The morning commute in urban areas with heterogeneous trip lengths

ISTTT22, Northwestern University

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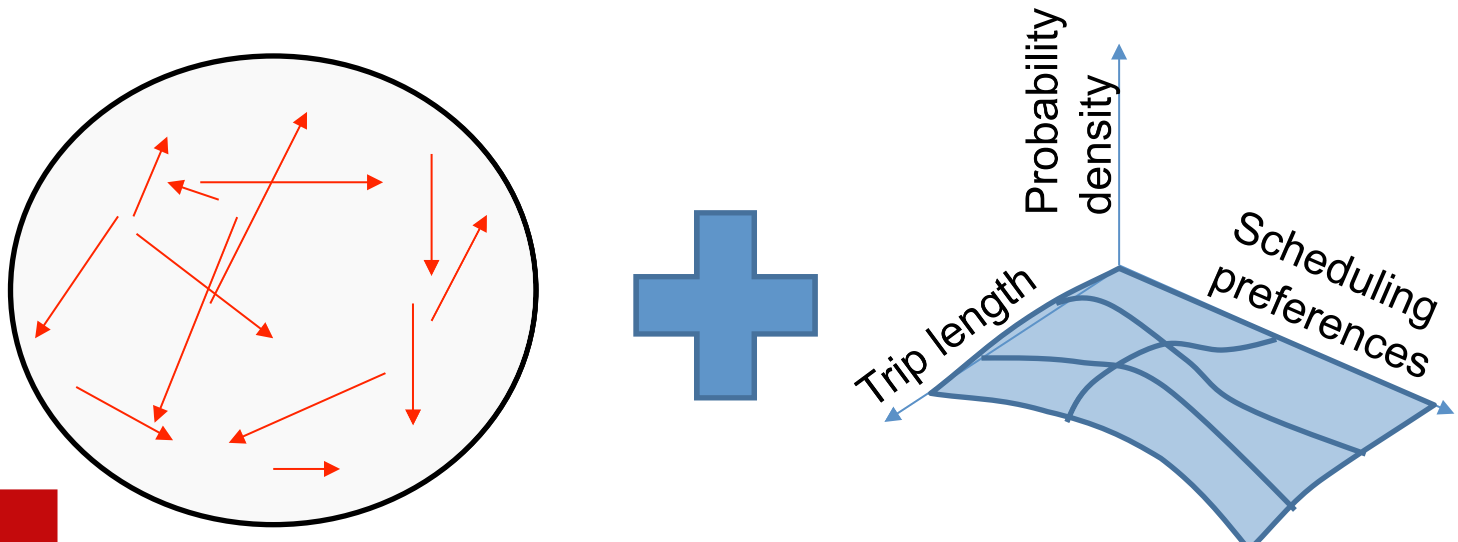
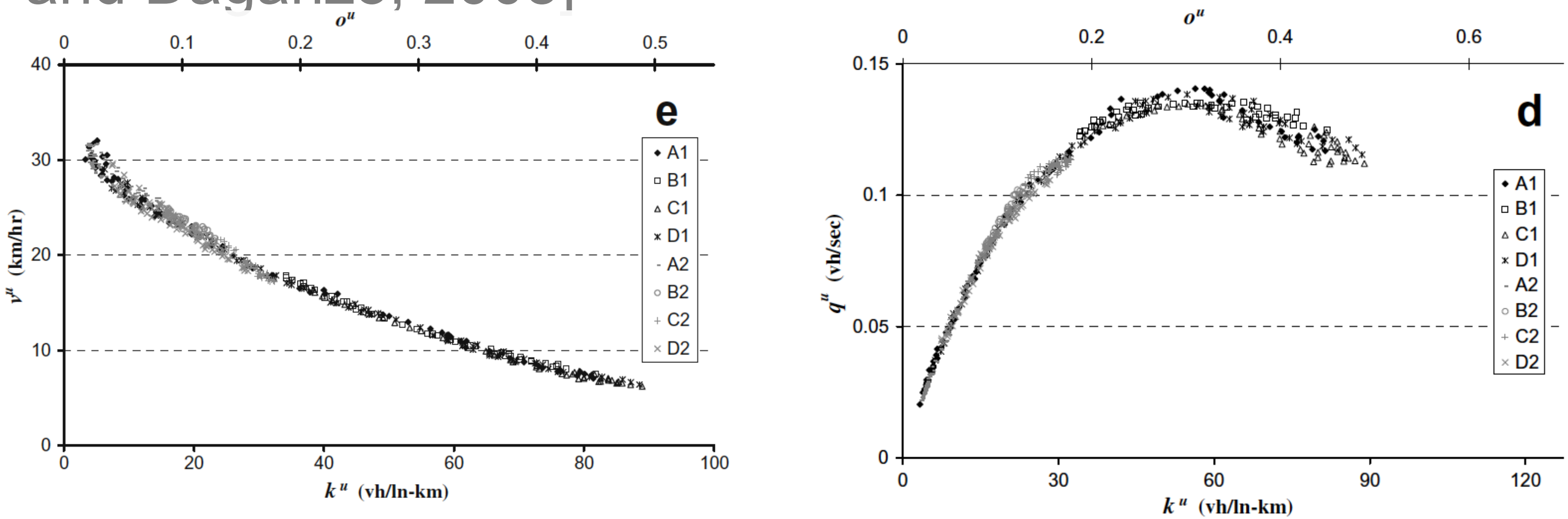
In a nutshell

- Vickrey (1969): the morning commute with a local bottleneck.
- How to extend it to a city?



In a nutshell

- The speed MFD or production MFD (NFD) [Geroliminis and Daganzo, 2008]

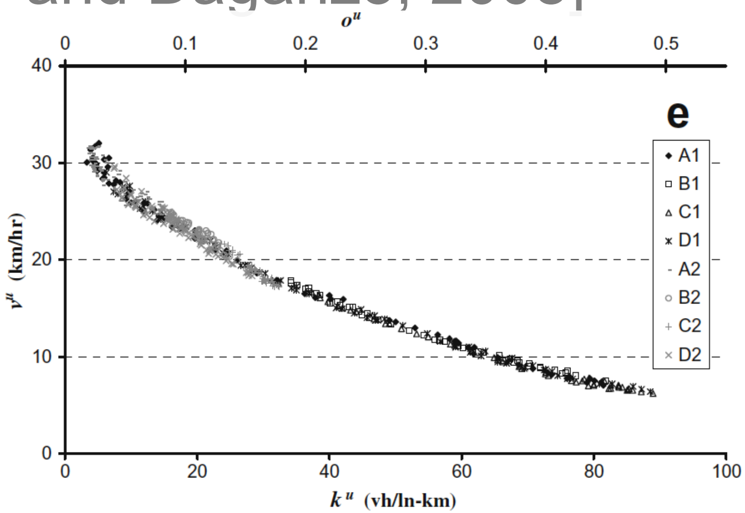


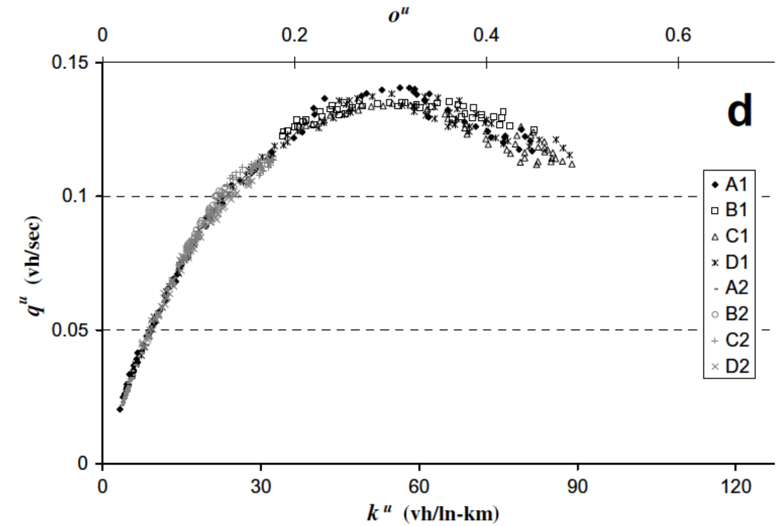
- Should not be confused with the outflow MFD (or NEF)

$$O(n)=nv(n)/L$$

In a nutshell

- The speed MFD or production MFD (NFD) [Geroliminis and Daganzo, 2008]





- Dynamics imposed by the fundamental equation:

$$\int_{t_d}^{t_a} v(n(t))\,\mathrm{d}t = l$$

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LITERATURE REVIEW

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Review of literature with MFD-like dynamics

Homogeneous users

Small and Chu (2003)

Geroliminis and Levinson (2009)

Fosgerau and Small (2013)

(Stochastic identical trip length)

Arnott (2013)

Arnott, Kokoza and Naji (2016)

Heterogeneous trip length

Fosgerau (2015)

Daganzo and Lehe (2015)

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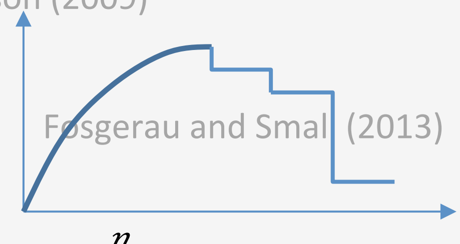
Review of literature with MFD-like dynamics

Homogeneous users

Small and Chu (2003)

$\tau(t) = L / v(t)$

Geroliminis and Levinson (2009)

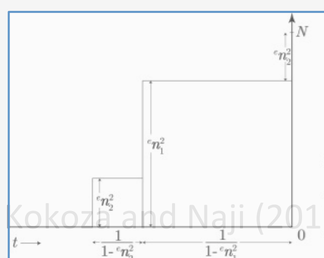


Fosgerau and Small (2013)

(Stochastic identical trip length)

Arnott (2013)

Arnott, Kokoza and Naji (2016)



Heterogeneous trip length

Fosgerau (2015)

Dagan and Lehe (2015)

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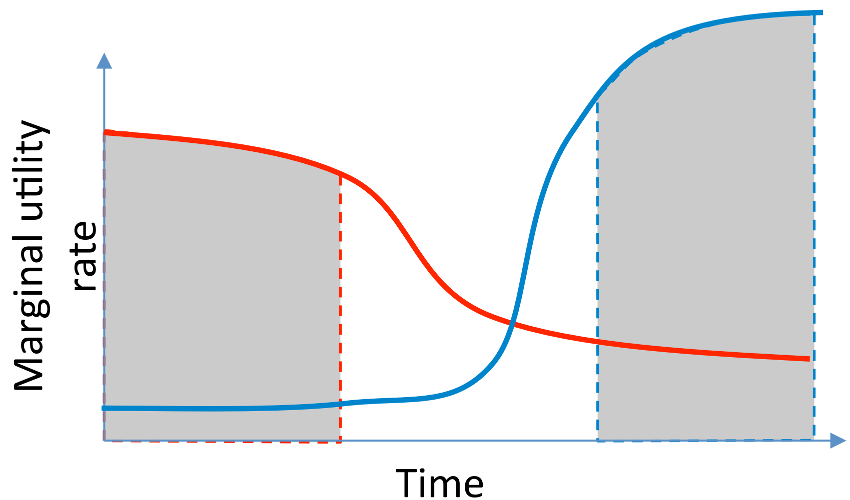
Scheduling preferences

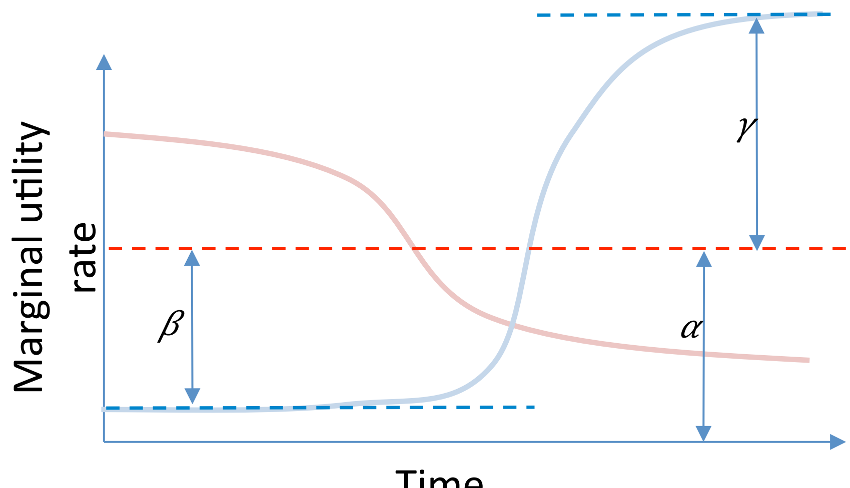
- Based on marginal utility rates:

$$U = \int_0^{t_d} h(t)\,\mathrm{d}t + \int_{t_d}^{t_a} 0\,\mathrm{d}t + \int_{t_a}^{0} w(t)\,\mathrm{d}t$$



Scheduling preferences

- Based on marginal utility rates
- No strong consensus on the shape despite several RP and SP studies [Small, 1982 ; Hendrickson and Planck, 1984 ; Tseng and Verhoef, 2008 ; Hjorth et al., 2015]
- Several functional forms proposed



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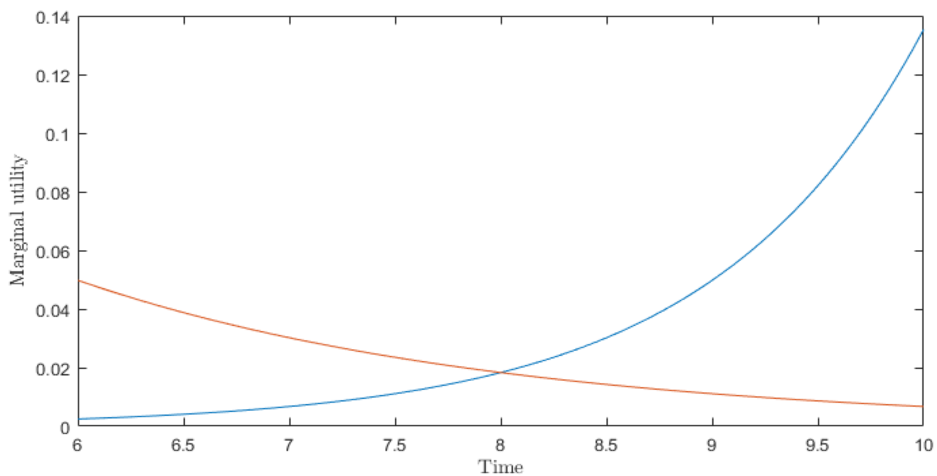

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Literature review (continued)

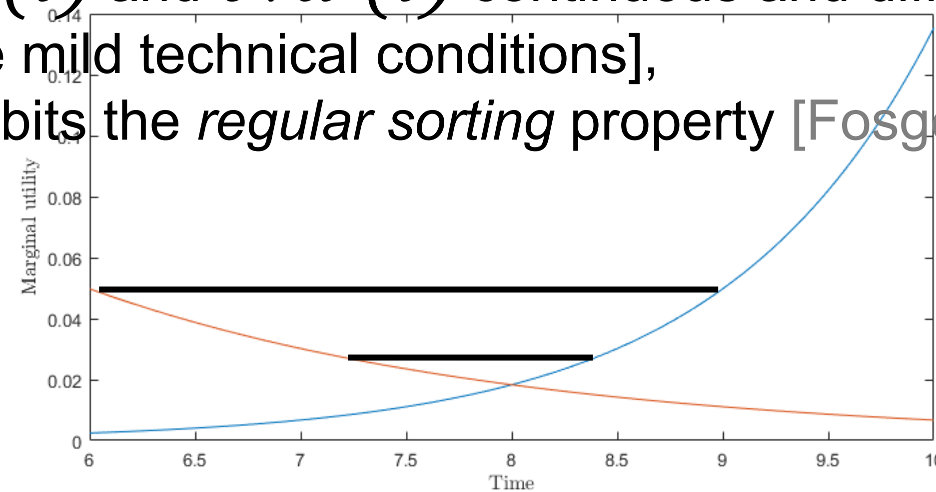
If Users differ only by trip length (l),

$d/dt\ (h/v)(t)<0$ and $d/dt\ (w/v)(t)>0$ for all times t,

$t_d(l)$ and $t_a(l)$ continuous and differentiable,

[some mild technical conditions],

Then UE exhibits the *regular sorting* property [Fosgerau, 2015]:



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ANALYTICAL RESULTS

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Continuity (proposition)

Assumptions:

A1
- Continuum of users
- At home: $h(t,\theta)$ [positive and continuous w.r.t. t]
- At work: $w(t,\theta,T^*)$ [continuous w.r.t. t, except possibly at $t = T^*$]

- l, T^* and $l|T^*$ are continuously distributed

$\Rightarrow n(t)$ and $v(t)$ continuous.

---------------------------------- Attention --

- Continuity result valid for a large set of h and w functions, including $\alpha-\beta-\gamma$ preferences.
- The next results require the same assumptions, plus $\alpha-\beta-\gamma$ preferences.

Fundamental relationships

$$u(t_a, t^*, l) = -\alpha\tau(t_a, l) - sp(t_a, t^*, l)$$

1st order condition of User Equilibrium:

$$\frac{\partial u}{\partial t_a}(t_a, t^*, l) = 0 \Leftrightarrow -\alpha\frac{\partial \tau}{\partial t_a}(t_a, l) - \frac{\partial sp}{\partial t_a}(t_a, t^*, l) = 0$$

$$\int_{t_a-\tau(t_a,l)}^{t_a} v(t)dt = l \quad \Rightarrow \quad \frac{\partial \tau}{\partial t_a}(t_a, l) = \frac{v(t_d) - v(t_a)}{v(t_d)}$$

$\Rightarrow$ For early users: $v(t_a(t_d, l)) = \frac{\alpha-\beta}{\alpha}v(t_d)$

$\Rightarrow$ For late users: $v(t_a(t_d, l)) = \frac{\alpha+\gamma}{\alpha}v(t_d)$

$\Rightarrow$ For on-time users: $\frac{\alpha-\beta}{\alpha}v(t_d) \leq v(t_a(t_d, l)) \leq \frac{\alpha+\gamma}{\alpha}v(t_d)$

Fundamental relationships

- 2nd order condition: if $t_a \neq t^*$ and $v(t)$ is differentiable[1], then at equilibrium

$$\frac{\partial^2 \tau}{\partial t_a^2}(t_a, l) \geq 0$$

Or, equivalently: $v'(t_d)/v(t_d)^2 \geq v'(t_a)/v(t_a)^2$

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[1] The exogenous differentiability assumption is avoided in the paper.

Intuition in the $(x, \ln(V))$ space…

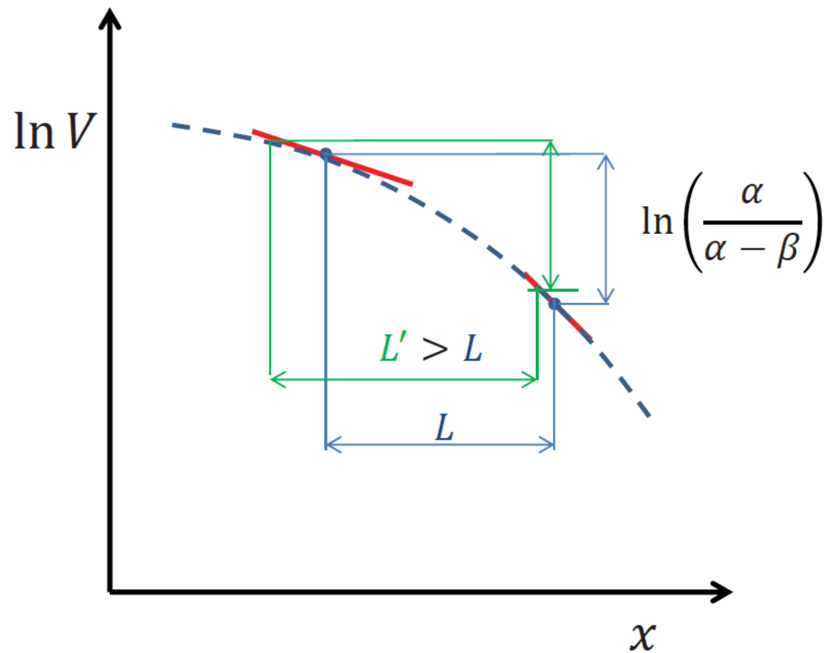
- Bijection: $x = f(t) = \int_0^t v(u)\,du$
- $V(x) = v(f^{-1}(x))$
- 1st order condition (FOC) for early users:

  $$\ln(V(x_d)) - \ln(V(x_a)) = \ln\left(\frac{\alpha}{\alpha-\beta}\right)$$

- 2nd order condition (SOC) for early users:

$$\frac{d\ln V}{dx}(x_a) \leq \frac{d\ln V}{dx}(x_d)$$

Intuition in the $(x, \ln(V))$ space...



SOC imposes a "point-to-point concavity" to $\ln(V)$ for early users

⇒ Locally FIFO

⇒ Trip-length serves as sorting criterion

Analytical results

- **Single peak assumption (A2)**: $n(t)$ strictly increasing before $t\downarrow p$, strictly decreasing after.
- A2+$(\alpha-\beta-\gamma)$ **=>** Early users travel before $t\downarrow p$, late users after (**Corollary of FOC + SOC).**

- **Proposition:** A1+A2+ $(\alpha-\beta-\gamma)$ => FIFO sorting among early (resp. Late) users having identical β/α (resp. γ/α). Longer trips start and finish earlier (resp. later).

- See also: Alternative approach based on exogenous assumptions only.

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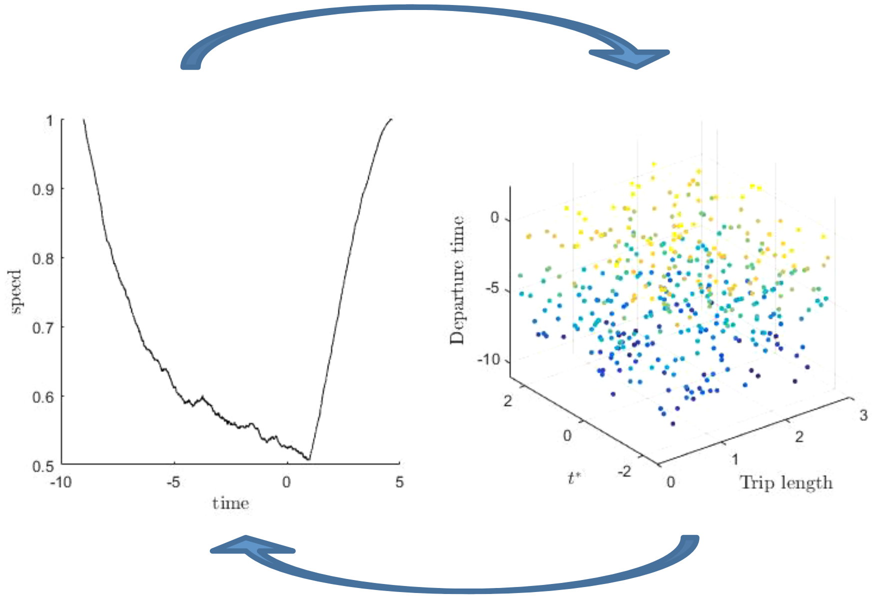
Adjustment mechanism



Simulation

SIMULATIONS

Simulation description

- Agent-based event-based dynamics
  - 4000 agents,
  - 10 families of $\alpha-\beta-\gamma$ preferences
  - l, $t\uparrow *$ uniformly distributed.

- MSA:
  - full knowledge of last iteration,
  - 2% update their decision at every iteration.

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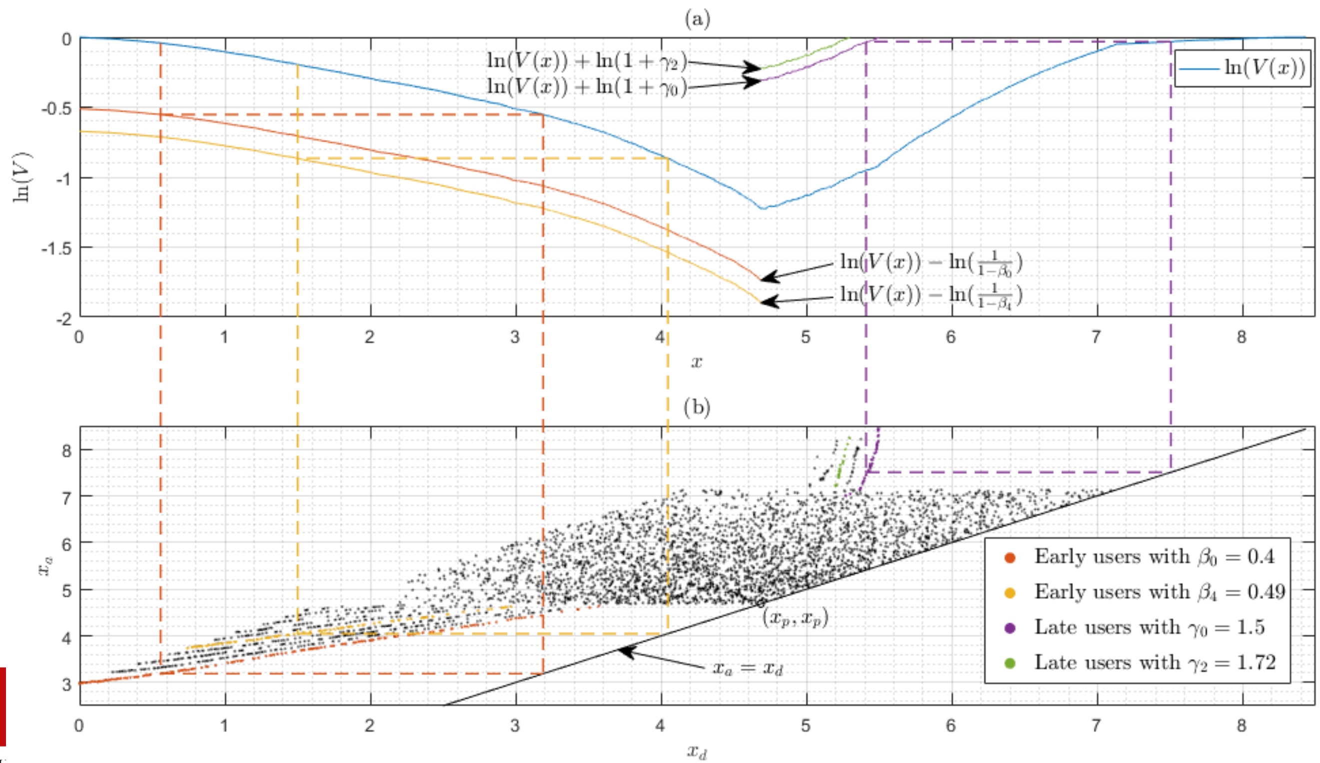
Lessons from simulations

- Confirms theoretical findings (continuity, FIFO, sorting).



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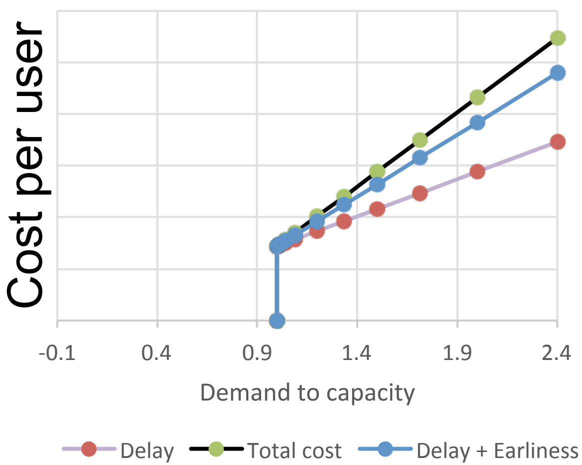
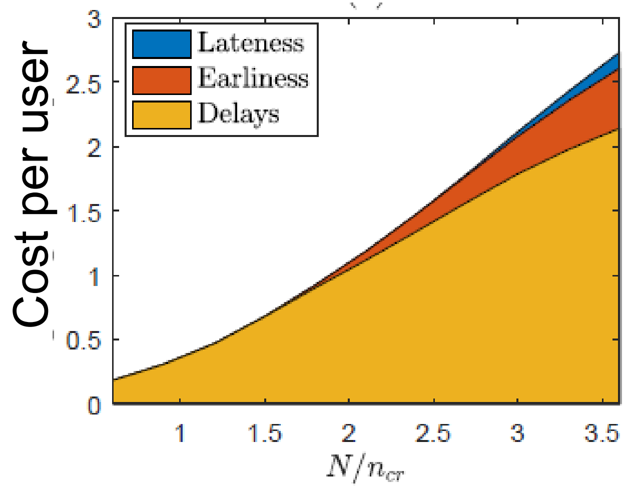
Influence of demand-to-capacity ratio

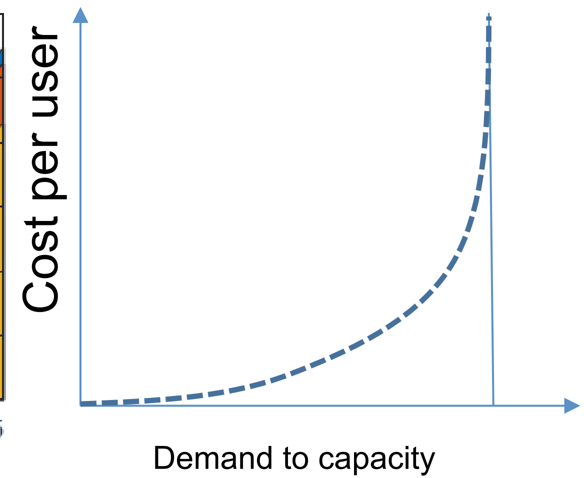
Vickrey (1969) (FIFO)



Present work (FIFO)



LIFO



- Smoothness
- Cost composition
- Instability and heavy congestion

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FUTURE RESEARCH DIRECTIONS

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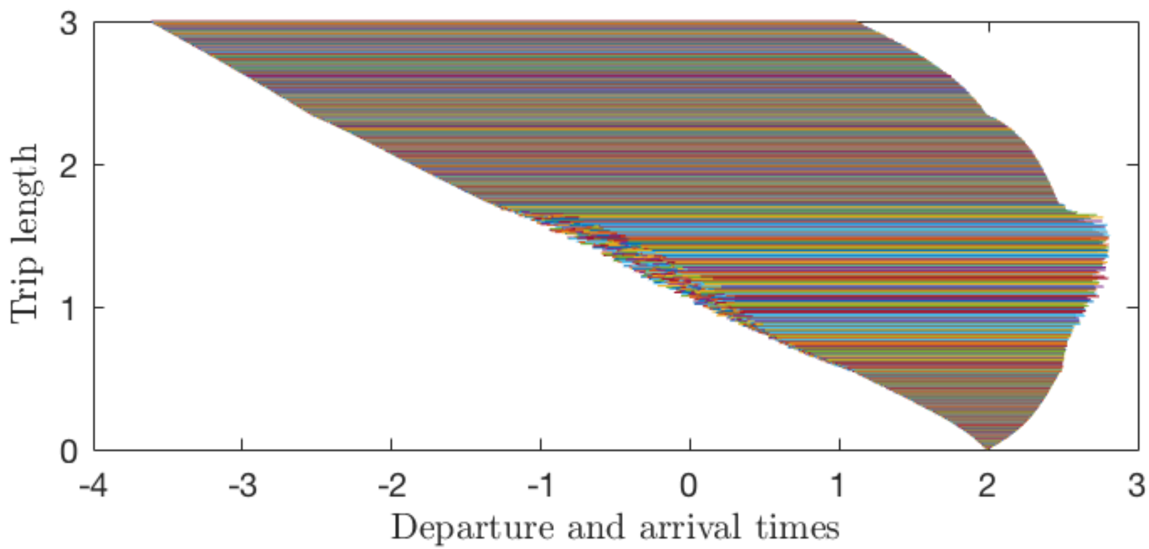
General scheduling preferences

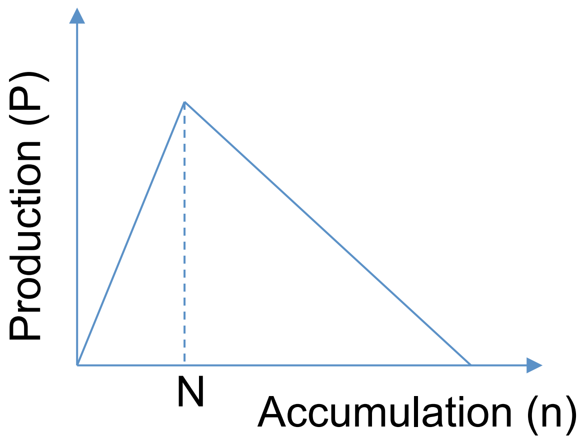
- Transition FIFO/LIFO

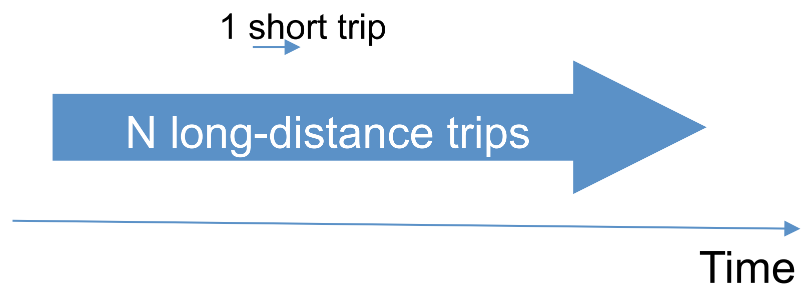


- Impact of congestion

Social optimum

- Applicability of enveloppe theorems?
- An example with hypercongestion





Further research directions

- Stability:
  - Depends on the adaptation mechanism,
  - Easy to observe with simulations,
  - Difficult to characterize analytically.

- Spatially heterogeneous settings.

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THANK YOU

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References

- Arnott, R., 2013. A bathtub model of downtown traffic congestion. Journal of Urban Economics 76, 110–121.
- Arnott, R.J., Kokoza, A., Naji, M., 2016. Equilibrium traffic dynamics in a bathtub model: A special case. Economics of Transportation 78, 38–52.
- Daganzo, C.F., Lehe, L.J., 2015. Distance-dependent congestion pricing for downtown zones. Transportation Research Part B: Methodological 75, 89–99.
- Fosgerau, M., Small, K.A., 2013. Hypercongestion in downtown metropolis. Journal of Urban Economics 76, 122–134.
- Fosgerau, M., 2015. Congestion in the bathtub. Economics of Transportation 4, 241–255.
- Geroliminis, N., Daganzo, C.F., 2008. Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. Transportation Research Part B: Methodological 42, 759–770.
- Geroliminis, N., Levinson, D., 2009. Cordon pricing consistent with the physics of overcrowding, in: Lam, W.H.K., Wong, S.C., Lo, H.K. (Eds.), Transportation and Traffic Theory 2009: Golden Jubilee. Springer US, pp. 219–240.
- Hendrickson, C., Plank, E., 1984. The flexibility of departure times for work trips. Transportation Research Part A: General 18, 25–36.
- Hjorth, K., Börjesson, M., Engelson, L., Fosgerau, M., 2015. Estimating exponential scheduling preferences. Transportation Research Part B: Methodological 81, Part 1, 230–251.
- Small, K.A., 1982. The scheduling of consumer activities: Work trips. The American Economic Review 72, 467–479.
- Small, K.A., Chu, X., 2003. Hypercongestion. Journal of Transport Economics and Policy 37, 319–352.
- Tseng, Y.Y., Verhoef, E.T., 2008. Value of time by time of day: A stated-preference study. Transportation Research Part B: Methodological 42, 607 – 618.
- Vickrey, W.S., 1969. Congestion Theory and Transport Investment. American Economic Review 59, 251–60.