# Stochastic dynamic switching in fixed and flexible transit services as market entry-exit real options

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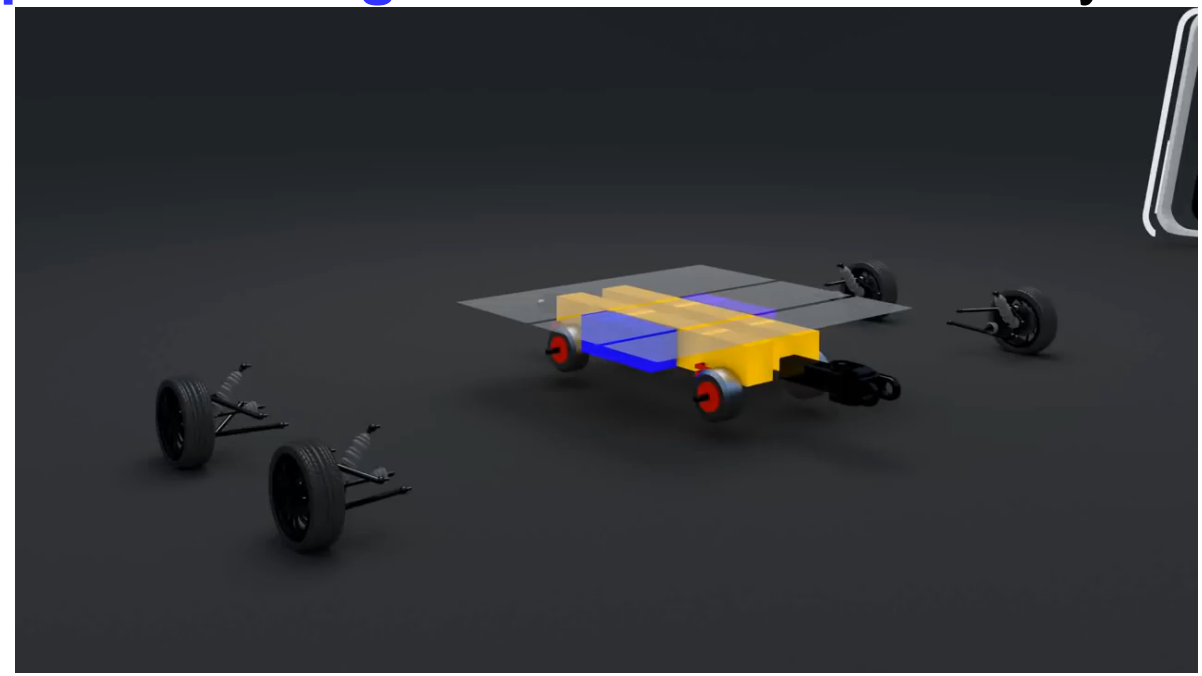
# Outline

- Background and literature review
- Problem definition
  - Problem illustration
  - Formal definition
- Proposed model
  - Dynamic switching between fixed and flexible transit
  - Model variation: modular vehicle sizes
- Model properties
- Computational evaluations

# Background

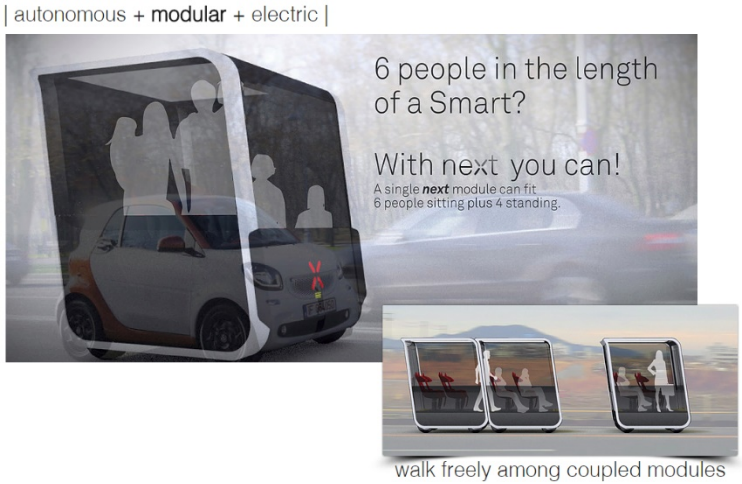
- **Data-driven** environment
- Demand **uncertainty**
- **Optimal timing and control** of transit systems



Shared autonomous fleets with modular vehicle size (source: www.next-future-mobility.com).

# Background and literature review

- Fixed-route and flexible-route systems
  Daganzo, 1984; Chang and Schonfeld, 1991a; Quadrifoglio and Li, 2009; Qiu et al, 2015



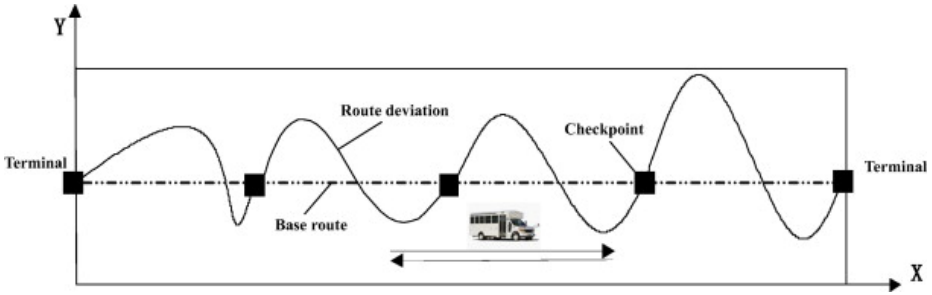

- Integrating fixed-route and flexible-route transit
  Chang and Schonfeld, 1991b; Kim and Schonfeld, 2013, 2014

- Artificial intelligence and autonomous vehicle technologies
  Dubai (Spera, 2016), Ackerman, 2016

# Background and literature review

## Static policies - Information and Communications Technologies (ICTs)-Dynamic flexible transit services

Djavadian and Chow, 2016




Qiu et al. 2014

## Some dynamic dual-mode transit fleet operating strategies

- Fixed route vs flexible service e.g. Kim and Schonfeld, 2013
- Vehicle size e.g. Fu and Ishkanov, 2004
- Headway control e.g. Thomas, 2007
- Idle vehicle relocation. e.g. Yuan et al., 2011
- Ridesharing options M to 1, M to Few, M to M.

# Contributions

The static fixed/flexible service policy

⬇

Stochastic & dynamic policy

- Modeling dynamic switching of transit service as **a market entry-exit real options model**;
- Presenting a variation of this model to address "**vehicle modularity**";

# Market entry-exit real options

Sødal et al. Review of Financial Economics, 2008

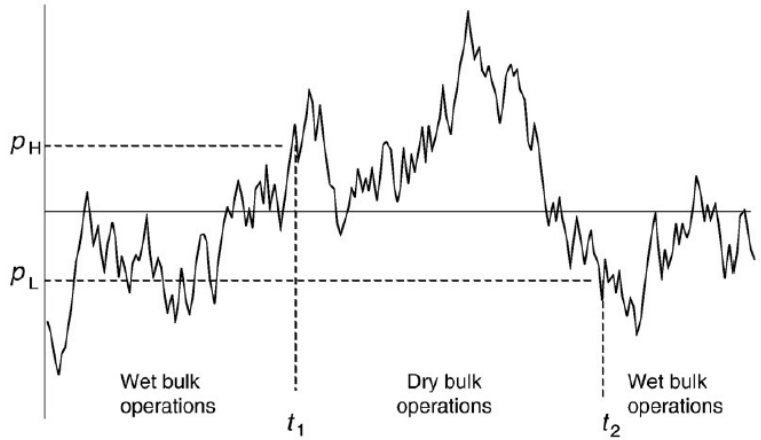
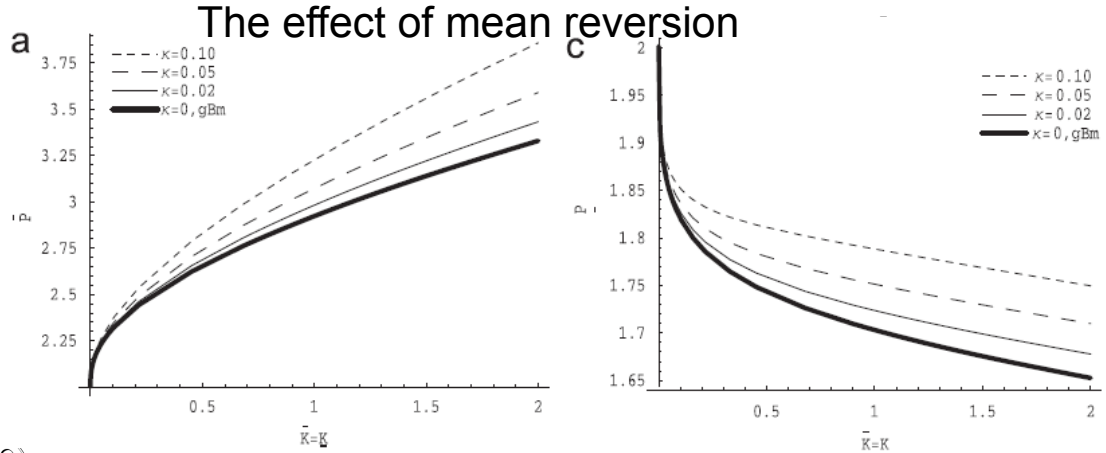


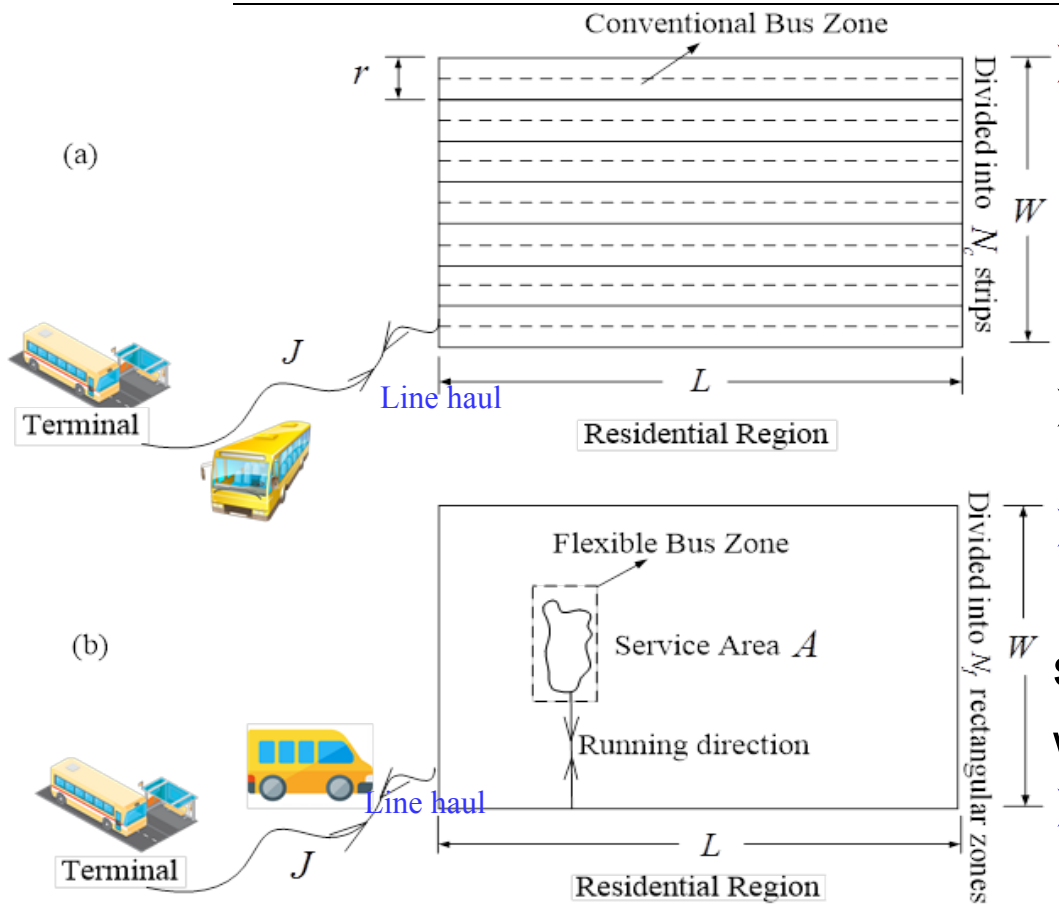

Fig. 2. Market switching.

Tsekrekos, Journal of Economic Dynamic & Control, 2010

The effect of mean reversion



# Problem illustration




Kim & Schonfeld (2014)

- An urban system: a single rectangular region of length $L$ and width $W$, and connected to a hub via line haul of length $J$
- Demand density is assumed to be uniformly distributed.
- The fixed-route conventional mode

subdivided into $N\downarrow c$ routes of width $r$ and length $L$

- The flexible service mode

subdivided into a grid of $N\downarrow f$ zones of area A.

- Dynamic choosing which mode in a specific time period based on real time data

# Fixed transit operating cost

- $C_{sc}$ = the sum of the bus operating cost $C_o$, user in-vehicle cost $C_v$, user waiting cost $C_w$, and user access cost $C_x$.

$$C_{sc}^*(Q)=3LWQ(v_w v_x aD_c/8V_x V_c LQ)^{1/3}+bLWQD_c/V_c+v_x LWQd/4V_x+v_i LWQM/V_c$$

If $S_c$ is fixed

$$C_{sc}^*(Q;S_c)=LWQD_c(a+bS_c)/V_c S_c+v_i LWMQ/V_c+W\sqrt{v_w S_c v_x LQ/2V_x}+v_x LWQ/4V_x(\sqrt{2V_x v_w S_c/v_x LQ}+d)$$

# Flexible transit service cost

- $C_{sf}$ = the sum of the bus operating cost $C_o$, user in-vehicle cost $C_v$, user waiting cost $C_w$, and user access cost $C_x$

$$C_{sf}^*(Q)=LWQ\left[\left(v_w a^2 k^2 D_f^2 (b+v_i/2)^2/V_f^2 uQ\right)^{1/5}+1.5\left(v_w a^2 k^2/V_f^4\right)^{1/5}\left(Y^2/uQ\right)^{1/3}+D_f(b+v_i/2)/V_f\right]$$

If $S_f$ is fixed

$$C_{sf}^*(Q;S_f)=LWD_f Q(a+bS_f)/V_f S_f+v_i LWD_f Q/2V_f+v_w S_f LW/2\left(v_w S_f V_f/Qk(a+bS_f)\sqrt{uS_f}+v_i/2\sqrt{S_f/u}\right)^{-2/3}+LWQ/V_f\left(k/\sqrt{uS_f}(a+bS_f)+v_i/2\sqrt{S_f/u}\right)\left(v_w S_f V_f/Q(a+bS_f)k\sqrt{uS_f}+v_i/2\sqrt{S_f/u}\right)^{1/3}$$

# Incremental cost savings function

- The immediate cost savings accrued from time $t$ to $t+dt$ when operating in one mode relative to the other

$$\Phi(Q(t)) = C_{sf}^{*}(Q(t); S_{f}) - C_{sc}^{*}(Q(t); S_{c})$$

Flexible Fixed

Demand uncertainty Stochastic process

# Demand density

- Demand density $Q(t)$ -**a mean-reverting Ornstein-Uhlenbeck** (O-U) process (Sarkar, 2003)

$$dQ=\mu(m-Q)dt+\sigma Q dw$$

$\mu$ is a mean reversion coefficient

$m$ is the long-term demand density

$\sigma$ is the process volatility

$dw \sim N(0,dt)$ is an increment in the Wiener process.

- **An optimal policy =** total realized operating cost + switching cost over an entire daily cycle is expected to be minimized.

Switching occurs instantaneously.

$$V(Q(t))=\max_{a(t)}\left(\Phi(a(t))+\rho EV(Q(t\right.$$

# Policy valuation

- Asset equilibrium pricing Dixit, 1989
- The option value of using flexible bus operating mode $V_0(Q)$

$$1/2\,\sigma^2 Q^2 V_0''(Q)+\mu(m-Q)V_0'(Q)-\rho V_0(Q)=0$$

- The option value of using conventional bus operating mode $V_1(Q)$

$$1/2\,\sigma^2 Q^2 V_1''(Q)+\mu(m-Q)V_1'(Q)-\rho V_1(Q)+\Phi(Q)=0$$

# Asset equilibrium conditions

- No single threshold (Due to switching cost)

When $F\uparrow+ = F\uparrow- = 0$, $Q\downarrow H = Q\downarrow L$

$Q\downarrow L$ -from fixed to flexible

$Q\downarrow H$ -from flexible to fixed

- Value matching

$$V\downarrow 0\ (Q\downarrow H) = V\downarrow 1\ (Q\downarrow H) - F\uparrow+$$

$$V\downarrow 1\ (Q\downarrow L) = V\downarrow 0\ (Q\downarrow L) - F\uparrow-$$

- Smooth pasting

$$V\downarrow 0\uparrow'\ (Q\downarrow H) = V\downarrow 1\uparrow'\ (Q\downarrow H)$$

$$V\downarrow 0\uparrow'\ (Q\downarrow L) = V\downarrow 1\uparrow'\ (Q\downarrow L)$$

$F\uparrow+$ is the cost assumed for switching from flexible bus service to conventional bus service

# Asset equilibrium conditions

- General solution of $V_0(Q)$ and $V_1(Q)$

$$V_0(Q)=[A_0 H(-\gamma_0, w_0, x)+B_0 (2\mu m/\sigma^2 Q)^{1-w_0} H(1-\gamma_0-w_0, 2-w_0, x)]Q^{\gamma_0}$$

$$V_1(Q)=[A_1 H(-\gamma_1, w_1, x)+B_1 (2\mu m/\sigma^2 Q)^{1-w_1} H(1-\gamma_1-w_1, 2-w_1, x)]Q^{\gamma_1} + E_t [\int_t^\infty \Phi(Q(s))e^{-\rho(s-t)} ds]$$

$H(\cdot)$ is the confluent hypergeometric function or Kummer function

$$H(\gamma,w,x)=1+\gamma/w\, x+\gamma(\gamma+1)x^2/w(w+1)2! +\gamma(\gamma+1)(\gamma+2)x^3/w(w+1)(w+2)3! +\ldots$$

The function has two asymptotic expressions.

# Asset equilibrium conditions

$F(X)=[Q_H - A_0 H_0 (Q_H)^{\gamma_0} + (\Delta_1 A_0 - A_1) H_1 (Q_H)^{\gamma_1} - E_t \int_t^\infty \Phi(Q(s) \mid Q(t)=Q_H) e^{-\rho(s-t)} ds + F + \partial A_0 M_0 (Q_H)^{\gamma_0} + (\Delta_1 A_0 - A_1) M_1 (Q_H)^{\gamma_1} + \partial E_t \int_t^\infty \Phi(Q(s) \mid Q(t)=Q_H) e^{-\rho(s-t)} ds / \partial Q \; \partial A_0 M_0 (Q_L)^{\gamma_0} + (\Delta_1 A_0 - A_1) M_1 (Q_L)$

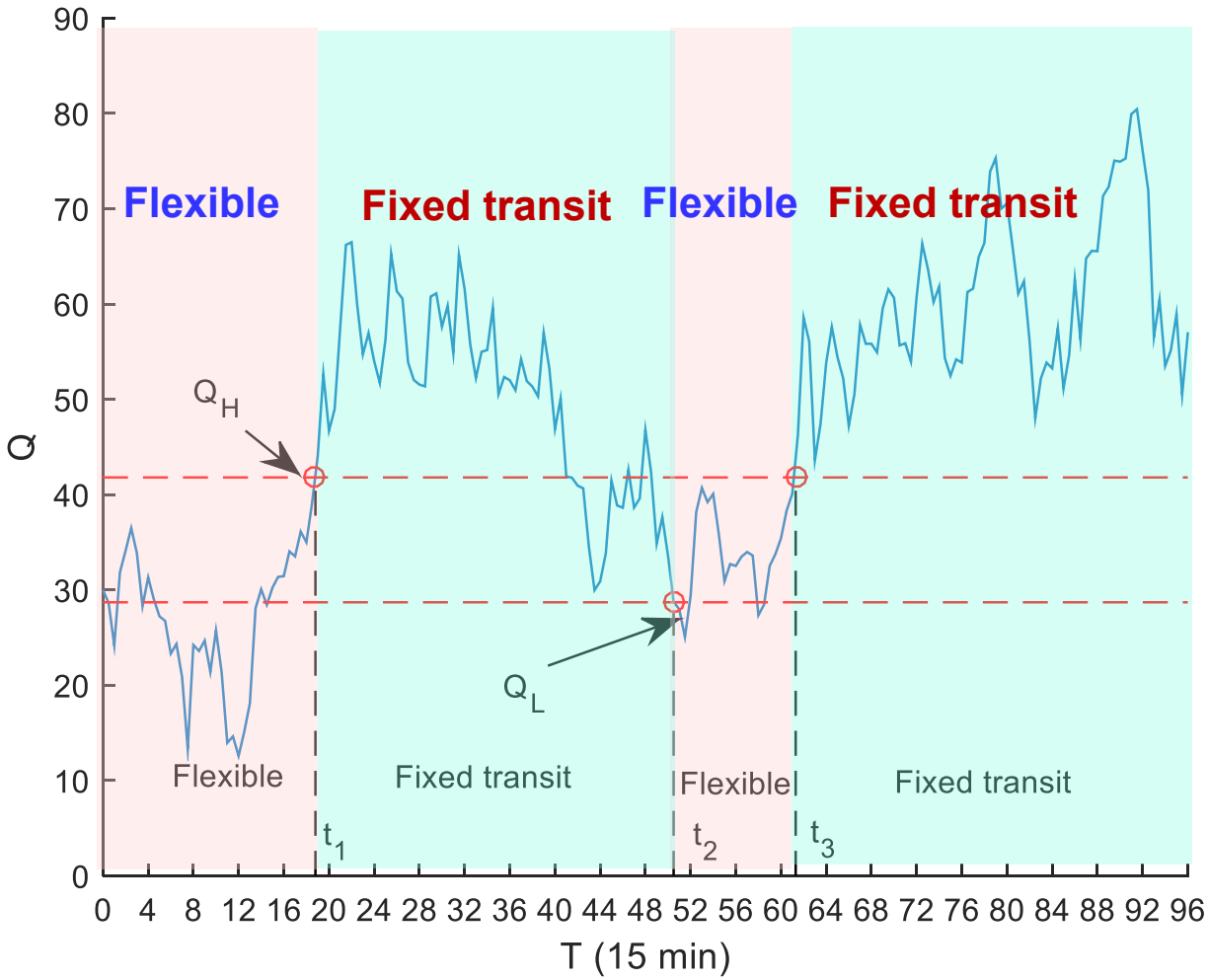
$Q_H$ is uniquely determined by solving

# Model properties

- Sensitivity of switching policy to transportation system parameters

| Solutions | | $Q(0)$=32 trips/mile$^2$/hr |
|---|---|---|
| Fixed Transit | Headway, $h_c$ | 0.42 |
| | Vehicle size, $S_c$ | 75 |
| | Fleet size, $F_c$ | 5 |
| | Route spacing, $r$ | 1.41 |
| | Total cost, $C_{sc}$ | 2881.1 |
| Flexible Transit | Headway, $h_f$ | 0.08 |
| | Vehicle size, $S_f$ | 7 |
| | Fleet size, $F_f$ | 58 |
| | Service zone, $A$ | 3.02 |
| | Total cost, $C_{sf}$ | 2883.0 |
| $\Phi(Q)$ | | 1.9 |
| $E_t\left[\int_t^\infty \Phi(Q)e^{-\rho(s-t)}\,ds\right]$ | | -39.4 |
| $V_0\,(Q(0))$ | | 23.5 |
| $V_1\,(Q(0))$ | | 17.4 |
| $Q_L$ | | 28.7 |
| $Q_H$ | | 41.8 |

# Model properties



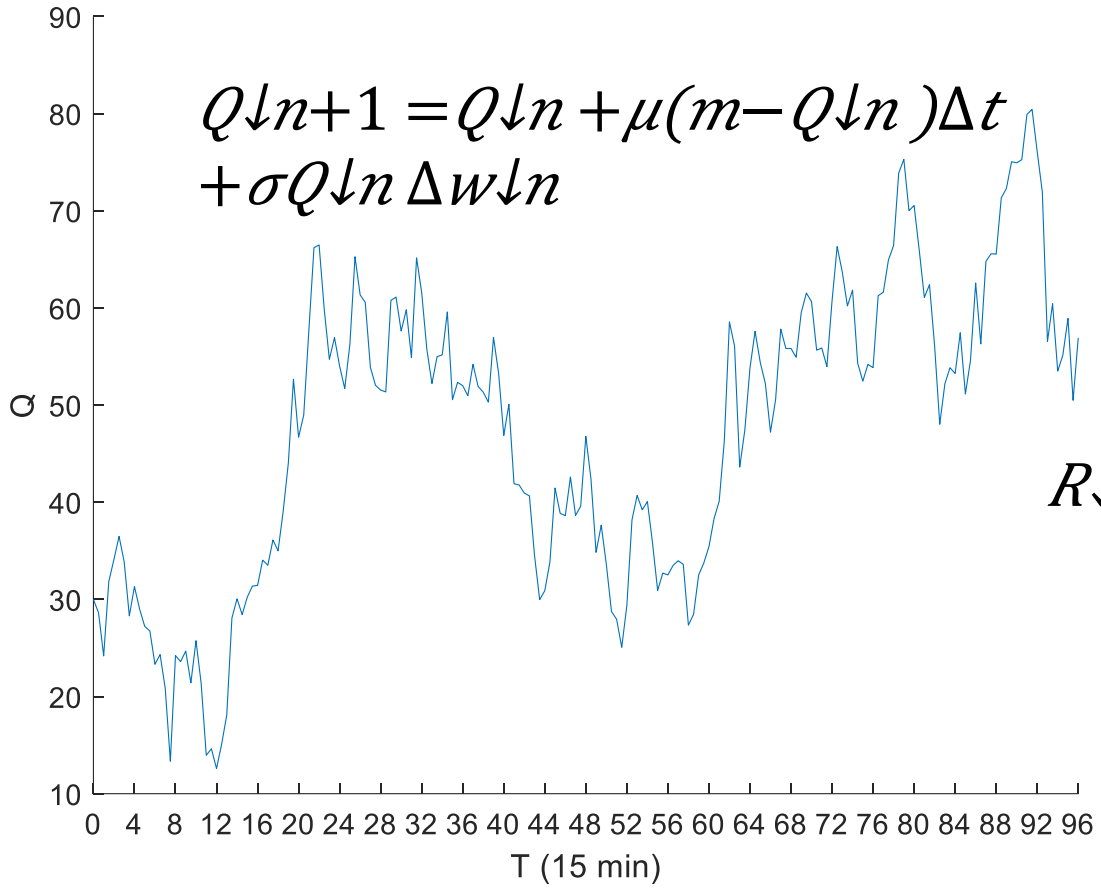
# Verification of policy

## Experimental design

For the same simulated demand density trajectory, each policy's outcome decisions are made as follows in three scenarios:

- Perfect information scenario: determine the optimal switching points deterministically as if the operator knew the demand outcome beforehand;
- Myopic policy scenario: determine the optimal switching points whenever the incremental cost threshold (Φ=0) is crossed；
- Proposed policy scenario based on market entry-exit switching option: switch to fixed transit whenever $Q(t)>Q\downarrow H$ or switch to flexible transit whenever $Q(t)<Q\downarrow L$ .

# Experimental design




- The performance of the proposed policy

$$\varpi(\pi) = R_{my} - \pi / R_{m}$$

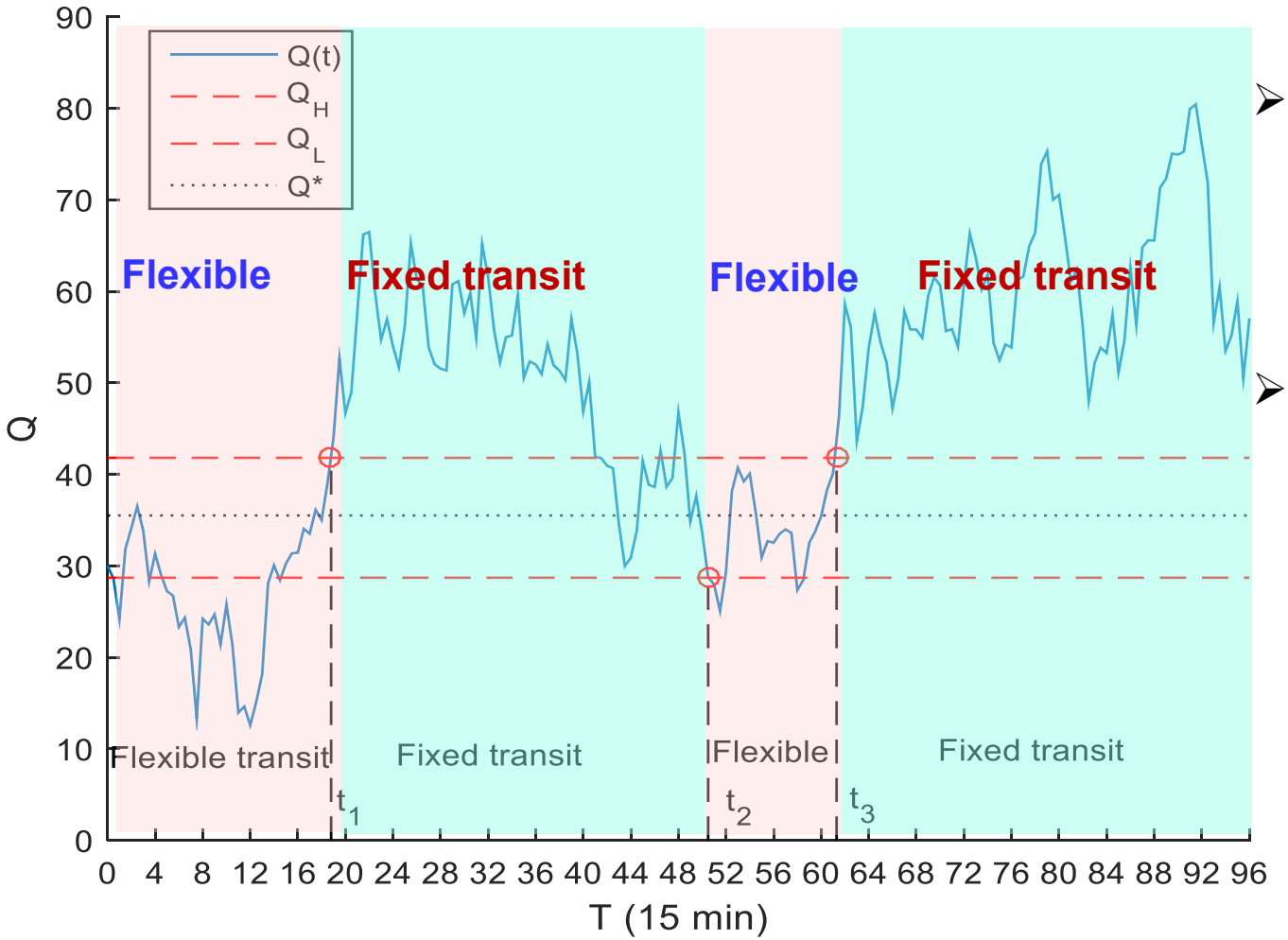
$R_{ph}$ perfect information s

$R_{my}$ myopic *scenario*

Fixed vehicle sizes $S_f = 8,\ S_c = 80$ seats/vehicle

Average demand density of 40 trips/mile$^2$/hr

# Experimental Results

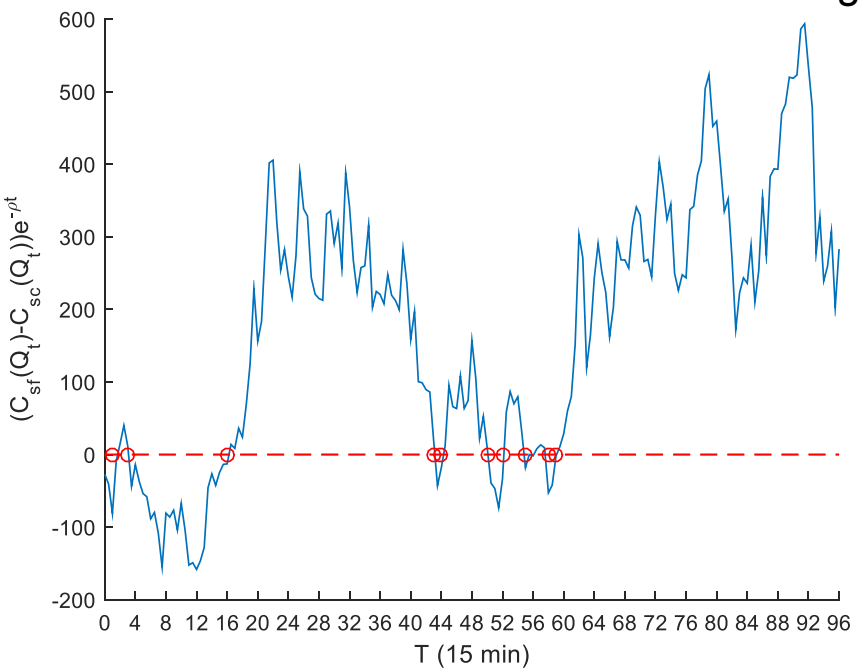



- When switching cost is 0, there is just one threshold $Q\uparrow* = 35.6$.
- When there is a switching cost $F\uparrow+ = F\uparrow- = 10$, then the optimal thresholds are $Q\downarrow L = 28.7$ $Q\downarrow H = 41.8$.

**Proposed policy**

# Experimental Results

the switching costs >the cumulative discounted cost saving



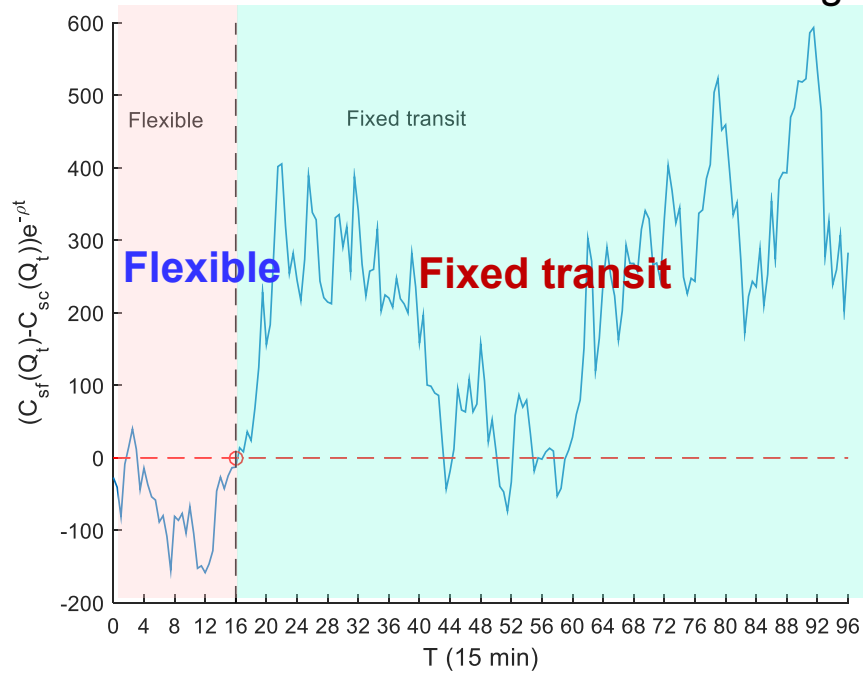




Fig. 8. (a) Myopic switching policy, and (b) perfect information switching policy.

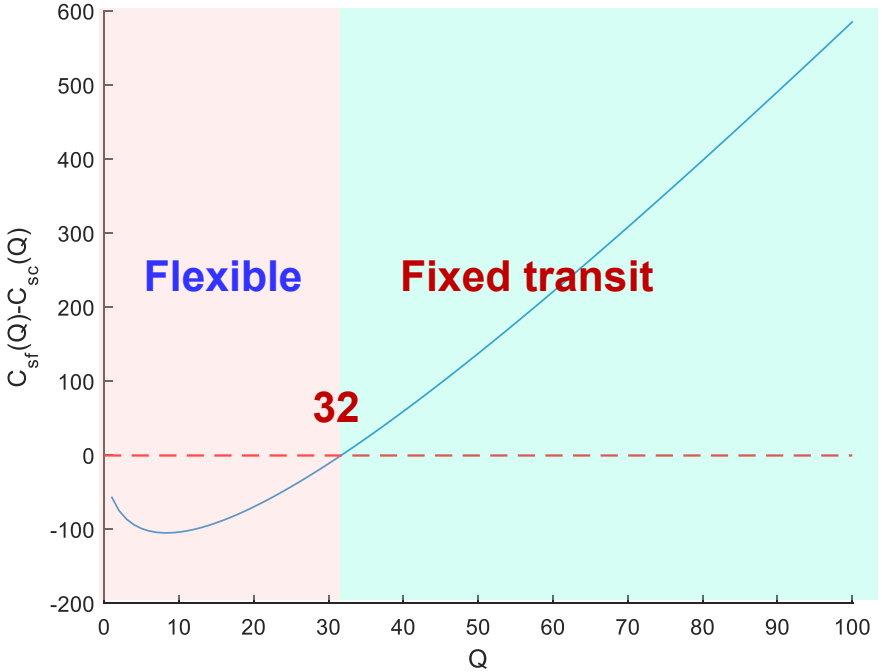
| | Perfect hindsight | Proposed policy | Myopic policy |
|---|---|---|---|
| **Total discounted cost** | 46093.77 | 46150.62 | 46293.73 |
| $\varpi(\pi)$ | 1.0000 | 0.7157 | 0.0000 |

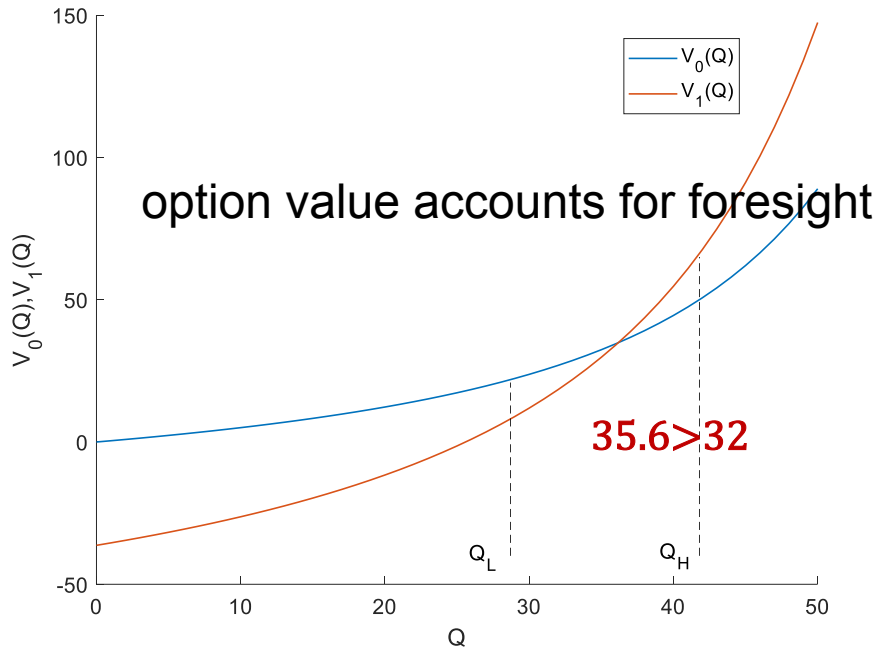
$$\varpi(\pi)=R\downarrow my^{\pi}-R\downarrow my^{\pi}/R\downarrow pn$$

The proposed policy can reduce the excess cost by 72% relative to myopic policy,

# Model properties

- Switching policy sensitivity to demand density




**28.7 41.8**

Fig. 4. (a) Incremental operational cost from fixed to flexible transit
(b) option value of conventional and flexible bus service with respect to demand density.

# Model properties

- Switching policy value sensitivity to stochastic process parameters **Hysteresis effect**

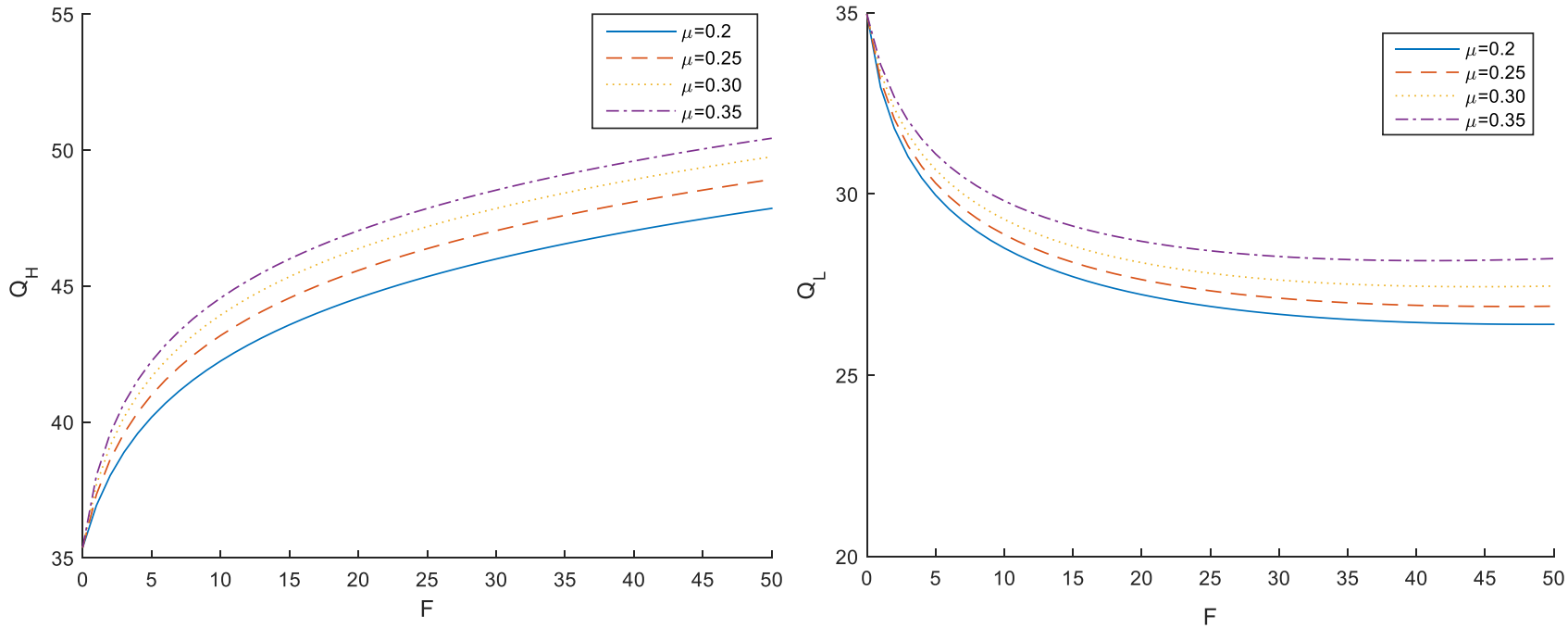



Fig. 5. Relation between (a) upper and (b) lower demand density threshold and switching cost for different reversion speed.

# Model properties

- Switching policy value sensitivity to stochastic process parameters

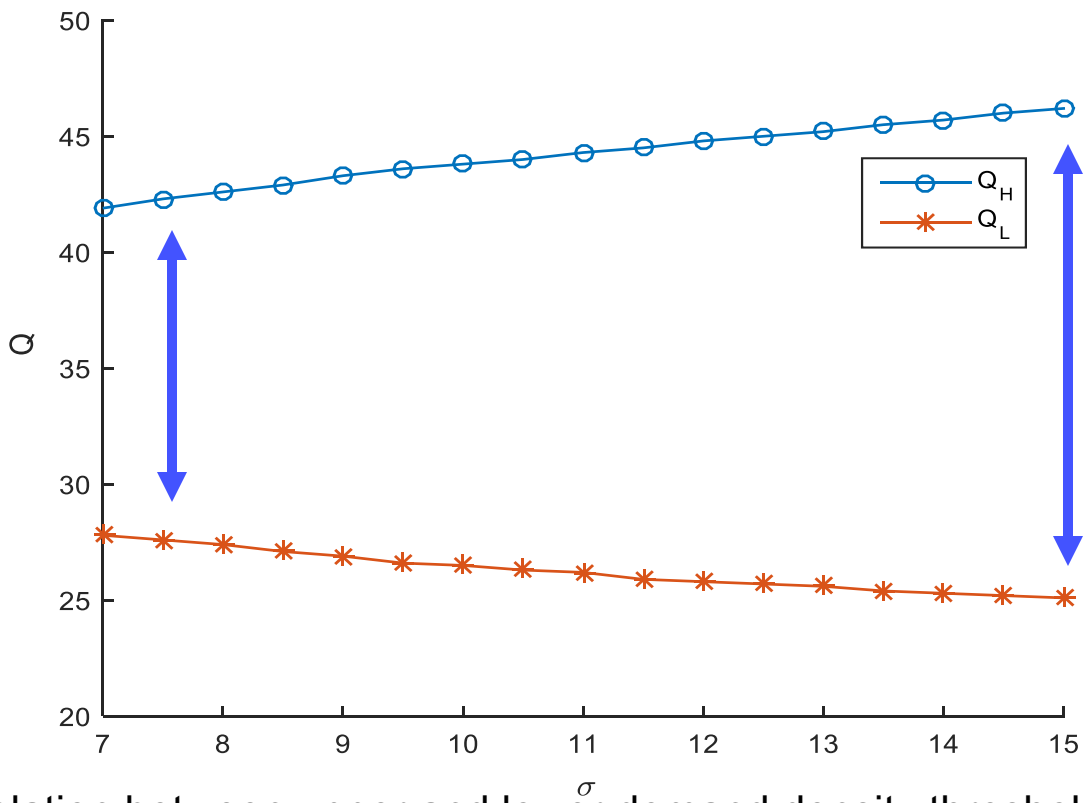



Fig. 6. Relation between upper and lower demand density threshold and volatility

# Model properties

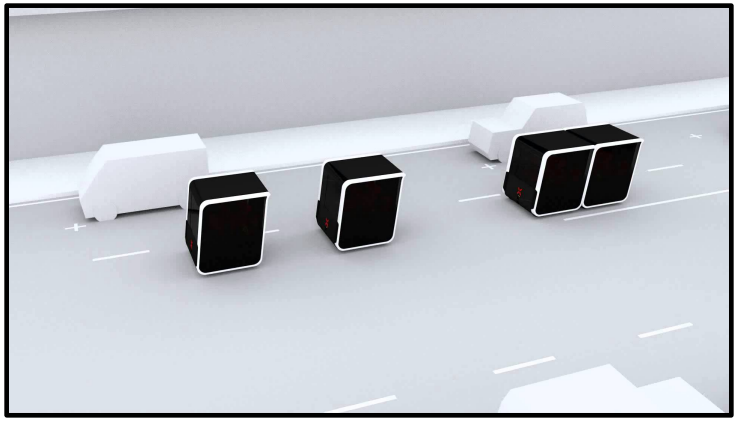
- Switching policy value sensitivity to stochastic process parameters

Table 5. Relation between discount rate and trigger demand density thresholds.

| Discount rate $\rho$ (%) | Single trigger demand density $Q\uparrow*$ | Upper demand density $Q\downarrow H$ (trips/mile2/hr) | Lower demand density $Q\downarrow L$ (trips/mile2/hr) |
|---|---|---|---|
| **5** | 35.2 | 41.0 (+5.8) | 28.3 (-6.9) |
| **7** | 35.6 | 41.8 (+6.2) | 28.7 (-6.9) |
| **9** | 36.1 | 42.6 (+6.5) | 29.1 (-7.0) |
| **11** | 36.7 | 43.4 (+6.7) | 29.5 (-7.2) |

not symmetric

# Model variation: modular vehicle size



This general structure can manage autonomous fleets such as the **Next Future Mobility system**

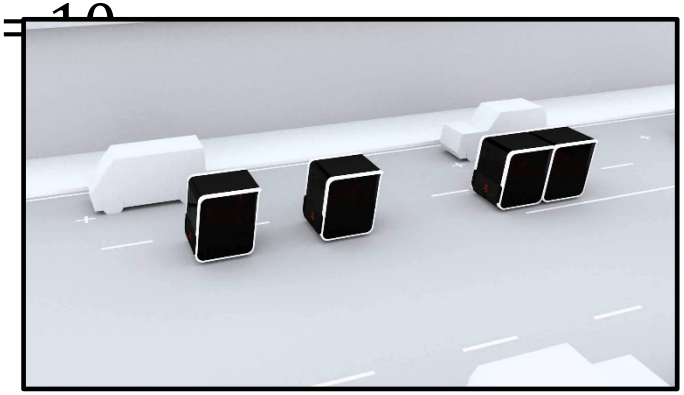
Shared autonomous fleets with modular vehicle size

- Two different vehicle sizes $S_1$ and $S_2$, $S_2 = 2S_1$
- $C_{sf1}(Q; S_1)$ = the cost of operating flexible service with vehicle size $S_1$
- $C_{sf2}(Q; S_2)$ for operating flexible service with vehicle size $S_2 = 2S_1$.
- The incremental cost savings function variation $\Phi / v$

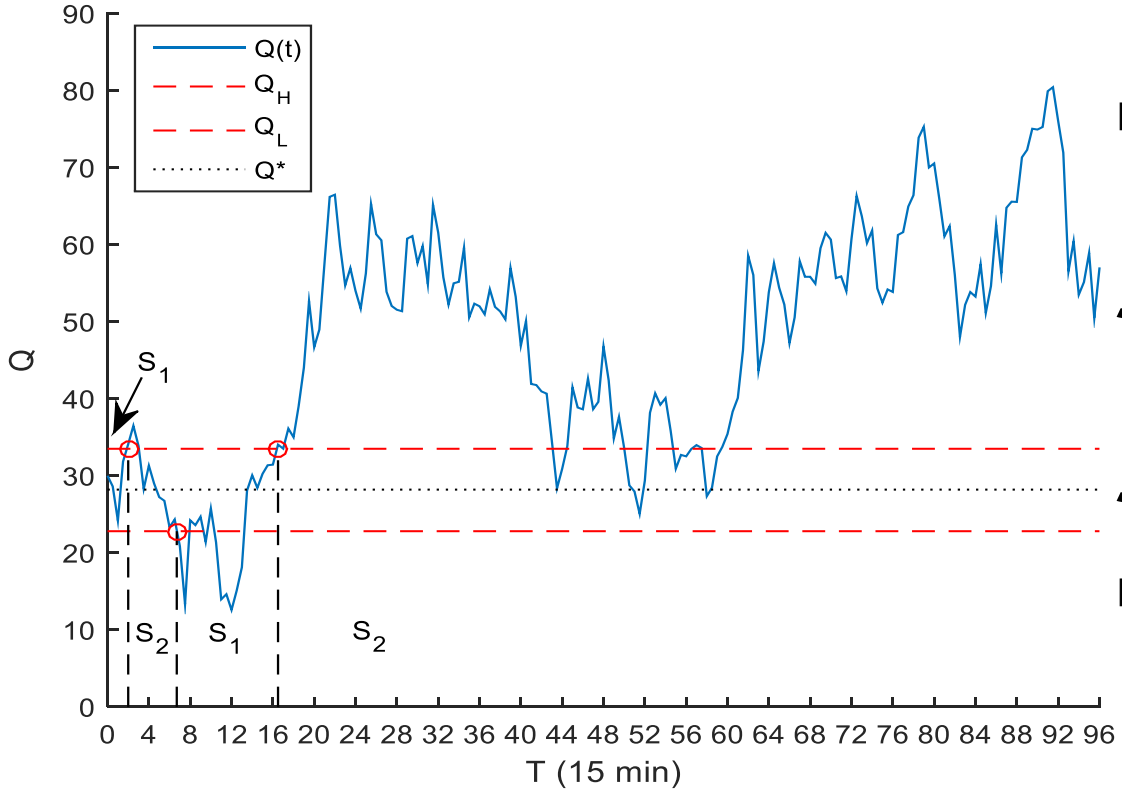
# Application to vehicle modularity

The experiment is designed to compute the **option premium** for the added flexibility to switch between two vehicle sizes: $S\downarrow 1 = 10$ and $S\downarrow 2 = 20$.

- **Scenario 1:** flexible service with only one fixed vehicle size $S\downarrow 0$ (static policy),
- **Scenario 2:** flexible service with two vehicle sizes in which the proposed policy is used to determine optimal switching, assuming the system initiates at $S\downarrow 1$ and having symmetric switching costs $F\downarrow S =$



# Application to vehicle modularity




- $F\downarrow S = 10$

  $Q\downarrow H = 33.5, S\downarrow 1 \rightarrow S\downarrow 2$

  $Q\downarrow L = 22.8, S\downarrow 2 \rightarrow S\downarrow 1$

- $F\downarrow S = 0$

  $Q\uparrow * = 28.2.$

Fig 10. Proposed switching policy for vehicle modularity.

| | Proposed policy | Static policy |
|---|---|---|
| **Cumulative total cost ($)** | 45946.64 | 46320.09 |

The flexibility to switch vehicle size in this case leads to an improvement over a static policy of $373.45 over the 24 hrs.

# Conclusion

- Optimize dynamic switching of transit service as **a market entry-exit real options model** with mean-reverting demand density;
- Adapt this model to analysis "vehicle modularity";
- Compare with Chang and Schonfeld (1991a):
  - A hysteresis effect is observable;
  - The values and sensitivity of the switching thresholds with respect to the transportation system conditions are not necessarily symmetric;
  - The cost-free switching threshold $\neq$ the deterministic cost savings threshold.
- Relative to a myopic policy, the performance of the proposed policy can eliminate up to 72% of the excess cost;
- An option premium exists for having the flexibility to switch between two vehicle sizes.

# Further research

- When the Ornstein–Uhlenbeck process is changed to a Geometric Brownian Motion process, the model may be applicable to long term regional planning and staging of flexible-route and fixed-route transit services.

- Further research should also be undertaken to address some shortcomings or simplifications assumed in this study.
  e.g. from a single many-to-one demand pattern to more realistic ones.

- Adding significant switching duration as a research extension should be feasible, given similar considerations of **construction duration** in other transit real options studies (e.g. Li et al, 2015).

- An empirical study using this methodology on a real transit service with performance validation would be highly valuable.

# THANK YOU!

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Paul Schonfeld: pschon@umd.edu

# Model properties

- Sensitivity of switching policy to transportation system parameters-local service speed

| Solutions | | Scaling factor $\alpha$ of basic value of $V_c$ and $V_f$ | | |
|---|---|---|---|---|
| | | $\alpha=0.8$ | $\alpha=1.0$ | $\alpha=1.2$ |
| Fixed Transit | Headway, $h_c$ | 0.43 | 0.42 | 0.39 |
| | Vehicle size, $S_c$ | 80 | 75 | 66 |
| | Fleet size, $F_c$ | 5 | 5 | 5 |
| | Route spacing, $r$ | 1.46 | 1.41 | 1.33 |
| | Total cost, $C_{sc}$ | 3044.0 (+5.65%) | 2881.1 | 2626.1 (-8.85%) |
| Flexible Transit | Headway, $h_f$ | 0.08 | 0.08 | 0.07 |
| | Vehicle size, $S_f$ | 8 | 7 | 7 |
| | Fleet size, $F_f$ | 62 | 58 | 50 |
| | Service zone, $A$ | 2.85 | 3.02 | 3.34 |
| | Total cost, $C_{sf}$ | 3153.2 (+9.37%) | 2883.0 | 2470.1 (-14.32%) |
| $\Phi(Q)$ | | 109.2 | 1.9 | -156 |
| $E_t\left[\int_t^\infty \Phi(Q)e^{-\rho(s-t)}\,ds\right]$ | | 654.8 | -39.4 | -989.0 |
| $V_0(Q(0))$ | | 16.5 | 23.5 | 125.8 |
| $V_1(Q(0))$ | | 35.9 | 17.4 | 76.7 |
| $Q_L$ | | 12.2 (-57.5%) | 28.7 | 68.9 (+140.1%) |

# Model properties

Table 4. Sensitivity of variable bus operating cost on fixed and flexible transit total cost with baseline-relative changes in parentheses.

| Solutions | | Scaling factor $\alpha$ of basic value of operational variable cost $b$ | | |
|---|---|---|---|---|
| | | $\alpha$=0.8 | $\alpha$=1.0 | $\alpha$=1.2 |
| **Fixed Transit** | Headway, $h_c$ | 0.42 | 0.42 | 0.42 |
| | Vehicle size, $S_c$ | 75 | 75 | 75 |
| | Fleet size, $F_c$ | 5 | 5 | 5 |
| | Route spacing, $r$ | 1.41 | 1.41 | 1.41 |
| | Total cost, $C_{sc}$ | 2873.1 (-0.28%) | 2881.1 | 2888.9 (+0.27%) |
| **Flexible Transit** | Headway, $h_f$ | 0.08 | 0.08 | 0.08 |
| | Vehicle size, $S_f$ | 8 | 7 | 7 |
| | Fleet size, $F_f$ | 57 | 58 | 58 |
| | Service zone, $A$ | 3.13 | 3.02 | 2.91 |
| | Total cost, $C_{sf}$ | 2799.0 (-2.91%) | 2883.0 | 2996.2 (+3.93%) |
| $\Phi(Q)$ | | -74.1 | 1.9 | 107.3 |
| $E_t\left[\int_t^\infty \Phi(Q)e^{-\rho(s-t)}ds\right]$ | | -802.4 | -39.4 | 995.5 |
| $V_0(Q(0))$ | | 86.4 | 23.5 | 19.8 |
| $V_1(Q(0))$ | | 44.5 | 17.4 | 33.5 |
| $Q_L$ | | 42.4 (+47.7%) | 28.7 | 21.5 (-25.1%) |

# Application to vehicle modularity

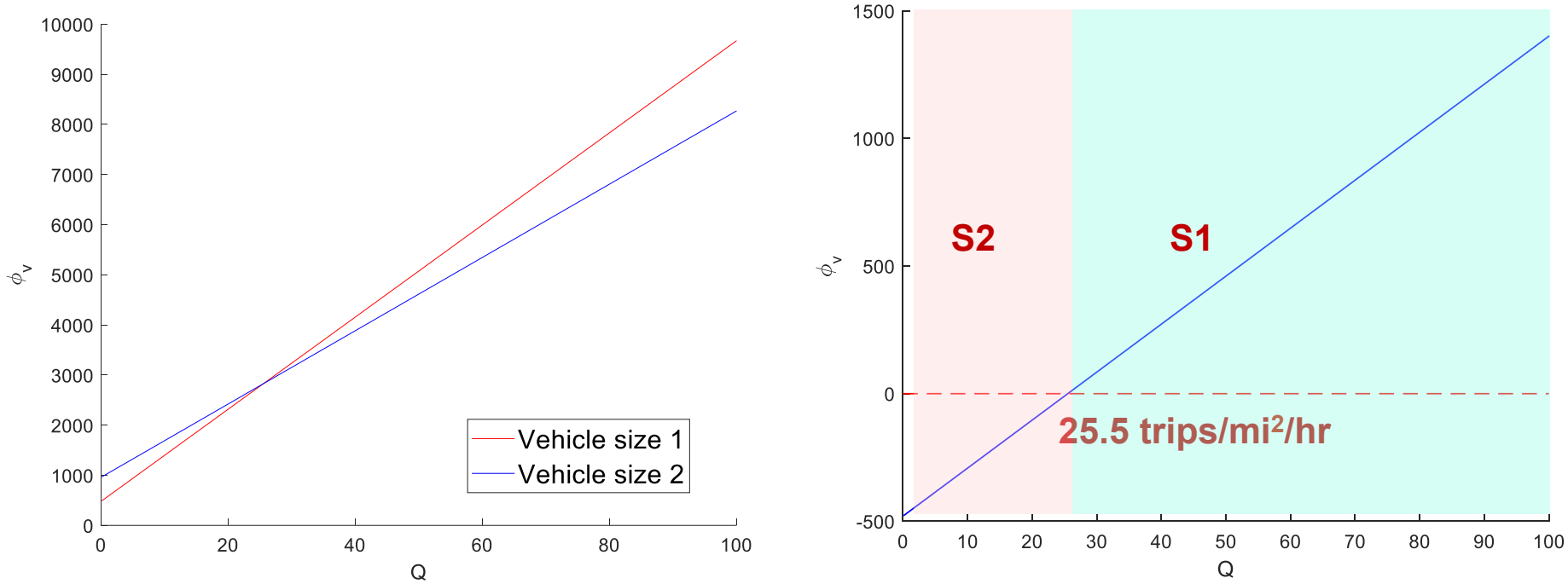


Fig. 9. (a) Total cost of two vehicle sizes
(b) cost savings of vehicle size 2 over vehicle size 1, with respect to demand density.

$$Q_{n+1} = Q_n + \mu(m - Q_n)\Delta t + \sigma Q_n \Delta w_n$$

$\Delta w_n$ are independent identitically distributed Wiener increments, normal variates with zero mean and variance $\Delta t$. Thus, $w_{t_{n+1}} - w_{t_n} = \Delta w_n \sim N(0, \Delta t) = \sqrt{\Delta t}\, N(0,1)$. $N(0,1)$ is the standard normal variate. In implementation, $\Delta t = 0.5$.

$$E_0[Q(t)] = m + (Q(0) - m)\exp(-\mu t)$$

$$Var_0[Q(t)] = m^2 \sigma^2 / 2\mu - \sigma^2 - \exp(-2\mu t)(Q(0) - m)^2 + 2m\sigma^2 (Q(0) - m)/\mu - \sigma^2 \exp(-\mu t) + 2\mu^2 (Q(0) - m)^2 - \mu\sigma^2 Q(0)(3Q(0) - 2m) + \sigma^4 Q^2(0)/(\mu - \sigma^2)(2\mu - \sigma^2) \exp(-(2\mu - \sigma^2)t)$$

O-U processes are generalizations of the geometric Brownian motion (GBM) that allow for stationarity, and have been shown to fit the data in investment models better than GBM despite the increased complexity (Tsekrekos, 2010).