Online Appendix
Reputational Bargaining and Inefficient Technology Adoption

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A Proof of Theorem 1: Existence of Equilibrium

First, we establish the existence of equilibrium in an auxiliary game where the buyer's initial offer belongs to $\mathbf{P}_b$ and the seller's initial offer belongs to $\mathbf{P}_s$. Then, we show that deviations to offers outside of these sets are unprofitable for the players, which establishes the existence of equilibrium where players can offer any price. We consider the case in which $\Theta = \{\theta_1, ..., \theta_n\}$ with $n \geq 2$, which will allow us to establish equilibrium existence for the more general model studied in Section 4.

Let $\sigma_b \in \Delta(\mathbf{P}_b)$ and $\sigma_s = (\sigma_\theta)_{\theta \in \Theta} : \mathbf{P}_b \to [\Delta(\mathbf{P}_s)]^n$ be a strategy for the buyer and the seller, respectively. We equip the probability spaces $\Delta(\mathbf{P}_b)$ and $\Delta(\mathbf{P}_s)$ with the topology of weak convergence, which is induced by the Prokhorov metric (Billingsley, 2013). Let $\pi : \mathbf{P}_b \times \mathbf{P}_s \to \Delta(\Theta)$ and $\varepsilon_s : \mathbf{P}_b \times \mathbf{P}_s \to [0, 1]$ be a system of beliefs for the buyer, assigning, respectively, a probability distribution over the rational-type seller's production cost and a probability that the seller is a commitment type, after every pair of offers $(p_b, p_s) \in \mathbf{P}_b \times \mathbf{P}_s$. In order to avoid confusion, throughout this section, we use $\pi_j = \pi(\theta_j)$ to denote the (exogenous) prior probability that the seller is rational and has production cost θ_j. Likewise, let $\varepsilon_b : \mathbf{P}_b \to [0, 1]$ denote the seller's posterior belief that the buyer is a commitment type after observing offer $p_b \in \mathbf{P}_b$.

Fix a continuation equilibrium in the war-of-attrition game $\Gamma(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$ that follows bargaining postures (p_b, p_s) with $p_b < p_s$ and beliefs given by $(\hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$. Such an equilibrium always exists given the results in Abreu and Gul (2000). Its construction for the case in which $p_b > \theta_1$ and $p_s < 1$ was provided in Lemmas 2 and 3 in the main text. If $p_b > \theta_1$ and $p_s = 1$, we set players' strategies to be equal to the limit of the strategies described in Lemmas 2 and 3 as $p_s \to 1$. Likewise, if $p_b \leq \theta_1$ and $p_s < 1$, we extend players' strategies in Lemmas 2 and 3 by taking the

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limit as $p_b \to \theta_1$. Otherwise, if $p_b \leq \theta_1$ and $p_s = 1$, there exists a continuation equilibrium in the war-of-attrition in which neither player concedes. Let $V_b(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$ and $V_\theta(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$ be the buyer's and type-θ seller's equilibrium continuation values in the war-of-attrition game $\Gamma(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$. Notice that our equilibrium construction implies that V_b and V_θ are continuous in each of their arguments.

Suppose the seller's strategy is σ_s and players' system of beliefs is induced by $(\pi, \varepsilon_s, \varepsilon_b)$. Then, the buyer's expected payoff from offering p_b is

$$U_b(p_b, \sigma_s, \varepsilon_b, \varepsilon_s, \pi) \equiv \sum_{j=1}^{n} \left(\sum_{p_s \leq p_b} \pi_j \sigma_{\theta_j}(p_s|p_b) \left(1 - \frac{p_s + p_b}{2}\right) \right.$$
$$\left. + \sum_{p_s > p_b} \pi_j \sigma_{\theta_j}(p_s|p_b) V_b(p_b, p_s, \varepsilon_b(p_b), \varepsilon_s(p_b, p_s), (1 - \varepsilon_s(p_b, p_s))\pi(p_b, p_s)) \right).$$

Likewise, we can write the seller's expected payoff from offering p_s after the buyer offers p_b when his type is $\theta \in \Theta$ as:

$$U_\theta(p_s, p_b, \varepsilon_b, \varepsilon_s, \pi) \equiv \begin{cases} \frac{p_s + p_b}{2} - \theta, & \text{if } p_s \leq p_b \\ V_\theta(p_b, p_s, \varepsilon_b(p_b), \varepsilon_s(p_b, p_s), (1 - \varepsilon_s(p_b, p_s))\pi(p_b, p_s)), & \text{if } p_s > p_b. \end{cases}$$

We define an equilibrium of the bargaining game to be an assessment $(\varepsilon_b, \varepsilon_s, \pi, \sigma_b, \sigma_s)$ such that strategies are *sequentially rational* and players' beliefs $(\varepsilon_b, \varepsilon_s, \pi)$ are consistent with Bayes' rule at every information set $(p_b, p_s) \in \mathbf{P}_b \times \mathbf{P}_s$. Since μ_b and μ_s have full support, all pairs of offers arise with positive probability and beliefs are thus always pinned down by Bayes' rule.

The proof follows from a fixed point argument, with the only caveat that we have to deal with the fact that final payoffs depend non-trivially on beliefs. We circumvent this by constructing a correspondence whose fixed point is an equilibrium *assessment*, specifying both strategies and beliefs, and then we show that such a fixed point exists.

Let $\mathcal{K} \equiv \Delta(\mathbf{P}_b) \times [\Delta(\mathbf{P}_s)]^{n|\mathbf{P}_b|}$ be the set of possible strategy profiles. Define $\Sigma : \mathcal{K} \rightrightarrows [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b| \times |\mathbf{P}_s|}$ to be the correspondence that maps every strategy profile $(\sigma_b, \sigma_s) \in \mathcal{K}$ to a system of beliefs $(\varepsilon_b, \varepsilon_s, \pi)$ obtained at every information set using Bayes' rule applied to the strategies (σ_b, σ_s). Since there is no off-path event when $\varepsilon > 0$, we know that $\Sigma(\sigma_b, \sigma_s)$ is non-empty, single-valued (and hence compact and convex), and upper hemi-continuous.

Next, we define the best response correspondence at every information set for a given strategy profile (being played at every other information set) and a given system of beliefs by

$$\mathrm{BR}_\theta(p_b, \varepsilon_b, \varepsilon_s, \pi) = \underset{\sigma_\theta \in \Delta(\mathbf{P}_s)}{\arg\max} \sum_{p_s \in \mathbf{P}_s} \sigma_\theta(p_s) U_\theta(p_s, p_b, \varepsilon_b, \varepsilon_s, \pi) \text{ for every } \theta \in \Theta, p_b \in \mathbf{P}_b,$$

$$\mathrm{BR}_b(\sigma_s, \varepsilon_b, \varepsilon_s, \pi) = \underset{\sigma_b \in \Delta(\mathbf{P}_b)}{\arg\max} \sum_{p_b \in \mathbf{P}_b} \sigma_b(p_b) U_b(p_b, \sigma_s, \varepsilon_b, \varepsilon_s, \pi).$$

And let $\mathrm{BR}_\theta(\varepsilon_b, \varepsilon_s, \pi) \equiv \prod_{p_b \in \mathbf{P}_b} \mathrm{BR}_\theta(p_b, \varepsilon_b, \varepsilon_s, \pi)$. Our equilibrium characterization implies that payoffs in the war-of-attrition game are continuous in beliefs $(\varepsilon_b, \varepsilon_s, \pi)$. Players' payoffs are also continuous in (p_b, p_s), which ensures that they are continuous in (σ_b, σ_s) by the Portmanteau Theorem (Theorem 2.1 in Billingsley, 2013). Moreover, compactness of $\mathbf{P}_b$ and $\mathbf{P}_s$ ensures that the spaces $\Delta(\mathbf{P}_b)$ and $\Delta(\mathbf{P}_s)$ are compact in the weak topology (Theorem 15.11 in Aliprantis and Border, 2006), and therefore so is the product space $\mathcal{K} \times [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b|}$. It then follows from the Maximum Theorem (Theorem 17.31 in Aliprantis and Border, 2006) that best-response correspondences are upper-hemi-continuous with nonempty and compact values. Linearity of payoffs in σ_b and σ_θ ensures that they are also convex-valued.

Let $\mathcal{M} \equiv [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \mathcal{K}$, which is a locally convex Hausdorff space. Then, we can define the correspondence $E : \mathcal{M} \rightrightarrows \mathcal{M}$ to be:

$$E(\varepsilon_b, \varepsilon_s, \pi, \sigma_b, \sigma_s) = \Sigma(\sigma_b, \sigma_s) \times \mathrm{BR}_b(\sigma_s, \varepsilon_b, \varepsilon_s, \pi) \times \mathrm{BR}_{\theta_1}(\varepsilon_b, \varepsilon_s, \pi) \times ... \times \mathrm{BR}_{\theta_n}(\varepsilon_b, \varepsilon_s, \pi).$$

According to the Kakutani-Fan-Glicksberg fixed point theorem (Corollary 17.55 in Aliprantis and Border, 2006), E has a fixed point. Let $(\varepsilon_b^*, \varepsilon_s^*, \pi^*, \sigma_b^*, \sigma_s^*)$ be a fixed point of E. By construction, this assessment is an equilibrium of the game where the buyer's and the seller's offers are restricted to lie, respectively, in $\mathbf{P}_b$ and $\mathbf{P}_s$.

We conclude by using $(\varepsilon_b^*, \varepsilon_s^*, \pi^*, \sigma_b^*, \sigma_s^*)$ to construct an equilibrium of the original game where players can choose any offer in $[0,1]$. In this construction, the buyer's strategy is $\sigma_b^* \in \Delta[0,1]$. We extend the seller's strategy to specify a counteroffer for every $p_b \in [0,1]$ as follows:

$$\overline{\sigma}_\theta(\cdot|p_b) = \begin{cases} \sigma_\theta^*(\cdot|p_b), & \text{if } p_b \in \mathbf{P}_b \\ \delta_{\{1\}}, & \text{if } p_b \in [0,1] \setminus \mathbf{P}_b, \end{cases}$$

where $\delta_{\{1\}}$ is the Dirac measure on 1. Let $\overline{\sigma}_s \equiv (\overline{\sigma}_\theta)_{\theta \in \Theta}$. Since all pairs of offers in $[0,1]^2 \setminus \mathbf{P}_b \times \mathbf{P}_s$

are off-path, we extend players' systems of beliefs to be:

$$\overline{\varepsilon}_b(p_b) = \begin{cases} \varepsilon_b^*(p_b), & \text{if } p_b \in \mathbf{P}_b \\ 0, & \text{if } p_b \in [0,1] \setminus \mathbf{P}_b \end{cases} \qquad \overline{\varepsilon}_s(p_b, p_s) = \begin{cases} \varepsilon_s^*(p_b, p_s), & \text{if } (p_b, p_s) \in \mathbf{P}_b \times \mathbf{P}_s \\ 0, & \text{if } p_s \in [0,1] \setminus \mathbf{P}_s \\ \frac{\varepsilon \mu_s(p_s)}{\varepsilon \mu_s(p_s) + (1-\varepsilon) \sum_{j=1}^n \pi_j \overline{\sigma}_{\theta_j}(p_s|p_b)}, & \text{otherwise} \end{cases}$$

We extend $\pi^*(p_b, p_s)$ to assign probability one to type θ_1 for all $(p_b, p_s) \in [0,1]^2$ such that $p_s \notin \mathbf{P}_s$. Let $\overline{\pi} : [0,1]^2 \to \Delta(\Theta)$ be such extension. We argue that the assessment $(\overline{\varepsilon}_b, \overline{\varepsilon}_s, \overline{\pi}, \sigma_b^*, \overline{\sigma}_s)$ is an equilibrium. In order to do so, we first check that the buyer does not benefit from deviating to an offer $p_b \in [0,1] \setminus \mathbf{P}_b$. If she does so, the seller demands the entire surplus, and the resulting payoff is 0 which is worse than her equilibrium payoff guarantee of $1 - p_{\theta_n}$.

Finally, we verify sequential rationality for the seller. Following $p_b \in \mathbf{P}_b$, it suffices to check that offers $p_s \in [0,1] \setminus \mathbf{P}_s$ do not dominate $\sigma_\theta^*(\cdot|p_b)$. This is straightforward when $p_b > \theta_n$, since demanding $p_s \in [0,1] \setminus \mathbf{P}_s$ implies revealing rationality and is therefore equivalent to conceding immediately. When $p_b \in \mathbf{P}_b \cap [0, \theta_n]$, deviating to $p_s \in [0,1] \setminus \mathbf{P}_s$ also involves immediate concession for types θ such that $\theta < p_b$ due to our construction of off-path beliefs. Further, because the buyer believes that she's facing the (rational) type θ_1 with probability one, we can set the buyer's strategy to be such that she delays concession indefinitely after the seller demands $p_s \in [0,1] \setminus \mathbf{P}_s$, which makes this offer dominated for types $\theta > p_b$ as well. After the buyer offers $p_b \in [0,1] \setminus \mathbf{P}_b$, she is perceived to be rational with probability one. Hence, there is a continuation equilibrium where the seller demands the entire surplus (as stipulated by $\overline{\sigma}_s$) and the buyer concedes immediately. Therefore, players' strategies are sequentially rational. This concludes the proof that $(\overline{\varepsilon}_b, \overline{\varepsilon}_s, \overline{\pi}, \sigma_b^*, \overline{\sigma}_s)$ is an equilibrium of the bargaining game.

B Proof of Theorem 2: Existence of Equilibrium

We modify the existence proof of Theorem 1 in order to allow for an additional stage of the game in which the seller decides which production technology to adopt. In that stage, the seller's action space is Θ, which determines the distribution of the seller's production cost in the bargaining game. In order to establish existence for the more general case studied in Section 4, we focus on the case where $\Theta = \{\theta_1, ..., \theta_n\}$. Let c_j denote the cost of choosing technology j for every $j \in \{1, ..., n\}$.

Similar to the existence proof of Theorem 1 in Online Appendix A, we start by showing existence in the modified game where players' bargaining postures are restricted to belong to the set $\mathbf{P}_b \times \mathbf{P}_s$. Throughout, we use the notation introduced in that proof. Let the payoff of the seller from choosing technology $\theta \in \Theta$, given the strategies at the bargaining stage $(\sigma_b, \sigma_s) \in \mathcal{K}$ and players' system of beliefs $(\varepsilon_b, \varepsilon_s, \pi) \in [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b| \times |\mathbf{P}_s|}$ be given by

$$U_s(\theta_j, \sigma_b, \sigma_s, \varepsilon_b, \varepsilon_s, \pi) = U_{\theta_j}(\sigma_s, \sigma_b, \varepsilon_b, \varepsilon_s, \pi) - c_j.$$

We construct a correspondence whose fixed point is an equilibrium of the game with endogenous technology adoption. Let $\hat{\pi} \in \Delta(\Theta)$ denote a strategy of the seller at the investment stage of the game. Let $\mathcal{J} \equiv \Delta(\Theta) \times \mathcal{K}$ be the set of possible strategy profiles in the bargaining game with investment. Define $\Sigma^I : \mathcal{J} \rightrightarrows [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b| \times |\mathbf{P}_s|}$ to be the correspondence mapping every action profile $(\hat{\pi}, \sigma_b, \sigma_s) \in \mathcal{J}$ to a system of beliefs $(\varepsilon_b, \varepsilon_s, \pi)$ obtained at every information set using Bayes' rule applied to the strategies $(\hat{\pi}, \sigma_b, \sigma_s)$. This is always well defined under our assumption that all irrational types $(p_b, p_s) \in \mathbf{P}_b \times \mathbf{P}_s$ occur with strictly positive probability. Thus, $\Sigma^I(\sigma_b, \sigma_s)$ is non-empty, single-valued, and upper-hemi-continuous.

Let the seller's best response correspondence at the investment stage $\mathrm{BR}_0 : \mathcal{K} \times [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \rightrightarrows \Delta(\Theta)$ be given by

$$\mathrm{BR}_0(\sigma_b, \sigma_s, \varepsilon_b, \varepsilon_s, \pi) = \underset{\hat{\pi} \in \Delta(\Theta)}{\arg\max}\ U_s(\hat{\pi}, \sigma_b, \sigma_s, \varepsilon_b, \varepsilon_s, \pi).$$

Since U_θ is continuous, which has been established in Online Appendix A, the correspondence BR_0 is non-empty, compact, convex, and upper hemi-continuous. Therefore, the *equilibrium correspondence* $E^I : [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \mathcal{J} \rightrightarrows [0,1]^{|\mathbf{P}_b|} \times [0,1]^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \Delta(\Theta)^{|\mathbf{P}_b| \times |\mathbf{P}_s|} \times \mathcal{J}$ defined as

$$E^I(\varepsilon_b, \varepsilon_s, \pi, \hat{\pi}, \sigma_b, \sigma_s) =$$
$$\Sigma^I(\hat{\pi}, \sigma_b, \sigma_s) \times \mathrm{BR}_0(\sigma_b, \sigma_s, \varepsilon_b, \varepsilon_s, \pi) \times \mathrm{BR}_b(\sigma_s, \varepsilon_b, \varepsilon_s, \pi) \times \mathrm{BR}_{\theta_1}(\sigma_b, \varepsilon_b, \varepsilon_s, \pi) \times \ldots \times \mathrm{BR}_{\theta_n}(\sigma_b, \varepsilon_b, \varepsilon_s, \pi),$$

admits a fixed point according to the Kakutani-Fan-Glicksberg fixed point theorem. Let $(\varepsilon_b^*, \varepsilon_s^*, \pi^*, \hat{\pi}^*, \sigma_b^*, \sigma_s^*)$ be a fixed point of E^I. This fixed point is an equilibrium of the game with price grids $\mathbf{P}_b \times \mathbf{P}_s$. Using the extension provided in Online Appendix A, one can apply the same argument to verify that the assessment $(\overline{\varepsilon}_b, \overline{\varepsilon}_s, \overline{\pi}, \hat{\pi}^*, \sigma_b^*, \overline{\sigma}_s)$ is an equilibrium of the game where players' can choose any bargaining posture from the unit interval.

C Generalization of Lemma 1

We generalize Lemma 1 in Appendix A to environments with more than two possible production costs. Let $\Theta \equiv \{\theta_1, ..., \theta_n\}$ with $0 < \theta_1 < \theta_2 < ... < \theta_n < 1$. Fix any equilibrium, let $\hat{\varepsilon}_b(p_b)$ be the probability that the buyer is the commitment type after she offers p_b. We state the generalization as Lemma C.1, which will be used in the subsequent proofs for Theorems 3 and 4.

Lemma C.1. *In any equilibrium, after the buyer offers $p_b \in \mathbf{P}_b$ with $\hat{\varepsilon}_b(p_b) < 1$, every type of the seller with production cost no less than p_b will demand* 1.

Proof. Suppose by way of contradiction that after the buyer offers p_b, there exists a type $\theta_k \geq p_b$ that demands some $p_s < 1$ with positive probability. Our richness assumption implies that $(p_s, 1) \cap \mathbf{P}_s$ is non-empty. For every $p'_s \in (p_s, 1) \cap \mathbf{P}_s$, there exists θ_j with $\theta_j < p_b$ such that type θ_j offers p'_s with positive probability. This is because otherwise, the buyer will rule out types lower than p_b after observing p'_s and will therefore concede immediately. If this is the case, then type θ_k has a strict incentive to deviate by offering p'_s instead of p_s. Without loss of generality, let θ_j be the highest type that (i) has cost strictly lower than p_b and (ii) offers p'_s with positive probability. In equilibrium, it is optimal for type θ_j to offer p'_s and then concede after the rational-type buyer finishes conceding.

After the seller offers p_s and p'_s, respectively, let T and T' be the times at which the rational-type buyer finishes conceding, let c_b and c'_b be the buyer's concession probabilities at time 0, and let A and A' be the discounted probability of trade when the buyer is rational and the seller never concedes. On the one hand, the high-cost type θ_k weakly prefers p_s to p'_s, which implies that

$$A(p_s - \theta_k) \geq A'(p'_s - \theta_k). \tag{C.1}$$

On the other hand, it is optimal for the low-cost type θ_j to offer p'_s and concede at time T', so he prefers this strategy to offering p_s and conceding at time T. This incentive constraint implies that:

$$e^{-rT'}\hat{\varepsilon}_b(p_b)(p_b - \theta_j) + (1 - \hat{\varepsilon}_b(p_b))A'(p'_s - \theta_j) \geq e^{-rT}\hat{\varepsilon}_b(p_b)(p_b - \theta_j) + (1 - \hat{\varepsilon}_b(p_b))A(p_s - \theta_j),$$

which is equivalent to

$$(e^{-rT'} - e^{-rT})\frac{\hat{\varepsilon}_b(p_b)}{1 - \hat{\varepsilon}_b(p_b)}(p_b - \theta_j) \geq A(p_s - \theta_j) - A'(p'_s - \theta_j). \tag{C.2}$$

Inequality (C.1) implies that

$$A(p_s-\theta_j)-A'(p_s'-\theta_j)=A(p_s-\theta_k)-A'(p_s'-\theta_k)+(A-A')(\theta_k-\theta_j)\geq(A-A')(\theta_k-\theta_j),$$

and therefore,

$$(e^{-rT'}-e^{-rT})\frac{\hat{\varepsilon}_b(p_b)}{1-\hat{\varepsilon}_b(p_b)}(p_b-\theta_j)\geq(A-A')(\theta_k-\theta_j). \tag{C.3}$$

Since $p_s<p_s'$, inequality (C.1) implies that $A>A'$. Inequality (C.3) together with $A>A'$ implies that $T'<T$, which further implies that $T>0$. As a result, there exists a type with production cost strictly lower than p_b who will offer p_s with positive probability in equilibrium. This is because otherwise, the buyer will concede immediately following p_s, which contradicts our earlier conclusion that $T>0$.

Let θ be the lowest type that offers p_s in equilibrium and let θ' be the lowest type that offers p_s' in equilibrium. Our earlier conclusion implies that $\theta,\theta'<p_b$. Consider the payoffs of type θ and type θ' under the following two strategies (i) offering p_s and conceding right after time 0, and (ii) offering p_s' and conceding right after time 0. Type θ weakly prefers the first strategy, which gives:

$$c_b(p_s-p_b)+p_b-\theta\geq c_b'(p_s'-p_b)+p_b-\theta.$$

Type θ' weakly prefers the second strategy, which gives:

$$c_b(p_s-p_b)+p_b-\theta'\leq c_b'(p_s'-p_b)+p_b-\theta'.$$

The two inequalities together imply that

$$c_b(p_s-p_b)=c_b'(p_s'-p_b). \tag{C.4}$$

Since $p_s'>p_s$, the above inequality implies that $c_b\geq c_b'$, where equality holds if and only if $c_b=c_b'=0$. Recall our formulas for players' concession rates. After the seller offers p_s, the buyer will concede at rate

$$\lambda_b(\theta)=\frac{r(p_b-\theta)}{p_s-p_b} \tag{C.5}$$

when type θ seller is conceding. After the seller offers p'_s, the buyer will concede at rate

$$\lambda'_b(\theta) = \frac{r(p_b - \theta)}{p'_s - p_b} \tag{C.6}$$

These expressions imply that (i) $\lambda_b(\theta) > \lambda'_b(\theta)$ for every $\theta < p_b$, and (ii) $\lambda_b(\theta') > \lambda_b(\theta)$ and $\lambda'_b(\theta') > \lambda'_b(\theta)$ for every $\theta' < \theta < p_b$.

Let $\widetilde{\Lambda}(t)$ be the buyer's concession rate at time t when the seller offers p_s and let $\widetilde{\Lambda}'(t)$ be the buyer's concession rate at time t when the seller offers p'_s, which are well-defined except for a finite number of points. Let $\Lambda(t) \equiv \lim_{t^* \downarrow t} \widetilde{\Lambda}(t^*)$ and let $\Lambda'(t) \equiv \lim_{t^* \downarrow t} \widetilde{\Lambda}'(t^*)$.

First, we show that $\Lambda(\eta) > \Lambda'(\eta)$ for $\eta = 0$ as well as every η that is sufficiently close to 0. This is because otherwise, the lowest type that offers p_s, denoted by θ, is strictly greater than the lowest type that offers p'_s, denoted by θ'. If this is the case, then type θ has a strict incentive to deviate by offering p'_s and conceding at time $\varepsilon \approx 0$. This is because (C.4) implies that type θ is indifferent between offering p_s and conceding at time 0 and offering p'_s and conceding at time 0, from which he obtains his equilibrium payoff since he offers p_s and concedes at time 0 with positive probability. By definition, type θ' is indifferent between offering p'_s and conceding at time 0 and offering p'_s and conceding at time $\varepsilon > 0$, for ε small enough. Therefore, type θ strictly prefers offering p'_s and conceding at time ε to his equilibrium strategy, which leads to a contradiction.

Let t^* be the smallest $t \in \mathbb{R}_+$ such that $\Lambda(t) < \Lambda'(t)$. Such t^* exists since otherwise, $\Lambda(t) \geq \Lambda'(t)$ for every t and the previous step implies that $\Lambda(t) > \Lambda'(t)$ for some t. Since $c_b \geq c'_b$, the rational-type of the buyer will finish conceding sooner when the seller offers p_s compared to the case in which the seller offers p'_s. This contradicts our earlier conclusion that $T > T'$.

Given the existence of such t^*, we know that the type of seller who starts to concede at time t^* is *strictly* greater under offer p_s, which we abuse notation and denote it by θ, compared to that under offer p'_s, which we abuse notation and denote it by θ'. This is because $\lambda_b(\theta) > \lambda'_b(\theta)$ for every $\theta < p_b$ and both $\lambda_b(\cdot)$ and $\lambda'_b(\cdot)$ are strictly *decreasing* functions of θ. Let $A(t^*)$ be the discounted probability that the buyer concedes to the seller before time t^* when the seller offers p_s. Let $B(t^*)$ be the probability that the buyer has *not* conceded by time t^* when the seller offers p_s. Let $A'(t^*)$ and $B'(t^*)$ denote the same variables when the seller offers p'_s. Type θ' weakly prefers offering p'_s and conceding at t^* to offering p_s and conceding at t^*, which gives:

$$A'(t^*)(p'_s - \theta') + B'(t^*)e^{-rt^*}(p_b - \theta') \geq A(t^*)(p_s - \theta') + B(t^*)e^{-rt^*}(p_b - \theta'). \tag{C.7}$$

Notice that $A(t^*) + e^{-rt^*}B(t^*)$ depends only on the expected delay in trade when the seller offers p_s and concedes at time t^*, and $A'(t^*) + e^{-rt^*}B'(t^*)$ depends only on the expected delay in trade when the seller offers p'_s and concedes at time t^*. The definition of t^* implies that $\Lambda(t) \geq \Lambda'(t)$ for every $t < t^*$ with strict inequality for every t that is close enough to 0. Therefore, less delay is incurred when the seller offers p_s, which implies that

$$A(t^*) + e^{-rt^*}B(t^*) > A'(t^*) + e^{-rt^*}B'(t^*) \tag{C.8}$$

Since $\theta > \theta'$, inequalities (C.7) and (C.8) together imply that

$$A'(t^*)(p'_s - \theta) + B'(t^*)e^{-rt^*}(p_b - \theta) > A(t^*)(p_s - \theta) + B(t^*)e^{-rt^*}(p_b - \theta). \tag{C.9}$$

Inequality (C.9) suggests that type θ strictly prefers offering p'_s and conceding at time t^*, to offering p_s and conceding at time t^*. However, type θ is supposed to play the latter strategy with positive probability in equilibrium. This implies that type θ has a strictly profitable deviation, which leads to a contradiction.

The above contradiction implies that when the buyer offers $p_b \in \mathbf{P}_b$ that is offered with positive probability by the rational-type buyer, every type of the seller with cost weakly greater than p_b will demand 1 with probability 1. □

D Proof of Theorem 3

We follow the same steps as in the Proof of Theorem 1. Fix $\pi \in \Delta(\Theta)$, and let $(\sigma_b, \sigma_s, \tau_b, \tau_s)$ be an equilibrium strategy profile of the bargaining game. Let

$$\hat{\varepsilon}_b(p_b) = \frac{\varepsilon\mu_b(p_b)}{\varepsilon\mu_b(p_b) + (1-\varepsilon)\sigma_b(p_b)}, \tag{D.1}$$

$$\hat{\varepsilon}_s(p_b, p_s) = \frac{\varepsilon\mu_s(p_s)}{\varepsilon\mu_s(p_s) + (1-\varepsilon)\sum_{j=1}^{n}\pi\{\theta_j\}\sigma_s(p_s|\theta_j, p_b)}, \tag{D.2}$$

$$\hat{\pi}_j(p_b, p_s) = \frac{(1-\varepsilon)\pi\{\theta_j\}\sigma_s(p_s|\theta_j, p_b)}{\varepsilon\mu_s(p_s) + (1-\varepsilon)\sum_{j=1}^{n}\pi\{\theta_j\}\sigma_s(p_s|\theta_j, p_b)}, \quad \text{for every } j \in \{1, ..., n\}. \tag{D.3}$$

Fix (p_b, p_s) and the resulting $(\hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi}_1, ..., \hat{\pi}_n)$. We characterize equilibrium strategies in the continuation game $\Gamma(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$, when $\theta_1 < p_b < p_s < 1$. Let

$$m \equiv \max\{j \in \{1, ..., n\} : \theta_j < p_b\},$$

and for every $j \in \{1, ..., m\}$, let $\lambda_b^j \equiv \frac{r(p_b - \theta_j)}{p_s - p_b}$, and

$$T_s^j \equiv \frac{-\log(\hat{\varepsilon}_s + \sum_{i>j} \hat{\pi}_i)}{\lambda_s}, \quad T_b \equiv \frac{-\log(\hat{\varepsilon}_b) - \sum_{j=1}^{m-1} (\lambda_b^j - \lambda_b^{j+1}) T^j}{\lambda_b^m},$$

$$L \equiv \frac{-\lambda_s \log \hat{\varepsilon}_b}{-\sum\limits_{j=1}^{m} (\lambda_b^j - \lambda_b^{j+1}) \log(\hat{\varepsilon}_s + \hat{\pi}_{j+1})},$$

$$\hat{c}_s^j \equiv 1 - \left(\hat{\varepsilon}_b^{-\lambda_s} \prod_{i=j}^{m} (\hat{\varepsilon}_s + \hat{\pi}_{i+1})^{\lambda_b^i - \lambda_b^{i+1}} \right)^{1/\lambda_b^j}, \quad \hat{c}_b = 1 - \hat{\varepsilon}_b \exp\left\{ \sum_{i=1}^{m} \lambda_b^j (T_s^i - T_s^{i-1}) \right\}.$$

The next series of lemmas extend the equilibrium characterization of the game $\Gamma(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$ to the case in which Θ has more than two elements. The arguments are exactly the same as in the two-type setting. Let

$$j^* \equiv \min\{j \in \{1, ..., m\} : \hat{c}_s^j < \sum_{i \le j} \hat{\pi}_i\}.$$

Lemma D.1. *Fix offers (p_b, p_s) with $1 > p_s > p_b > \theta_1$. In any equilibrium of $\Gamma(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$, the buyer concedes with positive probability at time zero if and only if $L > 1$ and the seller concedes with positive probability at time 0 if and only if $L < 1$. Players' concession probabilities at time 0 are $c_b \equiv \max\{0, \hat{c}_b\}$ and $c_s \equiv \max\{0, \hat{c}_s^{j^*}\}$, respectively.*

Let $T^j = T_s^j + \frac{\log(1-c_s)}{\lambda_s}$ for all $j \in \{j^*, ..., m-1\}$ and $T^m \equiv \min\left\{ \frac{-\log(\hat{\varepsilon}_b) - \sum_{j=j^*}^{m-1} (\lambda_b^j - \lambda_b^{j+1}) T_s^j}{\lambda_b^m}, T_s^m \right\}$.

Lemma D.2. *In every equilibrium of the war-of-attrition game $\Gamma(p_b, p_s, \hat{\varepsilon}_b, \hat{\varepsilon}_s, \hat{\pi})$ in which $p_b > \theta_1$ and $p_s < 1$, players' equilibrium concession times τ_b and $\tau_s(\theta)$ must satisfy:*

1. *For every $j \in \{j^*, ..., m\}$, the buyer concedes at rate λ_b^j when $t \in (T^{j-1}, T^j)$ with $T^{j^*-1} = 0$.*

2. *The seller with production cost $\theta \in \{\theta_{j^*}, ..., \theta_m\}$ concedes at rate λ_s when $t \in (T^{j-1}, T^j)$ with $T^{j^*-1} = 0$.*

3. *The seller never concedes if his production cost is strictly greater than θ_m.*

Next, we characterize players' concession probabilities at time 0 in the limit where $\varepsilon \to 0$. Consider an infinite sequence $\{\varepsilon^k\}_{k=0}^{+\infty}$ satisfying $\varepsilon^k \to 0$ as $k \to \infty$. Let (σ_b^k, σ_s^k) be players' equilibrium bargaining strategies when the ex ante probability of commitment types is ε^k, and $(\sigma_b^\infty, \sigma_s^\infty)$ be a subsequential limit. Let $(\hat{\varepsilon}_b^k, \hat{\varepsilon}_s^k, \hat{\pi}^k)$ be given by (D.1), (D.2) and (D.3) using $(\varepsilon^k, \sigma_b^k, \sigma_s^k)$, and let $\lim_{k\to\infty} \hat{\pi}_j^k = \hat{\pi}_j^\infty$ for every $j \in \{1, ..., n\}$ and $\hat{\varepsilon}_i^\infty \equiv \lim_{k\to\infty} \hat{\varepsilon}_i^k$ for every $i \in \{b, s\}$.

Lemma D.3. *Suppose $\{\varepsilon^k\}_{k=1}^\infty$ is such that $\varepsilon^k \to 0$ as $k \to \infty$. Let $(c_b^k, c_s^k)_{k=1}^\infty$ be given according to Lemma D.1 in the game $\Gamma(p_b, p_s, \hat{\varepsilon}_b^k, \hat{\varepsilon}_s^k, \hat{\pi}^k)$ with $\theta_1 < p_b < p_s < 1$, and let (c_b^∞, c_s^∞) be the limit as $k \to \infty$. Then*

1. *If $\lambda_b^j > \lambda_s$ for all $j \in \{1, ..., n\}$ such that $p_b > \theta_j$ and $\hat{\pi}_j^\infty(p_s, p_b) > 0$, then $c_s^\infty(p_b, p_s) = 1$.*

2. *If $\sigma_b^\infty(p_b) > 0$, $\hat{\pi}_j^\infty(p_s, p_b) > 0$, and $\lambda_s > \lambda_b^j$ or $p_b \leq \theta_j$ for some $j \in \{1, ..., n\}$, then $c_b^\infty(p_b, p_s) = 1$.*

3. *If $\sigma_b^\infty(p_b) > 0$ and $\hat{\varepsilon}_s^\infty(p_b, p_s) > 0$, then $c_b^\infty(p_b, p_s) = 1$.*

We now derive the seller's equilibrium strategy at the bargaining stage. Let $p_b \in \mathbf{P}_b \cap (\theta_1, p_{\theta_n}]$ be an offer in the support of the buyer's strategy. We show that all types $\theta \leq \theta_m$ (with m defined as above) offer the same price that is approximately $\max\{p_b, p_m(p_b)\}$ with $p_m(p_b) \equiv \max\{p \in \mathbf{P}_s : p \leq 1 + \theta_m - p_b\}$, and all types $\theta > \theta_m$ demand 1. According to Lemma C.1, for any p_b that belongs to the support of the buyer's strategy, all types with production cost $\theta > \theta_m$ will demand the entire surplus. Next, consider types $\theta \leq \theta_m$. Suppose first that $p_b < p_m(p_b)$. If type θ_m offers $p_s < p_m(p_b)$, then his payoff converges to $p_s - \theta_m$, by Part 2 of Lemma D.3. In order to prevent this type from deviating to $p_m(p_b)$, it must be that some type $\theta' < \theta_m$ offers $p_m(p_b)$ in equilibrium. Letting $\overline{\theta}$ be the highest type below θ_m that offers $p_m(p_b)$ in equilibrium, type θ_m's payoff from deviating to $p_m(p_b)$ is bounded below by

$$\left(c_b + (1 - c_b)\frac{p_b - \overline{\theta}}{p_m(p_b) - \overline{\theta}}\right)(p_m(p_b) - \theta_m) \geq \frac{p_s - \overline{\theta}}{p_m(p_b) - \overline{\theta}}(p_m(p_b) - \theta_m) > p_s - \theta_m,$$

Which gives rise to a contradiction. If otherwise type θ_m demands $p_s > p_m(p_b)$, then the fact that all types higher than θ_m demand 1 almost surely implies that type θ_m has to concede immediately after he demands p_s, which is again dominated by demanding $p_m(p_b)$ instead. Since type θ_m offers $p_s \in (p_m(p_b) - \nu, p_m(p_b) + \nu)$ with probability converging to one, it immediately follows that type $\theta < \theta_m$ finds it optimal to do so as well. On the other hand, if $p_b > p_m(p_b)$, by Part 1 of Lemma D.3, all types $\theta \leq \theta_m$ have to concede immediately when they offer anything higher than p_b, and thus any such strategy is equivalent to offering p_b.

Given the above, any offer $p_b \in \mathbf{P}_b \cap (\theta_1, p_{\theta_n}]$ that the buyer makes in equilibrium with probability bounded above 0 yields a payoff which converges to $\pi[\theta_1, \theta_m](1 - \max\{p_b, p_m(p_b)\})$, which is maximized at $p_b \in (\min\{p_{\theta_{i^*}}, \theta^*_{i+1}\} - \nu, \min\{p_{\theta_{i^*}}, \theta^*_{i+1}\} + \nu)$. Therefore, the buyer offer belongs to $(\min\{p_{\theta_{i^*}}, \theta^*_{i+1}\} - \nu, \min\{p_{\theta_{i^*}}, \theta^*_{i+1}\} + \nu)$ with probability converging to one when $\varepsilon \to 0$.

Given the above equilibrium strategies, the outcome is approximately efficient conditional on the seller's type being $\theta \leq \theta_{i*}$. Conditional on type $\theta > \theta_{i^*}$, we use the seller's incentive compatibility constraint to derive bounds on expected delay. First, in order to ensure that type θ_{i^*} does not benefit from deviating to demanding 1 after the buyer makes an offer in an arbitrarily small neighborhood of $\min\{p_{\theta_{i^*}}, \theta^*_{i+1}\}$ it must be that

$$\max\{p_{\theta_{i^*}}, 1 + \theta_{i^*} - \theta_{i^*+1}\} - \theta_{i^*} \geq (1 - \theta_{i^*})\mathbb{E}[e^{-r\tau_b} | \theta > \theta_{i^*}],$$

Which is also sufficient to ensure that types $\theta < \theta_{i^*}$ do not benefit from deviating. On the other hand, by an argument analogous to the one given in the proof of Theorem 1, when ε and ν are small, the condition preventing type $\theta > \theta_{i^*}$ from deviating to $p \in \mathbf{P}_s$ which is arbitrarily close to 1 after the buyer offers p_b in a small neighborhood of $\min\{p_{\theta_{i^*}}, \theta^*_{i+1}\}$ and waiting for the buyer to concede is

$$(1 - \theta)\mathbb{E}[e^{-r\tau_b} | \theta > \theta_{i^*}] \geq \frac{\max\{p_{\theta_{i^*}}, 1 + \theta_{i^*} - \theta_{i^*+1}\} - \theta_{i^*}}{1 - \theta_{i^*}}(1 - \theta).$$

The two conditions combined give rise to the expected welfare loss in (4.2). The proof of Theorem 3 is completed by showing equilibrium existence, which has been established in Online Appendix A.

E Proof of Theorem 4

Let V_θ denote type θ's equilibrium payoff net of the cost of adopting technologies. Let $\pi \in \Delta(\Theta)$ be the seller's equilibrium adoption decision.

We begin by showing the second part of Theorem 4. Suppose that condition (4.3) is satisfied and moreover C satisfies

$$c_1 - c_j < \frac{c_1(1-\theta_1) - (\theta_n - \theta_1)(1 - \min\{p_{\theta_1}, \theta_n\})}{(\theta_n - \theta_1)(\min\{p_{\theta_1}, \theta_n\} - \theta_1)}(\theta_j - \theta_1), \quad \forall j = 2, ..., n-1, \tag{E.1}$$

$$c_1 \in \left(\max\left\{\frac{1}{2}, \frac{1-\theta_n}{1-\theta_1}\right\}(\theta_n - \theta_1), \theta_n - \theta_1 \right). \tag{E.2}$$

The set of production costs satisfying (E.1), and (E.2) is open (in $\mathbb{R}^{n-1}$, given that we are fixing $c_n = 0$). It is also non-empty, given that the set of conditions are satisfied by C such that c_1 satisfies (E.2), and $(c_2, ..., c_{n-1})$ are sufficiently close to c_1.

Conditions (E.1) and (E.2) combined imply that $j^o = 1$. We construct an equilibrium where the seller adopts (θ_{j^o}, c_{j^o}) with probability strictly less than one. In particular, consider the adoption strategy for the seller where in the limit

$$\pi(\theta_{j^o}) = \frac{p_{\theta_n} - \theta_n}{\min\{p_{\theta_{j^o}}, \theta_n\} - \theta_{j^o}}, \quad \pi(\theta_n) = 1 - \pi(\theta_{j^o}).$$

Note that (4.3) implies that $\pi(\theta_{j^o}) < 1$.

Consider the buyer's strategy at the bargaining stage that assigns probability arbitrarily close to one to:

- offer p_{θ_n} with probability $\rho \in (0,1)$, and offer $\min\{p_{\theta_{j^o}}, \theta_n\}$ with probability $1 - \rho$.

The exact value of $\pi(\theta_{j^o})$ when $\varepsilon > 0$ and $\nu > 0$ is chosen so as to ensure that the buyer is indifferent between offering $p_b \in \mathbf{P}_b \cap (p_{\theta_n} - \nu, p_{\theta_n} + \nu)$, and making the screening offer $p_b \in \mathbf{P}_b \cap (\min\{p_{\theta_{j^o}}, \theta_n\} - \nu, \min\{p_{\theta_{j^o}}, \theta_n\} + \nu)$. This is well-defined by analogous arguments as in the proof of Theorem 2.

Moreover, as argued in the proof of Theorem 3, the buyer assigns vanishing probability to any other offer. We also showed there that, after the buyer offers $p_b \in \mathbf{P}_b \cap (p_{\theta_n} - \nu, p_{\theta_n} + \nu)$, an agreement is reached with vanishing delay; and after the buyer offers $p_b \in \mathbf{P}_b \cap (\min\{p_{\theta_{j^o}}, \theta_n\} - \nu, \min\{p_{\theta_{j^o}}, \theta_n\} + \nu)$, type θ_{j^o} trades with vanishing delay at a price that converges to $\hat{p} \equiv 1 + \theta_{j^o} - \min\{p_{\theta_{j^o}}, \theta_n\}$, and type θ_n demands 1 and trades with limiting expected delay equal to $\frac{\hat{p} - \theta_{j^o}}{1 - \theta_{j^o}}$. Therefore, in order to ensure that the seller is indifferent between adopting θ_{j^o} and θ_n, in the limit as $\varepsilon \to 0$ and $\nu \to 0$, ρ has to satisfy

$$\rho(p_{\theta_n} - \theta_n) + (1-\rho)\frac{\hat{p} - \theta_{j^o}}{1 - \theta_{j^o}}(1 - \theta_n) = \rho(p_{\theta_n} - \theta_{j^o}) + (1-\rho)(\hat{p} - \theta_{j^o}) - c_{j^o}$$
$$\iff \rho = \frac{c_{j^o}(1 - \theta_{j^o}) - (\theta_n - \theta_{j^o})(\hat{p} - \theta_{j^o})}{(\theta_n - \theta_{j^o})(1 - \hat{p})},$$

Where again the exact value of ρ when $\varepsilon > 0$ and $\nu > 0$ is pinned down by the seller's indifference, which is well-defined due to continuity of the seller's payoff with respect to ρ. $\rho \in (0, 1)$ is ensured by (E.2).

It remains to verify that the seller does not benefit from deviating to an alternative technology (θ_j, c_j), with $j \notin \{1, n\}$. To do so, we use to following auxiliary result. Let $V_\theta(p_b)$ be type θ's continuation payoff after the buyer offers $p_b \in [0, 1]$.

Lemma E.1. *In any equilibrium, $V_\theta(p_b)$ is weakly decreasing in θ.*

Proof. Take $\theta, \theta' \in \Theta$ and suppose that $\theta' > \theta$. Let $p_b \in [0, 1]$ be any offer by the buyer, and let $p \in [0, 1]$ be an offer in the support of type θ''s equilibrium strategy in the bargaining game after the buyer offers p_b. Let $T \in \mathbb{R}_+ \cup \{+\infty\}$ be any concession time in the support of $\tau_s(\theta', p_b, p)$. The continuation payoff of type-θ' following the buyer's offer p_b can be written as

$$V_{\theta'}(p_b) = \int_0^T e^{-rt}\, d\tau_b(p_b, p)(p - \theta') + \tau_b(p_b, p)[T, +\infty]e^{-rT}(p_b - \theta') \leq$$
$$\int_0^T e^{-rt}\, d\tau_b(p_b, p)(p - \theta) + \tau_b(p_b, p)[T, +\infty]e^{-rT}(p_b - \theta).$$

Because type θ can always mimic type θ''s strategy, it must be that the right-hand-side of the inequality is weakly less than $V_\theta(p_b)$. Therefore, type θ's equilibrium payoff is higher following every offer by the buyer. □

Using Lemma E.1, we have that the payoff from deviating to technology j with $j \notin \{1, n\}$ is at most

$$\rho(p_{\theta_n} - \theta_1) + (1-\rho)(\hat{p} - \theta_{j^o}) - c_{j^o} = V_{\theta_{j^o}} - c_{j^o}.$$

This verifies that the seller's adoption decision is sequentially rational.

In order to show the third part of Theorem 4, suppose that $p_{\theta_{j^o}} < \theta_n$ and that

$$c_{j^o} \in \left(\frac{\theta_n - \theta_{j^o}}{2}, \theta_n - \theta_{j^o}\right). \tag{E.3}$$

We argue that $\pi(\theta_{j^o})$ is bounded away from 1 in any equilibrium. Suppose by contradiction that $\pi(\theta_{j^o}) > 1 - \eta$ for all $\eta > 0$. By Lemma C.1, all sellers with type $\theta > p_{\theta_{j^o}}$ will demand 1 after the buyer offers $p_{\theta_{j^o}}$. As in the proof of Theorem 2, it then follows that the buyer's optimal offer when $\pi(\theta_{j^o})$ is high enough assigns probability converging to 1 to an arbitrarily small neighborhood of $p_{\theta_{j^o}}$, and thus the seller's limiting equilibrium payoff is $p_{\theta_{j^o}} - \theta_{j^o} - c_{j^o}$. On the other hand, by deviating to θ_n (the fact that $c_{j^o} + \theta_{j^o} < \theta_n$ implies that $\theta_{j^o} \neq \theta_n$), demanding $p_s \in \mathbf{P}_s$ arbitrarily close to 1 and waiting for concession after the buyer offers $p_b \in \mathbf{P}_b \cap (p_{\theta_{j^o}} - \nu, p_{\theta_{j^o}} + \nu)$, the seller can secure a limiting payoff of $\frac{1-\theta_n}{2}$. This is strictly better than his limiting equilibrium payoff $p_{\theta_{j^o}} - \theta_{j^o} - c_{j^o}$, under condition (E.3). This immediately implies that the equilibrium must feature strictly positive delay. If not, letting θ_j with $j \neq j^o$ be a technology in the support of the seller's adoption strategy, we have that $V_{\theta_j} - c_j = \mathbb{E}[p] - \theta_j - c_j < \mathbb{E}[p] - \theta_{j^o} - c_{j^o} \leq V(\theta) - c_{j^o}$, where $\mathbb{E}[p]$ is the equilibrium expected price. This contradicts the seller's willingness to adopt θ_j in equilibrium.

This establishes the third part of Theorem 4. The argument for the first part is given in the main text (Section 4). The existence of equilibrium has been established in Online Appendix B.

F Proofs of Corollary 4

Let θ_1, θ_2, c and $\hat{\theta}_1$ satisfy the conditions in Corollary 4. Denote $\kappa(\theta_1) \equiv \min\{p_{\theta_1}, \theta_2\}$. Let π, γ be the equilibrium adoption probability and the expected delay under $(\theta_1, \theta_2, c, \varepsilon, \nu)$, and $\hat{\pi}, \hat{\gamma}$ be analogously defined under $(\hat{\theta}_1, \theta_2, c, \varepsilon, \nu)$. Applying our characterization in Theorem 2, there are two cases to consider:

Case 1. If either $\theta_2 - \theta_1 > 1 - \theta_2$, or if $\theta_2 - \theta_1 < 1 - \theta_2$ and players coordinate on the inefficient equilibrium, then for ν and ε sufficiently small, Theorem 2 and equations (C.2) and (C.4) in Appendix C imply that $\pi \in (\pi^*(\theta_1) - \eta, \pi^*(\theta_1) + \eta)$, and $\gamma \in (\gamma^*(\theta_1) - \eta, \gamma^*(\theta_1) + \eta)$, where

$$\pi^*(\theta_1) \equiv \frac{p_{\theta_2} - \theta_2}{\kappa(\theta_1) - \theta_1}, \quad \gamma^*(\theta_1) \equiv \frac{(2\kappa(\theta_1) + \theta_2 - 2\theta_1 - 1)(\theta_2 - \theta_1 - c)}{2(\kappa(\theta_1) - \theta_1)(\theta_2 - \theta_1)}.$$

The conditions on $\hat{\theta}_1$ imply that, in any equilibrium, $\hat{\pi} \in (\pi^*(\hat{\theta}_1) - \eta, \pi^*(\hat{\theta}_1) + \eta)$ and $\hat{\gamma} \in (\gamma^*(\hat{\theta}_1) - \eta, \gamma^*(\hat{\theta}_1) + \eta)$. The result then follows from the fact that $\pi^*(\theta_1) > \pi^*(\hat{\theta}_1)$ and $\gamma^*(\theta_1) < \gamma^*(\hat{\theta}_1)$.

Case 2. If $\theta_2 - \theta_1 < 1 - \theta_2$ and players coordinate on the efficient equilibrium, then $\pi > 1 - \eta$ and $\gamma < \eta$. On the other hand, $\hat{\pi}$ and $\hat{\gamma}$ are given as in Case 1, and the result then follows immediately.

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