Lecture 6: Coasian Bargaining

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Spring Quarter, 2023

Bargaining: Overview

Two approaches to study bargaining:

- Cooperative approach (Nash, Shapley)
- Non-cooperative approach (Rubinstein)

Several key issues in the bargaining literature:

- How to incorporate incomplete information?
- Which insights are robust to different bargaining protocols?

Today: Revisit some classic models and results.

Next few lectures: Bargaining with reputation concerns.

Review: Rubinstein Bargaining Game

Two players decide how to divide a dollar.

- Time: $t = 0, \Delta, 2\Delta, \ldots$. Player i's discount factor $\delta_i \equiv e^{-r_i \Delta}$.

  Interpret Δ as period length and r_i as player i's interest rate.

In period $2k\Delta$, P1 makes an offer $\alpha_1 \in [0, 1]$.

- If P2 accepts, then the game ends.

  Payoffs: $\alpha_1 \delta_1^{2k}$ for player 1, and $(1 - \alpha_1)\delta_2^{2k}$ for player 2.

- If P2 rejects, then the game moves on to the next period.

In period $(2k + 1)\Delta$, P2 makes an offer $\alpha_2 \in [0, 1]$.

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Rubinstein's Theorem

Theorem: Rubinstein Bargaining Game

There exists a unique subgame perfect equilibrium.

On the equilibrium path, an agreement is reached in period 0.

Player 1*'s payoff is* $\frac{1-\delta_2}{1-\delta_1\delta_2}$. *Player* 2*'s payoff is* $\frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}$.

As the bargaining friction vanishes, i.e., $\Delta \to 0$

Player 1's payoff converges to:

$$\lim_{\Delta\to 0} \frac{1-e^{-r_2\Delta}}{1-e^{-(r_1+r_2)\Delta}} = \frac{r_2}{r_1+r_2}.$$

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Issues with Rubinstein Bargaining

1. Division of surplus is sensitive to the *bargaining protocol*.
   - What if P1 makes 2 offers in a row and then P2 makes 1 offer?
   - What if P1 makes 5 offers in a row and then P2 makes 7 offers?

We will come back to this issue in the next lecture.

2. How to introduce incomplete information?
   - The classic paper: Gul, Sonnenschein and Wilson (1986 JET).

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Model

- Time $t = 0, \Delta, 2\Delta, 3\Delta...$
- A buyer and a seller whose cost is 0.

  Common discount factor $\delta \equiv e^{-r\Delta}$, with $r > 0$.
- Buyer's value is v, with cdf $F : [\underline{v}, \overline{v}] \to [0, 1]$ with $\underline{v} \geq 0$.

  The results hold both for continuous F and discrete F.

- In period $t\Delta$, the seller makes an offer p_t,

  if the buyer accepts, then trade happens at price p_t and the game ends,

  if the buyer rejects, then the game moves on to period $(t + 1)\Delta$.
- **Important:** The uninformed player makes all the offers.

- If trade happens in period $t\Delta$ at price p, then the buyer's payoff is $(v - p)\delta^t$, and the seller's payoff is $p\delta^t$.

Skimming Property

Observation: It is more costly for high-value types to wait.

Lemma: Skimming Property

Suppose the buyer with value v' accepts price p_t at h^t with positive prob, he accepts price p_t at h^t with probability 1 when his value is $v'' > v'$.

If type v' buyer accepts p_t at h^t, then $v' - p_t \geq \delta U(v', h^t, p_t)$.

Since type v' can imitate the strategy of type v'' and vice versa,

$$0 < U(v'', h^t, p_t) - U(v', h^t, p_t) \leq v'' - v'.$$

If type v'' does not accept, then $v'' - p_t \leq \delta U(v'', h^t, p_t)$, we have:

$$\delta(v'' - v') \geq \delta U(v'', h^t, p_t) - \delta U(v', h^t, p_t) \geq (v'' - p_t) - (v' - p_t) = v'' - v'.$$

This leads to a contradiction.

Skimming Property

Lemma: Skimming Property

Suppose the buyer with value v' accepts price p_t at h^t with positive prob, he accepts price p_t at h^t with probability 1 when his value is $v'' > v'$.

The skimming property simplifies the search for equilibria:

- At every history, there exists $v^* \in [\underline{v}, \overline{v}]$ s.t. the buyer hasn't accepted the offer if and only if $v \leq v^*$.
- The seller's posterior belief is a truncation of his prior.

This is true in Coasian bargaining games but is not necessarily true in other dynamic games.

- Be careful when you use monotone methods in dynamic games since players also care about their continuation values.

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Lower Bound on Offered Price

Lemma: Lower Bound on Offered Prices

At every history of every equilibrium, the seller's offer is at least $\underline{v}$.

Let p^* be the supremum price s.t. all types will accept at all histories.

- The seller will not offer any price strictly less than p^*.

If $p^* < \underline{v}$, suppose the seller offers $p' \in \left(p^*, (1-\delta)\underline{v} + \delta p^*\right)$.

Since p^* is the lowest price the buyer can get tomorrow, the lowest type prefers to accept p' today instead of waiting for a lower price tomorrow.

The skimming property implies that all types want to accept p' today, which contradicts the definition of p^*.

Lower Bound on Offered Price

Lemma: Lower Bound on Offered Prices

At every history of every equilibrium, the seller's offer is at least $\underline{v}$.

Implication: Once the seller offers $\underline{v}$, all types of the buyer will accept.

Gap *vs* No Gap

The game's equilibrium outcome hinges on whether there is a gap between the buyer's lowest possible value and the seller's cost.

- The Gap Case: $\underline{v} > 0$.
- The No-Gap Case: $\underline{v} = 0$.

The Gap Case: $\underline{v} > 0$

Theorem: Coase Conjecture with Gap

Fix $r > 0$, $\underline{v} > 0$, and F. For every $\varepsilon > 0$, there exists $\overline{\Delta} > 0$ such that when $\Delta < \overline{\Delta}$, in every equilibrium of the bargaining game,

1. *Players reach an agreement before time ε with prob 1.*
2. *All trading prices are below $\underline{v} + \varepsilon$.*

The uninformed seller makes all the offers.

- He has all the bargaining power.

  However, he receives his lowest possible profit under incomplete info.

There is almost no inefficiency as the bargaining friction vanishes.

Coase Conjecture: Immediate Agreement and Low Prices

The seller has all the bargaining power, but he receives his lowest possible profit under incomplete info.

- Why? The seller faces a lack-of-commitment problem.
- He cannot commit not to lower the price tomorrow after learning that the buyer rejects his offer today.
- His future self competes with his current self a la Bertrand.

  This problem exacerbates when Δ becomes smaller.

What if the seller can commit to a price ex ante?

$$\max_{v^* \in [\underline{v}, \overline{v}]} \left\{ (1 - F(v^*))v^* \right\}$$

Take the FOC,

$$v^* = \frac{1 - F(v^*)}{f(v^*)}.$$

Proof: Two-Type Case

We prove the result when there are two types $v \in \{\overline{v}, \underline{v}\}$.

- The prior belief is $v = \overline{v}$ with probability $F \in (0, 1)$.

The low type buyer accepts if and only if the seller offers $\underline{v}$.

Let F_t be the ex ante prob of the following event:

- The seller's type is high and remains in the market at time $t\Delta$.

We know that $F_0 = F$ and F_0 decreases over time.

Strict incentive to offer $\underline{v}$ when $F_t \approx 0$

Lemma

There exists $\underline{F} > 0$ such that the seller strictly prefers to offer $\underline{v}$ at time $t\Delta$ when $F_t < \underline{F}$.

The seller's payoff from offering $\underline{v}$ is

$$\underline{v}(1 - F + F_t).$$

The seller's payoff from offering anything greater than $\underline{v}$ is at most

$$\delta\underline{v}(1 - F) + F_t\overline{v}.$$

Since $\underline{v} > 0$, the former is strictly greater than the latter when $F_t \approx 0$.

Proof: Bargaining Ends in Finite Time

Lemma

Fix $F_0 \in (0,1)$. There exist $t \in \mathbb{N}$ and $w > 0$ such that for every $s \in \mathbb{N}$ and $F_s > 0$, we have $F_{t+s} \leq \max\{0, F_s - w\}$.

Proof: If $F_{t+s} > \max\{0, F_s - w\}$, then the seller's payoff at time $s\Delta$ is at most:

$$\bar{v}w + \delta^t \left\{ \underline{v}(1 - F_0) + \bar{v}(F_s - w) \right\}.$$

The seller's payoff from offering $\underline{v}$ at time $s\Delta$ is $\underline{v}(1 - F_0 + F_s)$.

When $t \to +\infty$ and $w \to 0$, we have

$$\underline{v}(1 - F_0 + F_s) > \bar{v}w + \delta^t \left\{ \underline{v}(1 - F_0) + \bar{v}(F_s - w) \right\}.$$

The choice of t and w depend only on F_0 and is uniform for all $F_s \in [0, F_0]$.

Lemma

There exists $T \in \mathbb{N}$ such that the bargaining game ends before the Tth period, i.e., $F_T = 0$ for some T.

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Proof: If $F_{t+s} > \max\{0, F_s - w\}$, then the seller's payoff at time $s\Delta$ is at most:

$$\bar{v}w + \delta^t \Big\{ \underline{v}(1 - F_0) + \bar{v}(F_s - w) \Big\}.$$

The seller's payoff from offering $\underline{v}$ at time $s\Delta$ is $\underline{v}(1 - F_0 + F_s)$.

When $t \to +\infty$ and $w \to 0$, we have

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The choice of t and w depend only on F_0 and is uniform for all $F_s \in [0, F_0]$.

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There exists $T \in \mathbb{N}$ such that the bargaining game ends before the Tth period, i.e., $F_T = 0$ for some T.

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Backward Induction: Small Price Difference

Let $T\Delta$ be the last period of the bargaining game.

The seller's price in the Tth period is $p_T = \underline{v}$.

In the $T-1$th period, the high type prefers accepting p_{T-1} to waiting for p_T:

$$\bar{v} - p_{T-1} \geq \delta(\bar{v} - p_T),$$

which yields:

$$p_{T-1} \leq (1-\delta)\bar{v} + \delta p_T.$$

Similarly, the high type is indifferent between accepting p_{T-2} and waiting for p_{T-1}:

$$p_{T-2} = (1-\delta)\bar{v} + \delta p_{T-1}.$$

Hence, for every $t < T$, we have

$$p_t - p_{t+1} \leq (1-\delta)(\bar{v} - p_{t+1}) \leq (1-\delta)(\bar{v} - \underline{v}),$$

i.e., the price difference across periods must be small enough.

Backward Induction: Seller's Incentive

In the $T-1$th period, the seller prefers offering p_{T-1} to offering p_T:

$$(F_{T-1}-F_T)p_{T-1}+\delta(1-F_0+F_T)p_T \geq (F_{T-1}+1-F_0)p_T.$$

or equivalently

$$\underbrace{(F_{T-1}-F_T)(p_{T-1}-p_T)}_{\text{benefit from offering } p_{T-1}} \geq \underbrace{(1-\delta)(F_T+1-F_0)p_T}_{\text{cost of delay}}.$$

Recall that

$$p_{T-1}-p_T \leq (1-\delta)(\bar{v}-p_T),$$

we have:

$$F_{T-1}-F_T \geq \frac{(F_T+1-F_0)p_T}{\bar{v}-p_T} \geq \frac{(1-F_0)p_T}{\bar{v}-p_T}.$$

The fraction of high type who trades in the $T-1$th period must be large enough in order to compensate for the loss of delaying $\underline{v}(1-F_0)$.

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Similarly, we can find uniform lower bounds on $F_{T-2} - F_{T-1}$, $F_{T-3} - F_{T-2}$,..., which depend only on $1 - F_0, \underline{v}, \overline{v}$, but not on δ and Δ.

This suggests that $F_T, F_{T-1}, \ldots$ reaches F_0 in bounded number of periods.

- This leads to an upper bound on T that does not depend on Δ

As $\Delta \to 0$, we have $T\Delta \to 0$ and $p_0 \to \underline{v}$.

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The No-Gap Case

The Gap Case ($\underline{v} > 0$):

- The seller cannot resist the temptation to lower prices, and his profit is arbitrarily close to $\underline{v}$ as $\Delta \to 0$.
- Why? Offering $\underline{v}$ and obtaining $\underline{v}$ is tempting for the seller.

The No-Gap Case ($\underline{v} = 0$):

- There is no benefit from serving the lowest type, which helps the seller to commit to high prices.

The No-Gap Case

An equilibrium is stationary if the buyer's strategy depends on the history only through the current-period offer and her value.

Theorem: Coase Conjecture without Gap

Suppose $\underline{v} = 0$. *For every* $\varepsilon > 0$, *there exists* $\overline{\Delta} > 0$ *such that when* $\Delta < \overline{\Delta}$, *in every stationary equilibrium of the bargaining game,*

1. *Players reach an agreement before time* ε *with prob* 1.
2. *All trading prices are below* ε.

Ausubel and Deneckere (1989): Folk theorem in the No-Gap Case.

- Any payoff between 0 and the commitment payoff can arise in Perfect Bayesian equilibrium.
- The seller can credibly commit to delay trading with the lowest type.

Bargaining with Endogenous Investment

A model with endogenous and unobservable investment.

- Time $t = 0, \Delta, 2\Delta, 3\Delta...$
- Common discount factor $\delta \equiv e^{-r\Delta}$.
- The seller's production cost is 0. She makes all the offers.

- Before bargaining starts, the buyer decides whether to invest.

  If he does not invest, his value is $\underline{v}$.
  If he invests, then he pays a cost of c and his value becomes $\overline{v}$.

- Suppose $\underline{v} > 0$ and $c \in (0, \overline{v} - \underline{v})$,

  i.e., investment is costly and is socially efficient.

- The seller cannot observe the buyer's investment.

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Two Useful Benchmarks

1. Observable investment: The buyer never invests.
   - Equilibrium social welfare is $\underline{v}$.
2. Unobservable investment but take-it-or-leave-it offer:
   - The buyer invests with probability $\underline{v}/\overline{v}$.
   - The seller is indifferent between offering $\underline{v}$ and offering $\overline{v}$.
   - The buyer's equilibrium payoff is 0.
   - The seller's equilibrium payoff is $\underline{v}$.
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These inefficiencies are caused by the hold-up problem.

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Unobservable Investment and Coasian Bargaining

What if investment is unobserable and offers are frequent, i.e., $\Delta \to 0$?

- Recall that $c \in (0, \overline{v} - \underline{v})$.

In equilibrium, the buyer cannot invest with probability 1.

- Otherwise, his value is $\overline{v}$ for sure.
- The seller will charge him $\overline{v}$, so he has no incentive to invest.

In equilibrium, the buyer cannot invest with probability 0.

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Unobservable Investment and Coasian Bargaining

Hence, the buyer invests with interior probability $F \in (0, 1)$.

- He must be indifferent between investing and not investing.

A possible line of reasoning:

- The seller will never offer anything below $\underline{v}$.
- The buyer will accept $\underline{v}$ if the seller offers it.
- The Coase conjecture in the gap case implies that as $\Delta \to 0$, the seller's offer will fall to $\underline{v}$ within ε unit of time.
- The buyer's maximization problem at the investment stage:
  - If he does not invest, he gets 0.
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The buyer has a strict incentive to invest, which is a contradiction.

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The buyer has a strict incentive to invest, which is a contradiction.

Unobservable Investment and Coasian Bargaining

Hence, the buyer invests with interior probability $F \in (0, 1)$.

- He must be indifferent between investing and not investing.

A possible line of reasoning:

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The Coase Conjecture Revisited

What did GSW show?

Theorem: Coase Conjecture with Gap

Fix F. For every $\varepsilon > 0$, there exists $\overline{\Delta} > 0$ such that when $\Delta < \overline{\Delta}$, in every equilibrium of the bargaining game,

1. *Players reach an agreement before time ε with prob 1.*
2. *All trading prices are below $\underline{v} + \varepsilon$.*

What's going on in the game with endogenous investment?

- The value distribution F is endogenous, and hence, it may depend on the parameters such as Δ.

When $\Delta \to 0$, the prob that F assigns to $\underline{v}$ may also go to 0.

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Unobservable Investment and Coasian Bargaining

Let $\alpha \equiv \frac{c}{\overline{v}-\underline{v}} \in (0,1)$.

- The low type's payoff must be 0.
- The high type's payoff must be $\alpha(\overline{v}-\underline{v})$ s.t. the buyer is indifferent between investing and not investing.

Recall that the seller's offered prices must satisfy:

$$p_t - p_{t+1} = (1-\delta)(\overline{v} - p_{t+1}).$$

The price in the Tth period must be $\underline{v}$.

Type $\overline{v}$ must find it optimal to accept $p_{T-1} \approx \underline{v}$ in the $T-1$th period, which gives:

$$\alpha(\overline{v}-\underline{v}) \approx e^{-r\Delta T}(\overline{v}-\underline{v})$$

Hence, $e^{-r\Delta T} \approx \alpha$, i.e., ΔT does not converge to 0 as $\Delta \to 0$ since T depends on F, which depends endogenously on Δ.

How can T explode as $\Delta \to 0$?

Recall: The seller's incentive to offer p_{T-1} instead of p_T in the Tth period.

$$F_{T-1} - F_T \geq \frac{(1-F_0)p_T}{\overline{v} - p_T}.$$

Similarly, $F_{t-1} - F_t$ is bounded below by a linear function of $1 - F_0$.

Therefore, $F_{T-1} - F_T$ can be very small only when $1 - F_0 \to 0$, i.e., when the low type is sufficiently unlikely.

Hence, the buyer's investment prob $\to 1$ as $\Delta \to 0$.

Lesson: Unobservable investment and frequent offers lead to almost efficient investment.

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Expected Delay & Social Welfare

The seller prefers p_{T-1} to p_T at time $(T-1)\Delta$:

$$F_{T-1} - F_T \geq \frac{F_T + (1-F)}{\bar{v} - p_T} p_T.$$

The seller prefers p_{T-2} to p_{T-1} at time $(T-2)\Delta$:

$$F_{T-2} - F_{T-1} \geq (F_{T-1} - F_T)\left\{\delta + \frac{p_{T-1}}{\bar{v} - p_{T-1}}\right\}$$

... ...

The seller prefers p_{t-1} to p_t at time $(t-1)\Delta$:

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Since p_t decreases in t, when Δ is close to 0, $\exists\, \gamma > 1/\delta$ s.t.

$$\delta + \frac{p_t}{\bar{v} - p_t} > \gamma \text{ for every } t \in \mathbb{N}.$$

The fraction of buyer accepting the seller's offer declines exponentially over time, at a rate bounded away 1, and this bound is uniform for all Δ.

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$$e^{-r\Delta T} = \delta^T \approx \alpha.$$

The previous slide implies that there exists $\gamma > 1$ s.t.

$$F_{t-1} - F_t \geq \gamma(F_t - F_{t+1}),$$

and

$$\sum_{t=1}^{T}(F_{t-1} - F_t) \approx 1.$$

As $\Delta \to 0$ (or $\delta \to 1$), since $F_{t-1} - F_t$ decays at a rate higher than $1/\delta$,

- For every $\varepsilon > 0$, let $T_{\varepsilon,\Delta} \equiv \lfloor \varepsilon/\Delta \rfloor$, $\exists\, \overline{\Delta} > 0$ such that $\forall\, \Delta < \overline{\Delta}$:

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Lesson: Players trade before time ε with prob close to 1.

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Coase Conjecture: Negative Selection

Coase conjecture: The remaining types are undesirable (e.g., types with low values).

The optimal price when the seller can commit satisfies:

$$v^* = \frac{1 - F(v^*)}{f(v^*)}.$$

When the distribution is truncated at $\widehat{v}$, s.t. the new distribution G satisfies:

$$G(v) = \frac{F(v)}{F(\widehat{v})} \quad \text{and} \quad g(v) = \frac{f(v)}{F(\widehat{v})}.$$

Since

$$\frac{1 - G(v)}{g(v)} = \frac{1 - \frac{F(v)}{F(\widehat{v})}}{\frac{f(v)}{F(\widehat{v})}} = \frac{F(\widehat{v}) - F(v)}{f(v)} < \frac{1 - F(v)}{f(v)},$$

the optimal monopoly price decreases, so the seller faces a lack-of-commitment problem.

Positive Selection

What if the remaining types are high types?

- You charge a price, and those who stay in the game must keep paying the price you charge.

Solving for the ex ante optimal price for the seller:

$$\max_{v^* \in [\underline{v}, \bar{v}]} \left\{ (1 - F(v^*))v^* \right\}$$

Take the FOC,

$$v^* = \frac{1 - F(v^*)}{f(v^*)}.$$

After the types below v^* leave, what will happen?

Tirole (2016): Positive Selection

The ex ante optimal price satisfies:

$$v^* = \frac{1 - F(v^*)}{f(v^*)}.$$

After the types below v^* leave, the truncated distribution H satisfies:

$$H(v) = \frac{F(v) - F(v^*)}{1 - F(v^*)} \quad \text{and} \quad h(v) = \frac{f(v)}{1 - F(v^*)}.$$

Since

$$\frac{1 - H(v)}{h(v)} = \frac{1 - \frac{F(v) - F(v^*)}{1 - F(v^*)}}{\frac{f(v)}{1 - F(v^*)}} = \frac{1 - F(v)}{f(v)}.$$

The optimal price remains the same, i.e., the seller faces no lack-of-commitment problem.

Application: Conversion Game

After the Muslim conquest of Egypt, the Muslim rulers levied a poll tax for non-Muslims.

- You pay a lump sum every year if you are a copt (i.e., not a Muslim).
- You can avoid paying this tax if you convert, which is irreversible.

Presumably, a copt is more likely to convert if they are poorer.

- The remaining copts should be richer (or more religious).

Empirical findings:

- The poll tax does not increase over time.
- Most of the conversion happened in the first few centuries.

Question: Why doesn't the tax increase over time?

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Next Few Lectures

Abreu and Gul (2000): The paper is NOT easy to read.

- Part 1: War of attrition with one commitment type.
- Part 2: War of attrition with multiple commitment types.
- Part 3: Bargaining game with frequent offers.

Kambe (1999): An alternative approach to reputational bargaining.

Compte and Jehiel (2002): Reputational bargaining with outside options.

Abreu and Pearce (2007): Bargaining with contracts.

Alison will present Che and Sakovics (2004):

- A dynamic theory of hold-up.