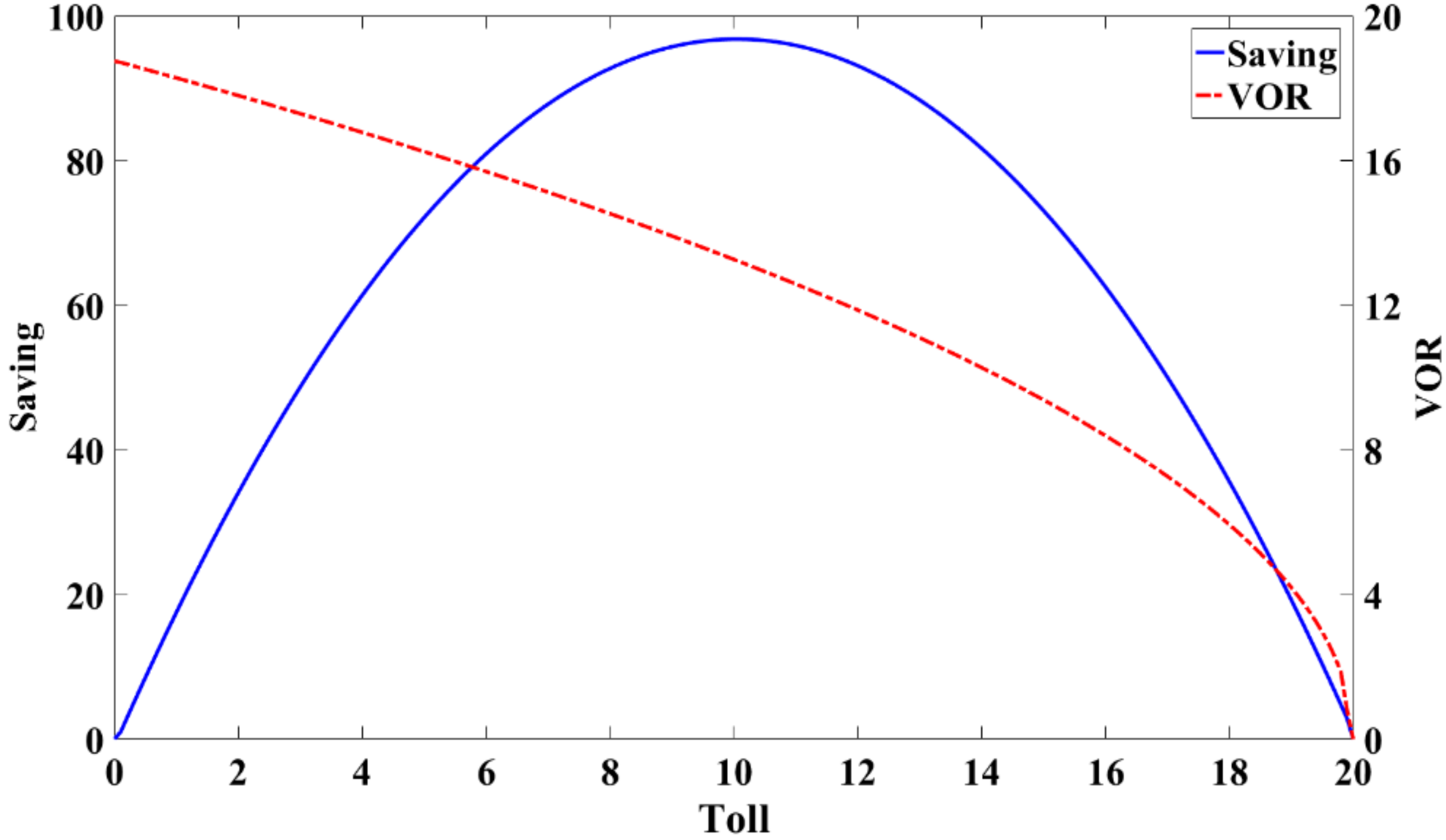
- Equilibrium costs and cost decomposition under different toll levels

Social Benefits



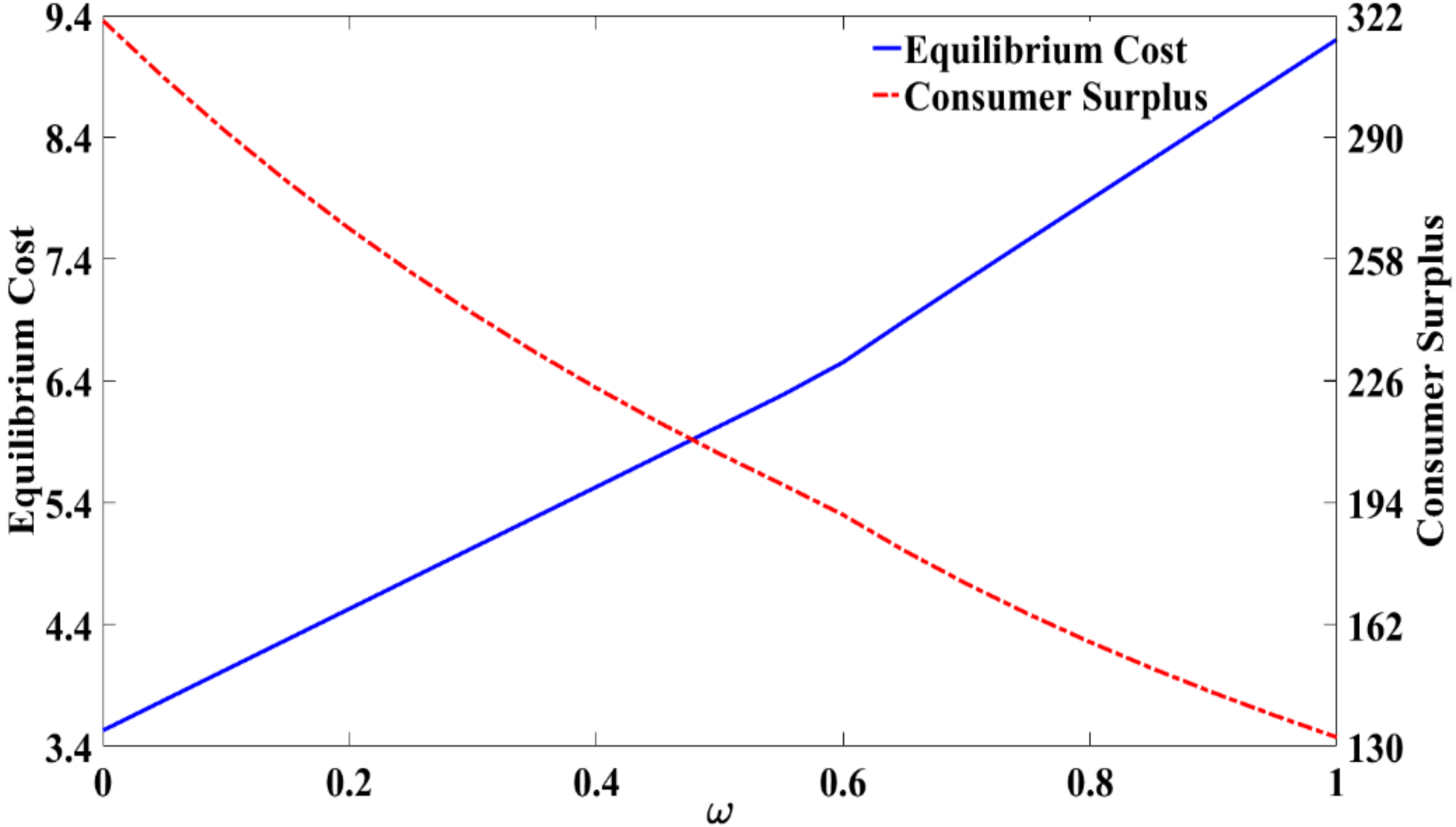
- Consumer surplus and total revenue at different toll level

Numerical Example



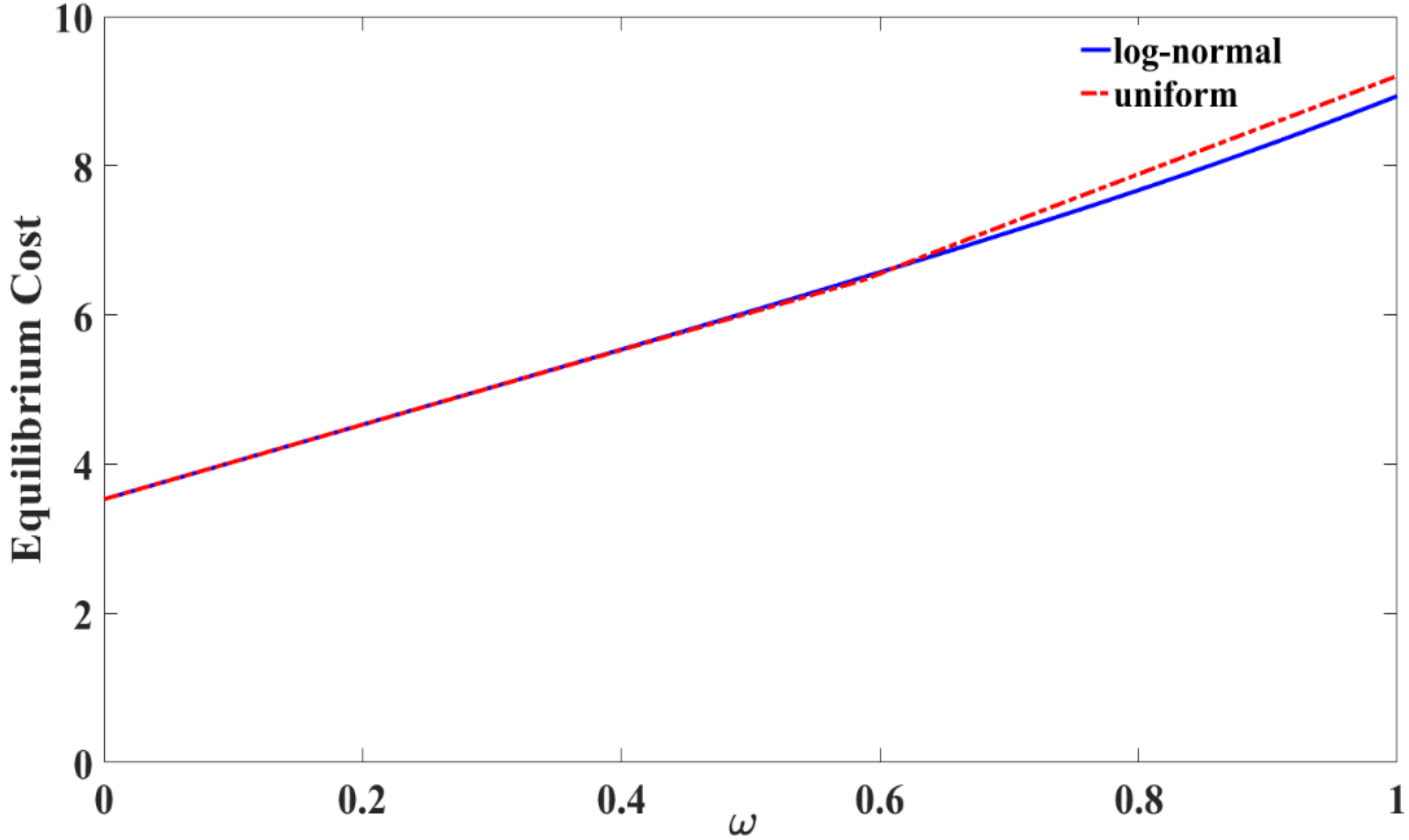
- Cost savings due to reliability improvements and evolution of Value of Reliability at different toll levels

Sensitivity Test of the Operation and Management Efficiency Parameter



- Impact of different travel time variance sensistity parameters to system v/c ratio on comsumer surplus and equilibrium cost at the system optimal point

Impact of Random Delay Distributions



- Equilibrium cost under uniform and log-normal endogenous random delay

Conclusions

- Both analytical analysis and numerical examples showed that when the level of travel time unreliability increases (large random delay), travelers have to depart earlier to accommodate their preferred WBTR.
- When the system operation and management improve, the random delay decreases with the same demand over capacity ratio and the average cost curve would shifts downward.
- When a toll is charged, the cost related to both travel time and travel time reliability reduces.
- VOR is monotonically decreasing as the toll increases, thus a moderate toll level is most effective in capturing reliability improvement.

Next Step

- Stepwise tolling scheme
- Further theoretical development of VOT and VOR
- Empirical analysis of VOT and VOR
- Signaling effects in dynamic tolling scheme

Thank You!

Questions and Comments?

Shanjiang Zhu, Ph.D., Assistant Professor

Civil, Environmental & Infrastructure Engineering

George Mason University

szhu3@gmu.edu

http://civil.gmu.edu/people/shanjiang-zhu/