Generalised MFDs for Pedestrian Networks

Theory and Applications

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Statement of contribution

- Provide evidence for the existence **generalised Macroscopic Fundamental Diagram** for pedestrian networks for general pedestrian areas or networks, similar to the NFD for cars
- Analytically show **relation between local pedestrian Fundamental Diagram and generalised P-MFD**
- Show impact **spatial density variation** on overall network performance
- Propose several applications of P-MFD

Complexity of Pedestrian Dynamics!

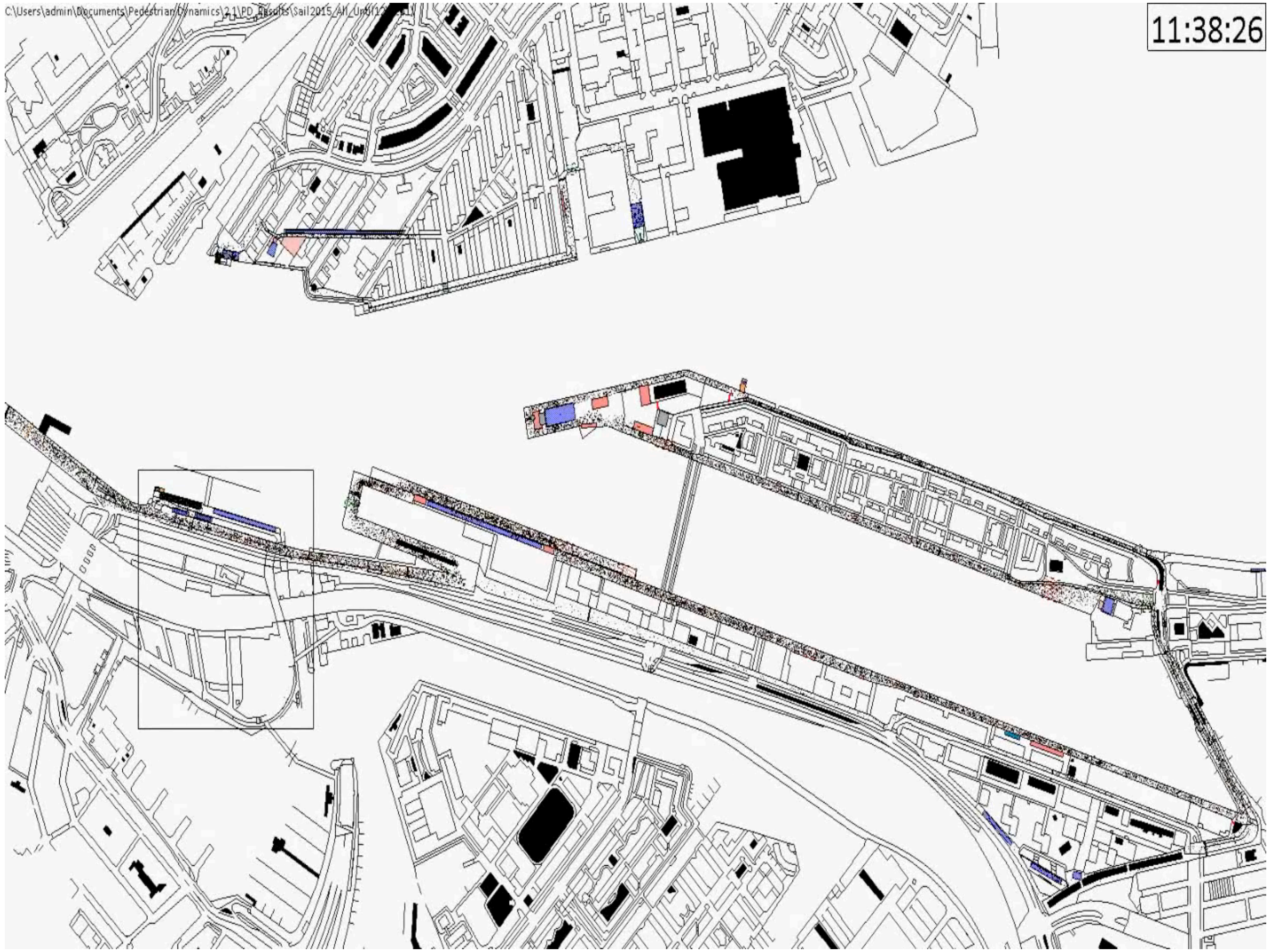
Efficient self-organisation

Faster = slower effect, collapse self-organisation

Blockades and turbulence

Reduced efficiency op network operations

Motivation of work



- On a detailed level, pedestrian flow operations notoriously are hard to model and predict due to inherent complexity (self-organisation and its breakdown, context relevance, …)
- **Proposition:** simple spatially coarser models can be derived that are accurate at network level, like NFD concept for cars
- Gain insights in pedestrian network dynamics, and use these for design and crowd management purposes + applications similar to NFD!

Existence of pedestrian fundamental diagram?

- Pedestrian flow operations are complex and dynamic, and they are (at least) two dimensional
- Does concept of a fundamental diagram make sense?
- Attention points:
  - Definition of flow / speed (in relation to flow direction)
  - Incorporation of composition (e.g. destination, type of pedestrian)
- Quick look at theory and empirics

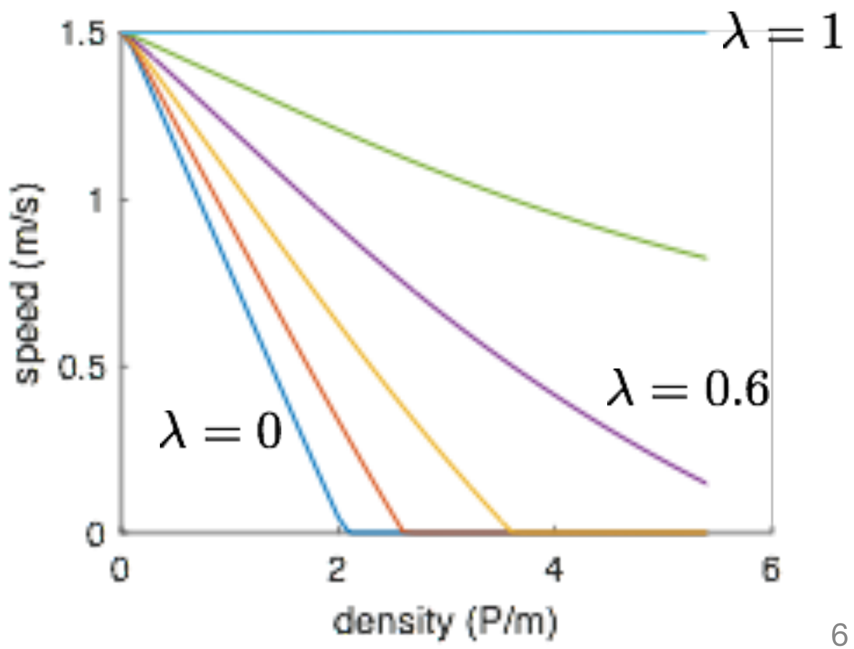
Theoretical derivation of fundamental diagram

- Start from (simple) behavioural assumptions, e.g.:
  - Relation between walking speed and area needed by pedestrian, e.g.:

$$A_p = A^0 + T \cdot v_p \qquad \rightarrow \qquad v(\rho) = \min\left(v^0, \frac{1}{T}\left(\frac{1}{\rho} - \frac{1}{\rho_{jam}}\right)\right)$$

  - Micromodel (e.g. Social-Forces, NOMAD) assumed in equilibrium (zero accel.):

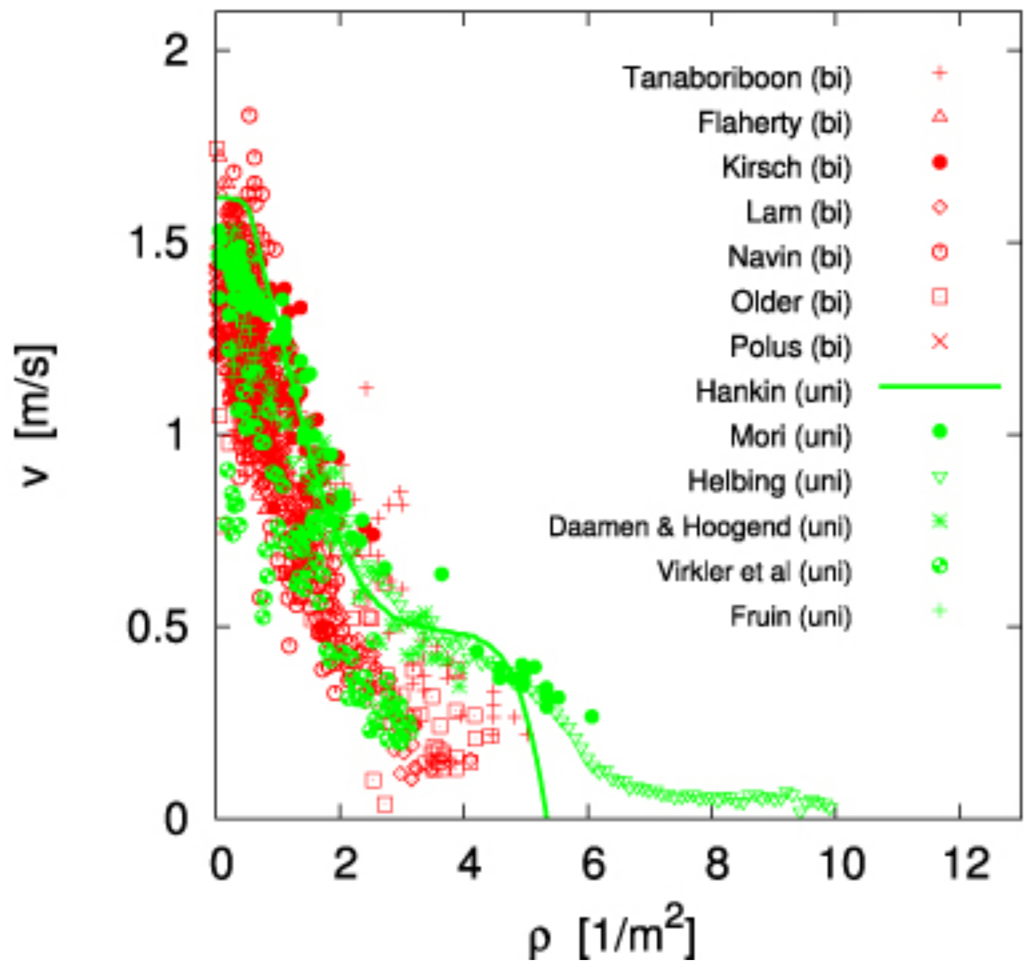
$$v(\rho) = \max\left(0, v^0 - \tau \cdot A \cdot (1-\lambda) \cdot \exp(-\beta\rho)\right)$$



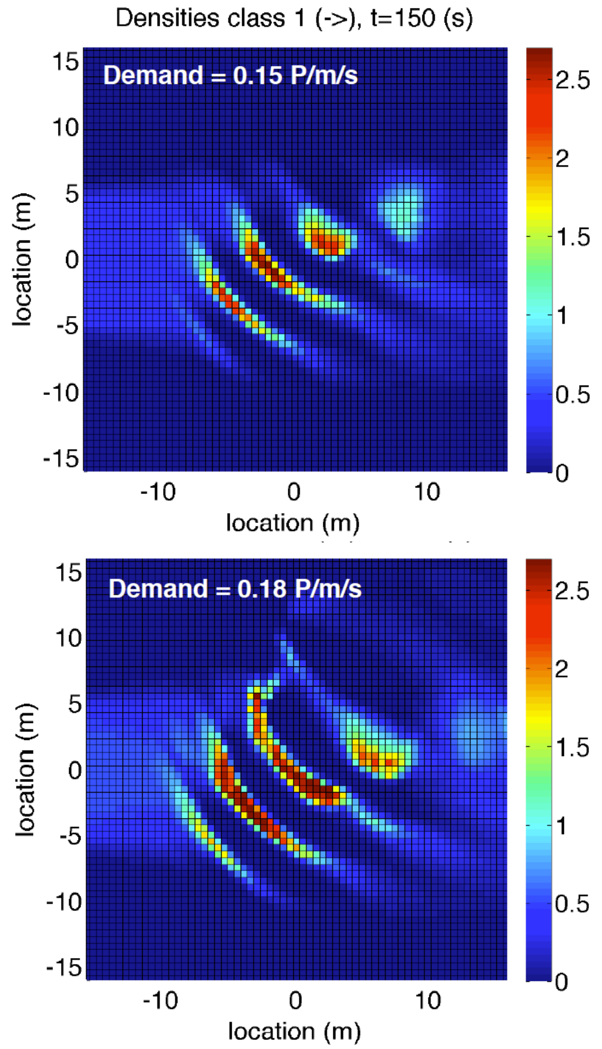
- Based on reasonable behaviour, fundamental diagram pedestrian flow appears useful concept
- What about empirics?

Fundamental diagram for pedestrians

- Accurate pedestrian flow data has only been available since two decades
- Weidmann (1993) was on of the first to formulate an FD based on data
- Since then, different (empirical) studies (often experimental) have been performed (TU Delft, Hermes group, Monash, …)
- Next to FDs, these studies also provided insights into fascinating pedestrian flow phenomena discussed before



Recent refinements of the FD



- Different scholars have looked into multi-class fundamental diagram generalisations (e.g. Hanseler, Nikolic, Hoogendoorn, etc.), showing impact of flow composition
- Example: multi-class (direction) continuum model derived from Social-Forces model yields multi-class interactions (as well as impact of density gradient) [ISTTT, 2015]

$$v_d(\rho_1, \ldots, \rho_D) = ||v_d^0 - \sum_\delta \beta_{\delta\to d} \cdot \nabla\rho_\delta|| - \sum_\delta \alpha_{\delta\to d} \cdot \rho_\delta$$

- **Side remark**: in conjunction with (two-dimensional) conservation of pedestrian equation, model mimics several phenomena observed in pedestrian flow operations

Existence of Pedestrian MFD?

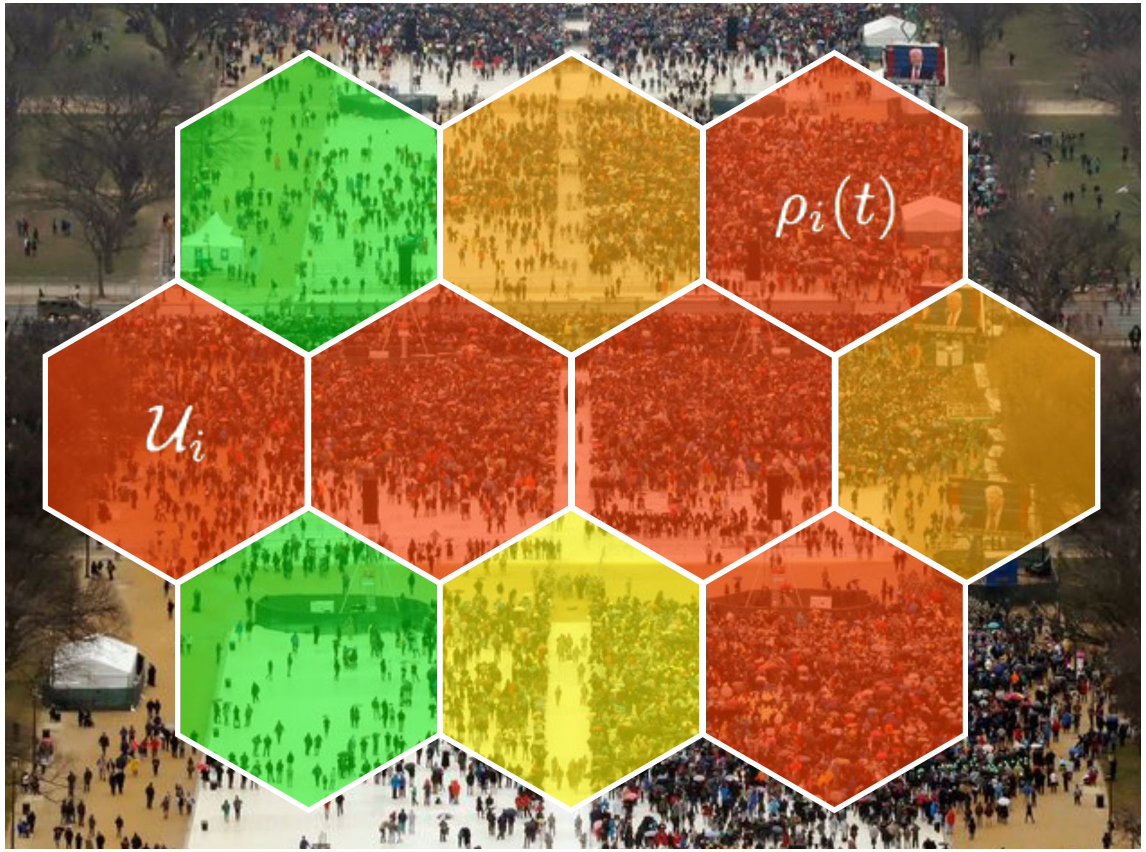
- FD is a meaningful for characterising pedestrian operations…

- Next: does the “pedestrian network flow equivalent” of the NFD exist?
- In remainder, we derive a generalised version of the **P-MFD**, where we express flow operations as function of **average density** over area and **spatial density variation**

$$\bar{q} = \bar{Q}(\bar{\rho}, \sigma)$$

Analytical derivation of P-MFD

Area $\mathcal{U}$



*) Illustration only: we consider walking pedestrians

- Suppose that w… t
  we partition int…
- For each suba…
  are described

$$q_i(t) = Q(\rho_i) = v^0 \rho_i \left(1 - \rho_i/\rho_{jam}\right)$$

- Then we can easily show that for the entire area $\mathcal{U}$ **we have the P-MFD**:

$$\bar{q}(t) = Q(\bar{\rho}(t)) - (v^0/\rho_{jam}) \cdot \sigma^2(t)$$

- where $\sigma^2(t) = \frac{1}{m}\sum_m(\rho_i - \bar{\rho})^2$ denotes the spatial variation in density

In sum: in this case, the P-MFD exist! It is given by the FD with a correction due to the spatial variation!

Analytical derivation of P-MFD

- For Greenshieds FD, we find a linear influence of spatial density variation
- For other FDs, analytical relations can sometimes be determined given that we consider additional assumptions (e.g. on spatial distribution of density)
- Example: use Underwood's FD for the local FD:

$$Q(\rho) = \rho e^{b_0 + b_1 \rho}$$

with $b_0 = \ln v^0$ and $b_1 < 0$

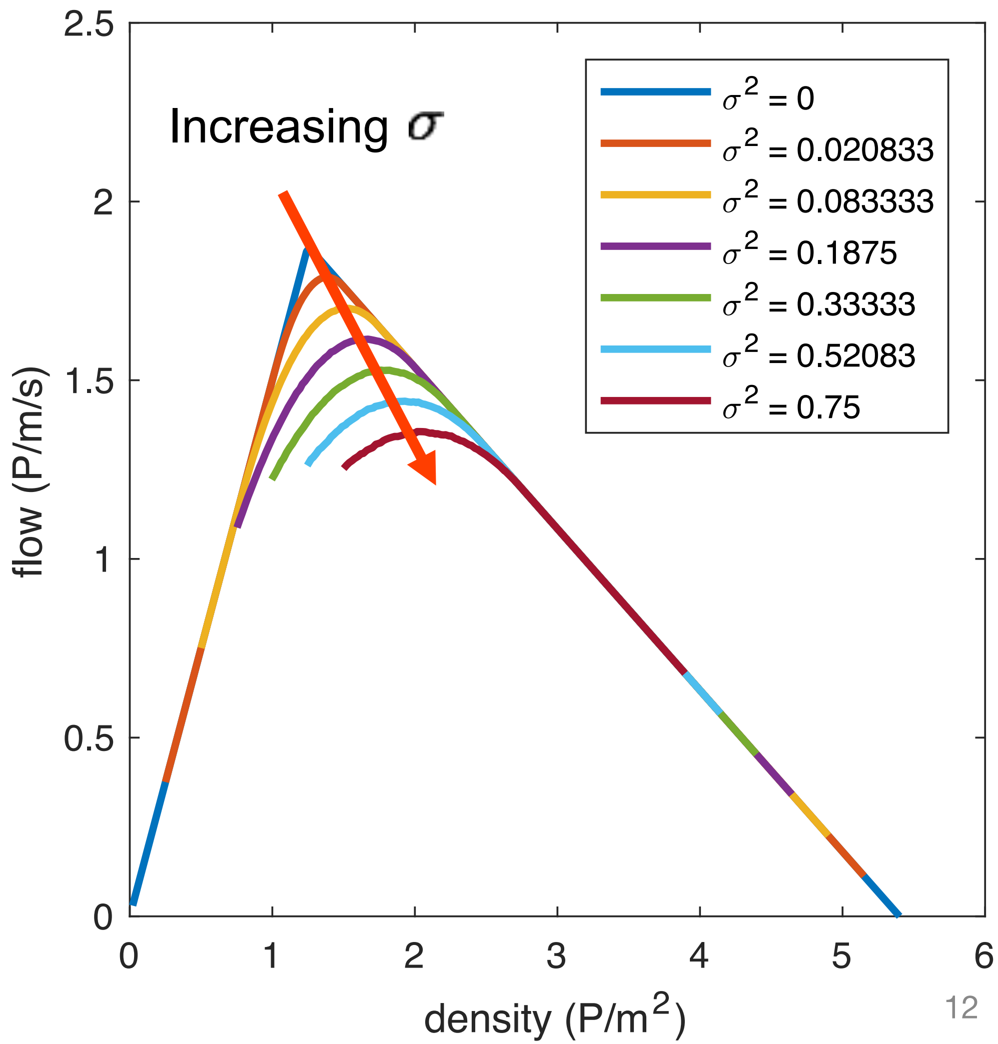
- Assuming a uniform distribution over interval $[\bar{\rho} - \sigma\sqrt{3}, \bar{\rho} + \sigma\sqrt{3}]$ for subareas $\mathcal{U}_i$ allows determining (Taylor series) approximation (for exact result, see paper):

$$\bar{q} = Q(\bar{\rho}) + b_1 \left(\frac{5}{2} U(\bar{\rho}) + \frac{b_1}{2} Q(\bar{\rho}) \right) \sigma^2$$

Monte-Carlo simulation-based approach

Note that the capacity and the critical density changes with changing σ

- s, we need to resort to the P-MFDs
- se means drawing c ubareas $\mathcal{U}_i$, and d ing $q = \frac{1}{m}\sum Q(\rho_i)$, $\bar{\rho}$ and σ
- Example shows Daganzo's FD determined using simulation
- Again we see how the spatial variation changes the FD
- Conclusion? $\bar{Q} = \bar{Q}(\bar{\rho}, \sigma)$ with $\partial \bar{Q} / \partial \sigma < 0$



What about its applicability in more realistic, dynamic circumstances

- If equilibrium conditions cannot be guaranteed, do we still observe a meaningful P-MFD?
- How do features like self-organisation, spontaneous flow breakdown, faster = slower effects, etc., undermine equilibrium assumption underlying theoretical results

NOMAD simulation cases: crossing flow

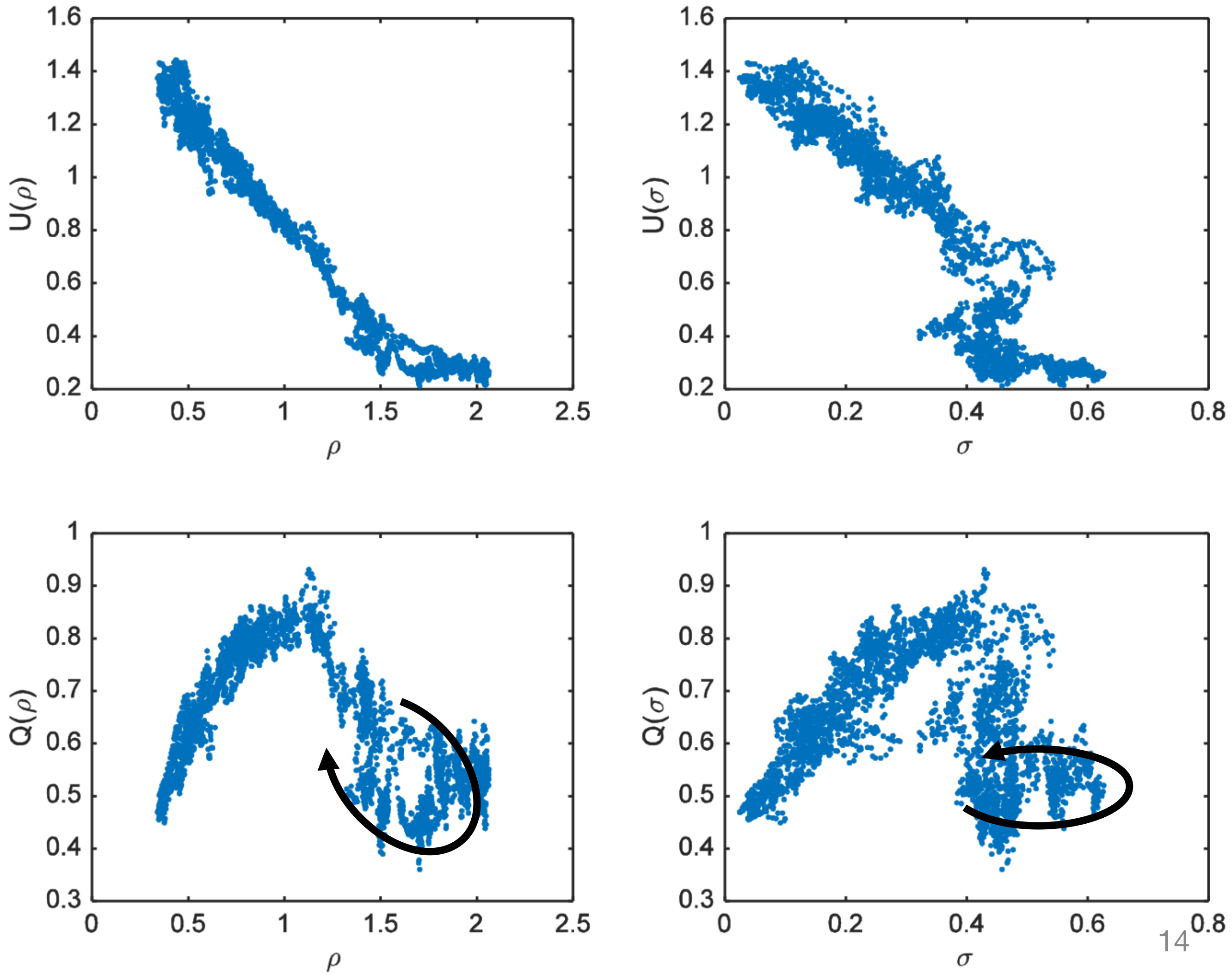
Note that results are in line with theoretical result based on Greenshields FD

- ... etwork s... average c... ariation
- H... ars to o... densities
- Two-regime P-MFD, good fit!
- Free branch ($\bar{\rho} < 1.2$)

$$\bar{Q}(\bar{\rho}, \sigma) = 1.43\bar{\rho} - 0.62\bar{\rho}^2 - 0.23\sigma^2$$

- Congested branch:

$$\bar{Q}(\bar{\rho}, \sigma) = 0.87 - 0.14\bar{\rho} - 0.19\sigma^2$$

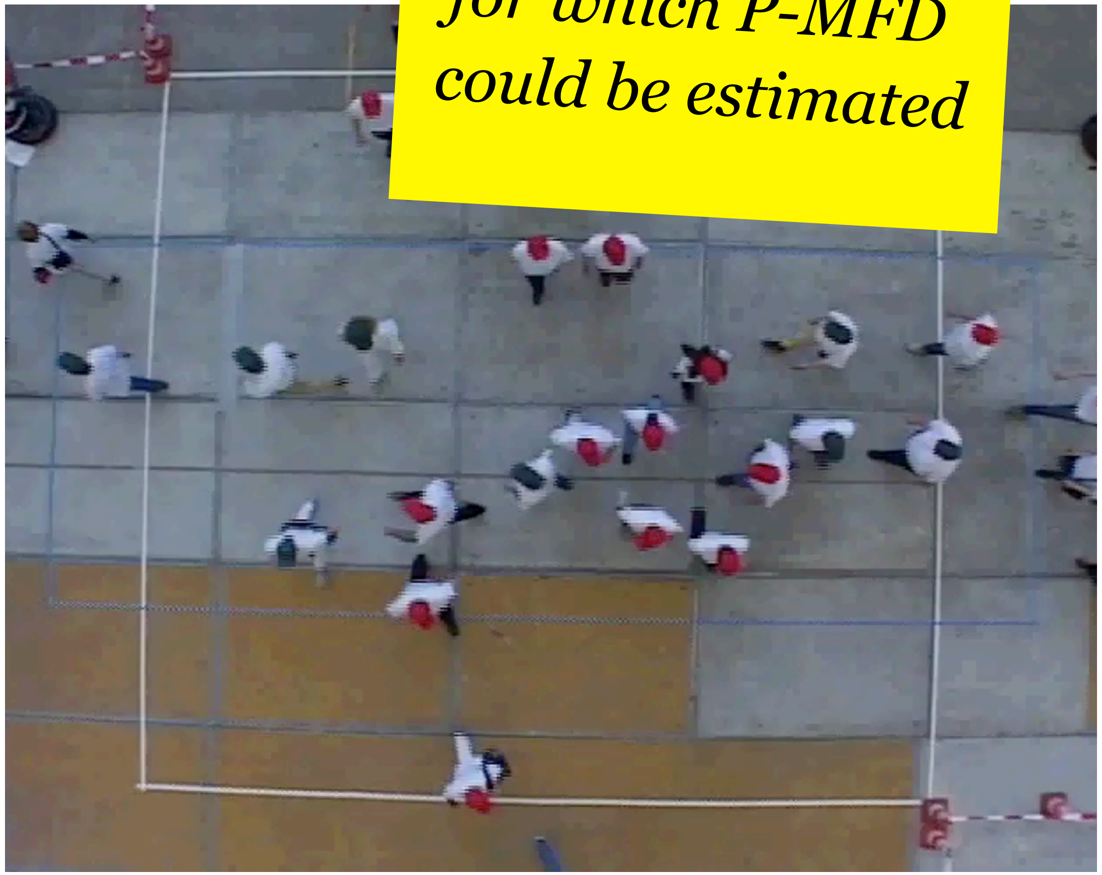


Crossing flow experiment

- Data from the 2002 experiments
- Crossing flow showing self-organisation of diagonal stripes
- No congestion, only free-flow branch of P-MFD could be established:

$$Q(\bar{\rho}, \sigma) = 1.63\bar{\rho} - 0.41\bar{\rho}^2 - 0.28\sigma^2$$

- Result very similar to result from simulation experiment



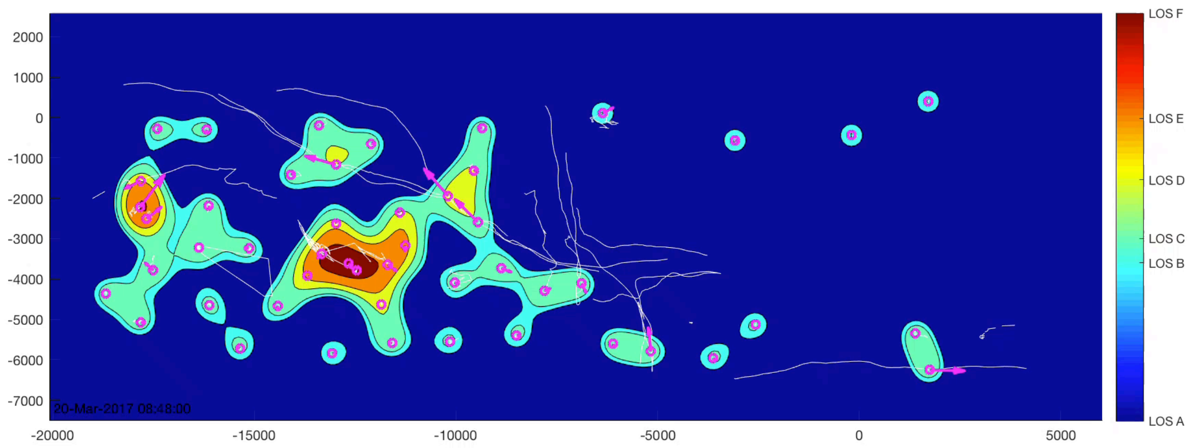
Paper discusses many other cases for which P-MFD could be estimated

- Application to crowd management (sweet-spot control) to keep flow operations in area efficient, e.g. using gating:

$$q_{max}(t) = q_{max}(t-1) - K \cdot (\bar{\rho}(t) - \bar{\rho}_{crit}) \quad \text{with} \quad \bar{\rho}_{crit} = \bar{\rho}_{crit}(\sigma^2)$$

The sweetspot (critical area density) is function of σ

- Example application: monitoring average density and spatial density variation at platform and limit inflow (turnstiles, escalator)
- *Utrecht platform example (real data)*



Applications of P-MFD

allegro

- Coarse modelling of network flow operations, where dynamics of (sub-)area are described via P-MFD:

$$\frac{dn_i}{dt} = \sum_j f_{ji}(t) - F_i(n_i(t), \sigma_i(t))$$

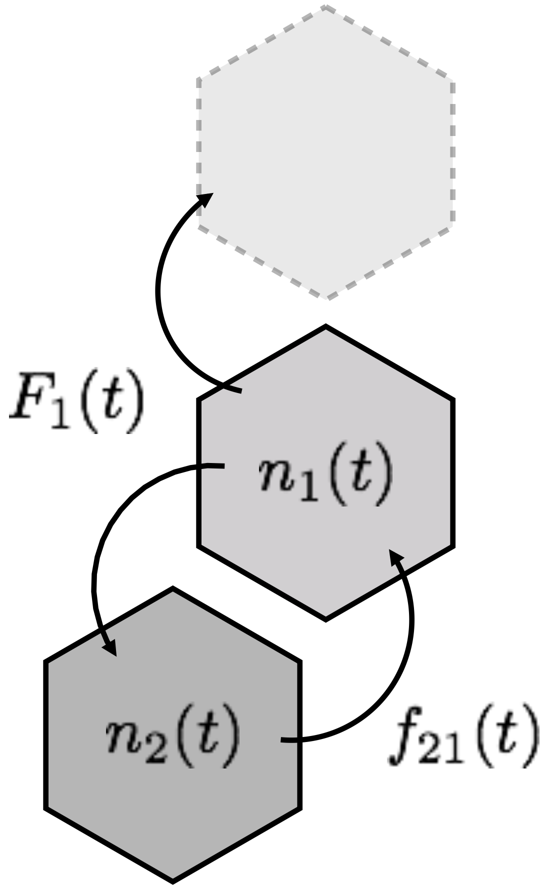
where

$$f_{ji}(t) = \phi_{ji}(t) \cdot F_j(n_j(t), \sigma_j(t))$$

- For crossing-flow simulation we found (approximate!) relation by looking at the hysteresis loop data (very preliminary!!!!)

$$\sigma(\bar{\rho}, \dot{\bar{\rho}}) = 0.277 \cdot \bar{\rho} - 0.039 \cdot \dot{\bar{\rho}}$$

- Relation shows spatial variation relates to density, and density changes (lower spatial variation during build up congestion)



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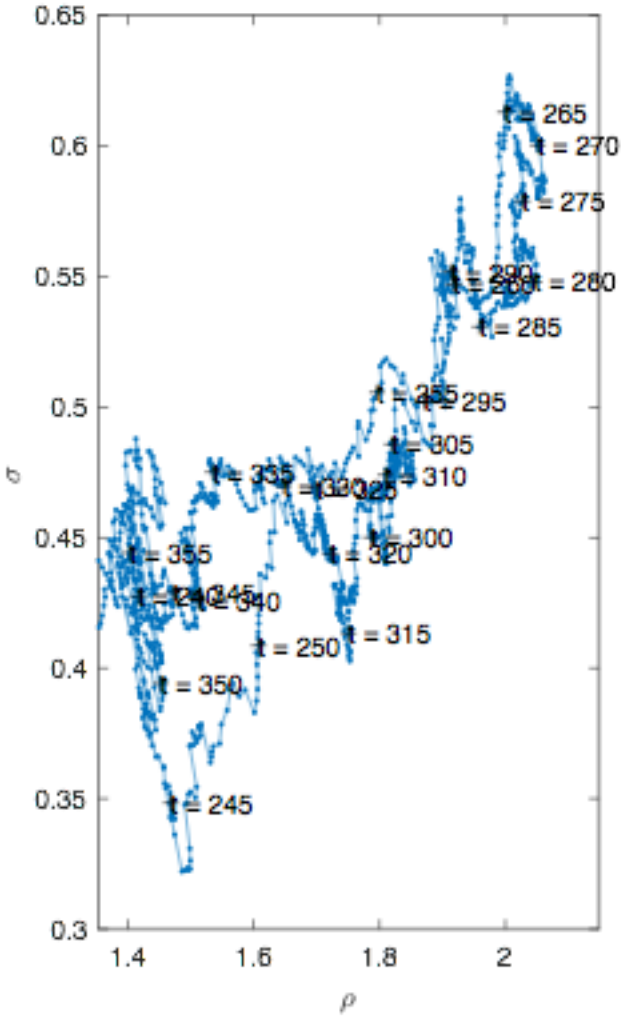
$$\sigma(\bar{\rho}, \dot{\bar{\rho}}) = 0.277 \cdot \bar{\rho} - 0.039 \cdot \dot{\bar{\rho}}$$

- Relation shows spatial variation relates to density, and density changes (slightly higher spatial variation during congestion)



Statement of contribution + Q&A

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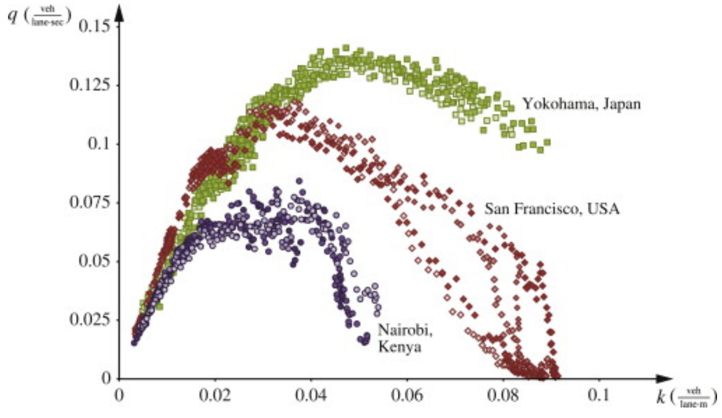

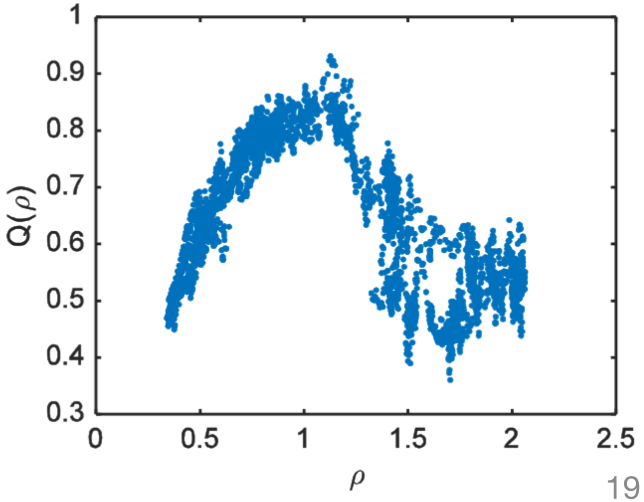

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June 2018, exact dates soon to follow!

Co-hosts: Victor Knoop, Dorine Duives, Paola Goatin, Ricardo Colombo, Alexandre Bayen