# Lecture 14: Behavioral Social Learning

Harry PEI
Department of Economics, Northwestern University

Spring Quarter, 2021

# Motivation

Social learning with homogenous preferences and rational Bayesian agents.

- Agents' actions are asymptotically efficient if

  their private signals are unbounded (Smith and Sorensen),

  or their action space is sufficiently rich (Lee).

Critiques of rational social learning models.

- Requires too much sophistication (e.g., double-counting problems).

Attempts to relax the rationality assumption.

- DeGroot (1974), Golub and Jackson (2010), Molavi et al. (2012,2018): Non-Bayesian rule-of-thumb learning rules.
- Eyster and Rabin: Agents are Bayesian but fails to recognize the double-counting problem when aggregating different sources of info.

# Social Learning in a Doubly Rich Setting

Social learning with unbounded signals and a continuum of actions.

- Time $t = 1, 2, \ldots$ One agent arriving in each period.
- State $\omega \in \{0, 1\}$, equally likely.
- Action of agent $t$: $a_t \in [0, 1]$.
- Agent $t$ observes $s_t$ and $\{a_1, \ldots, a_{t-1}\}$ and then chooses $a_t$.
- Agent $t$'s payoff is $-(a_t - \omega)^2$, so $a_t \equiv \mathbb{E}[\omega | s_t, a_1, \ldots, a_{t-1}]$.
- Agent $t$'s private signal $s_t \sim G(\cdot|\omega)$, conditionally independent.
  - ↪ The signal structure can be represented by the distribution over private beliefs conditional on $\omega$, i.e., $F_\omega \in \Delta[0, 1]$.
  - ↪ We assume that for every $\omega$, $F_\omega(0) = 0$, $F_\omega(1) = 1$, $F_\omega$ is differentiable, and has continuous and positive density $f_\omega$.
  - ↪ Unbounded private beliefs, no perfectly revealing signal.
- Let $p_t \equiv \mathbb{E}[\omega | s_t]$.

  The log likelihood ratio (LLR) is $l_t \equiv \log \frac{p_t}{1-p_t}$.

# Benchmark: Sophisticated Bayesian Social Learning

Suppose agents are Bayesian and rational (i.e., fully sophisticated).

**Theorem: Smith and Sorensen (2000)**

*Suppose the agents' private signals are unbounded, then conditional on every* $\omega \in \{0,1\}$, $a_t \to \omega$ *almost surely.*

**Theorem: Lee (1993)**

*In environments where* $a_t \in [0,1]$, *then conditional on every* $\omega \in \{0,1\}$, $a_t \to \omega$ *almost surely.*

Sophisticated Bayesian agents' actions are asymptotically efficient.

# Naive Bayesian Agents

Form of naivete: Best response trailing naive inference (BRTNI)

- Each player best responds to the belief that each of her predecessors follows their own signal.
- This reasoning neglects the fact that their predecessors also make inferences from their own predecessors' actions.

What are the beliefs of these naive players?

- Player 1's posterior log likelihood ratio is $l_1$.
- Player 2's posterior log likelihood ratio is $l_1 + l_2$.
- Player 3's posterior log likelihood ratio is $2l_1 + l_2 + l_3$.

  If P3 is rational, then they would ignore P1's action and their posterior should be $l_1 + l_2 + l_3$.
- Player 4's posterior log likelihood ratio is $4l_1 + 2l_2 + l_3 + l_4$.

... ... ...

# Naive Bayesian Agents

What are players' beliefs if they engage in BRTNI?

- Player 1's posterior log likelihood ratio is $l_1$.
- Player 2's posterior log likelihood ratio is $l_1 + l_2$.
- Player 3's posterior log likelihood ratio is $2l_1 + l_2 + l_3$.
- Player 4's posterior log likelihood ratio is $4l_1 + 2l_2 + l_3 + l_4$.

... ...

- Player $n$'s posterior log likelihood ratio is $l_n + \sum_{\tau=1}^{n-1} 2^{n-1-\tau} l_\tau$.

Players over-weight the private signals of early players.

- P1's private signal $s_1$ should have weight $1/t$ in Player t's belief.
- When players are naive, $s_1$ has weight $1/2$ in all players' beliefs.

It is hard to correct early players' mistakes even with rich action spaces and unbounded private signals (will affect the asymptotic outcome).

# Inefficiencies in All Periods

**Theorem**

*When players are naive, for every $r < 1$, there exists $\delta > 0$ such that*

$$\Pr\left(a_t > r \text{ for all } t \in \mathbb{N} \middle| \omega = 0\right) > \delta.$$

Naive players' actions are bounded away from efficiency in *all periods* with positive probability.

- Intuition: Since naive agents over-weigh the signals of earlier agents, it is hard to correct earlier players' mistakes.

Next: How the proof incorporates this intuition.

# Proof

**Theorem**

*When players are naive, for every $r < 1$, there exists $\delta > 0$ such that*

$$\Pr\left(a_t > r \textit{ for all } t \in \mathbb{N} \middle| \omega = 0\right) > \delta.$$

Let $L_t$ be the log likelihood ratio after a naive player observes all predecessors' actions but before observing their own private signal.

- Given that the prior is uniform, $\log \frac{a_t}{1-a_t} = L_t + l_t$.

One can show by induction that $L_n = 2L_{n-1} + l_{n-1}$ for every $n \in \mathbb{N}$.

- Public LLR in period $n-1$: $\sum_{\tau=1}^{n-2} \log \frac{a_\tau}{1-a_\tau}$.
- Player $n-1$'s action satisfies $\log \frac{a_{n-1}}{1-a_{n-1}} = L_{n-1} + l_{n-1}$.
- Public LLR in period $n$ is $L_{n-1} + \log \frac{a_{n-1}}{1-a_{n-1}} = 2L_{n-1} + l_{n-1}$.

# Proof

**Theorem**

*When players are naive, for every $r < 1$, there exists $\delta > 0$ such that*

$$\Pr\left(a_t > r \text{ for all } t \in \mathbb{N} \middle| \omega = 0\right) > \delta.$$

When players are naive, $L_n$ satisfies $L_n = 2L_{n-1} + l_{n-1}$.

- Pick an arbitrary $r \in (1/2, 1)$, let $R \equiv \log \frac{r}{1-r} > 0$.
- **Question:** What if $L_2 > 3R$ and $l_t \geq -tR$ for every $t \in \mathbb{N}$?

  Since $L_2 = l_1$, LLR of P1's action is greater than $3R$.

  Since P2's LLR is $L_2 + l_2$ and $l_t \geq -2R$, we have $\log \frac{a_2}{1-a_2} > R$.

  Since $L_3 = 2L_2 + l_2 \geq 6R - 2R = 4R$ and $l_3 \geq -3R$, $\log \frac{a_3}{1-a_3} > R$.

**Lemma**

*If $L_2 > 3R$ and $l_t \geq -tR$ for every $t \in \mathbb{N}$, then $L_n > (n+1)R$ and $\log \frac{a_n}{1-a_n} > R$ for every $n \geq 2$ (which implies that $a_t > r$).*

# Proof

**Theorem**

*When players are naive, for every $r < 1$, there exists $\delta > 0$ such that*

$$\Pr\left(a_t > r \text{ for all } t \in \mathbb{N} \middle| \omega = 0\right) > \delta.$$

**Lemma**

*If $L_2 > 3R$ and $l_t \geq -tR$ for every $t \in \mathbb{N}$, then $L_n > (n+1)R$ and $\log \frac{a_n}{1-a_n} > R$ for every $n \geq 2$.*

Intuition behind the lemma: Suppose $\omega = 0$,

- $L_2 \geq 3R$ means that P1's signal is in favor of $\omega = 1$.
- $l_t \geq -tR$: mistake of P1's signal becomes harder to correct over time.

  Why? Since P1's signal carries a large weight, $l_t$ needs to be sufficiently negative in order to drive $\log \frac{a_t}{1-a_t}$ below $R$.

# Proof

**Theorem**

*When players are naive, for every $r < 1$, there exists $\delta > 0$ such that*

$$\Pr\left(a_t > r \textit{ for all } t \in \mathbb{N} \middle| \omega = 0\right) > \delta.$$

**Lemma**

*If $L_2 > 3R$ and $l_t \geq -tR$ for every $t \in \mathbb{N}$, then $L_n > (n+1)R$ and $\log \frac{a_n}{1-a_n} > R$ for every $n \geq 2$.*

Since $L_2 > 3R$ with positive prob conditional on $\omega = 0$, we only need to show that event

$$\left\{ l_t \geq -tR \text{ for every } t \in \mathbb{N} \right\}$$

occurs with prob bounded away from 0 conditional on $\omega = 0$.

# Proof

We need to show that

$$\Pr\left\{ l_t \geq -tR \text{ for every } t \in \mathbb{N} \middle| \omega = 0 \right\} > 0.$$

By Markov inequality,

$$\Pr\left\{ l_t < -tR \middle| \omega = 0 \right\} \leq \Pr\left\{ l_t^2 \geq (tR)^2 \middle| \omega = 0 \right\} \leq \frac{1}{(tR)^2}\mathbb{E}[l_t^2|\omega = 0].$$

Bound the value of $Q \equiv \mathbb{E}[l_t^2|\omega = 0]$ from above:

$$\mathbb{E}[l_t^2|\omega = 0] = \int_0^1 \left(\log \frac{s}{1-s}\right)^2 f_0(s)ds \leq \underbrace{\max\{f_0(s)|s \in [0,1]\}}_{\text{a bounded number}} \underbrace{\int_0^1 \left(\log \frac{s}{1-s}\right)^2 ds}_{=\pi^2/3}$$

Hence, there exists a bounded $Q$ such that for every $t \in \mathbb{N}$,

$$\Pr\left\{ l_t \geq -tR \middle| \omega = 0 \right\} \geq 1 - \frac{Q}{(tR)^2}.$$

# Proof

We need to show that

$$\Pr\left\{ l_t \geq -tR \text{ for every } t \in \mathbb{N} \middle| \omega = 0 \right\} > 0.$$

We have shown that there exists $Q > 0$ such that for every $t \in \mathbb{N}$,

$$\Pr\left\{ l_t \geq -tR \middle| \omega = 0 \right\} \geq 1 - \frac{Q}{(tR)^2}.$$

Let $\tau \in \mathbb{N}$ be such that $1 - \frac{Q}{(tR)^2} > 0$ for every $t > \tau$.

- We know that $\Pr\left\{ l_t \geq -tR \text{ for every } t \leq \tau \middle| \omega = 0 \right\} > 0$.
- We need to show that $\Pr\left\{ l_t \geq -tR \text{ for every } t > \tau \middle| \omega = 0 \right\} > 0$.

# Proof

We have shown that for every $t \in \mathbb{N}$,

$$\Pr\left\{ l_t \geq -tR \middle| \omega = 0 \right\} \geq 1 - \frac{Q}{(tR)^2}.$$

We need to show that $\Pr\left\{ l_t \geq -tR \text{ for every } t > \tau \middle| \omega = 0 \right\} > 0$.

$$\Pr\left\{ l_t \geq -tR \text{ for every } t > \tau \middle| \omega = 0 \right\} \geq \Pi_{t=\tau+1}^{+\infty} \left( 1 - \frac{Q}{(tR)^2} \right)$$

$$= \underbrace{\exp\left\{ \sum_{t>\tau} \log\left(1 - \frac{Q}{(tR)^2}\right) \right\} \geq \exp\left\{ \sum_{t>\tau} -\frac{Q}{(tR)^2} \right\}}_{\text{uses inequality } \log(1-x) \geq -x} \geq \exp\left( -\frac{Q\pi}{6R^2} \right)$$

To summarize, both $\Pr\left\{ l_t \geq -tR \text{ for every } t > \tau \middle| \omega = 0 \right\}$ and $\Pr\left\{ l_t \geq -tR \text{ for every } t \leq \tau \middle| \omega = 0 \right\}$ are bounded away from 0, which implies that $\Pr\left\{ l_t \geq -tR \text{ for every } t \middle| \omega = 0 \right\}$ is bounded away from 0.

# Limit Points of Naive Agents' Actions

**Theorem**

*When agents are naive, for every $r < 1$, there exists $\delta > 0$ such that*

$$\Pr\left(a_t > r \textit{ for all } t \in \mathbb{N} \middle| \omega = 0\right) > \delta.$$

The agents' limiting actions can be wrong, but what can they be?

**Theorem**

*When agents are naive, their beliefs (and hence their actions) converge almost surely to either* 0 *or* 1.

Lesson: If their beliefs are wrong in the long run, then they must be fully confident in the wrong state.

- Cannot happen when agents are Bayesian and sophisticated.

# Proof

To show that players' beliefs converge a.s. to 0 or 1, it is sufficient to show that $L_n$ diverges to $+\infty$ or $-\infty$ almost surely as $n \to +\infty$.

Recall the formula for $L_n$:

$$L_n = \sum_{t=1}^{n-1} \log \frac{a_t}{1-a_t} = \sum_{t=1}^{n-1} 2^{n-t-1} l_t$$

Therefore,

$$2^{1-n} L_n = \sum_{t=1}^{n-1} 2^{-t} l_t.$$

If we can show that $\sum_{t=1}^{n-1} 2^{-t} l_t$ converges as $n \to +\infty$, then $L_n$ must be diverging to $+\infty$ or $-\infty$.

# Proof

We need to show that $\sum_{t=1}^{n-1} 2^{-t} l_t$ converges as $n \to +\infty$.

**Kolmogorov Three-Series Theorem (Theorem 5.3.3 in Chung's textbook)**

*Suppose $\{X_n\}_{n\in\mathbb{N}}$ are independent random variables. Then $\sum_n X_n$ converges a.s. if the following conditions hold for some $A > 0$*

1. $\sum_n \Pr(|X_n| \geq A)$ *converges,*
2. $\sum_n \mathbb{E}[X_n \mathbf{1}\{|X_n| \leq A\}]$ *converges,*
3. $\sum_n Var\left(X_n \mathbf{1}\{|X_n| \leq A\}\right)$ *converges.*

Let $X_n$ be $2^{-n} l_n$ conditional on $\omega = 0$.

$$\sum \Pr(|X_n| \geq A | \omega = 0) = \sum \Pr(2^{-n}|l_n| \geq A | \omega = 0) = \sum \Pr(4^{-n} l_n^2 \geq A^2 | \omega = 0)$$

$$\leq \sum \frac{4^{-n}\mathbb{E}[l_n^2 | \omega = 0]}{A^2} \leq \frac{\mathbb{E}[l_n^2 | \omega = 0]}{A^2} \quad \text{(which is bounded)}$$

# Proof

**Kolmogorov Three-Series Theorem (Theorem 5.3.3 in Chung's textbook)**

*Suppose $\{X_n\}_{n\in\mathbb{N}}$ are independent random variables. Then $\sum_n X_n$ converges a.s. if the following conditions hold for some $A > 0$*

1. $\sum_n \Pr(|X_n| \geq A)$ *converges,*
2. $\sum_n \mathbb{E}[X_n \mathbf{1}\{|X_n| \leq A\}]$ *converges,*
3. $\sum_n Var\left(X_n \mathbf{1}\{|X_n| \leq A\}\right)$ *converges.*

Let $X_n$ be $2^{-n} l_n$ conditional on $\omega = 0$.

$$\sum \mathbb{E}\left[2^{-n} l_n \mathbf{1}\{2^{-n} l_n \leq A\} \middle| \omega = 0\right] \leq \sum \mathbb{E}\left[2^{-n} |l_n| \middle| \omega = 0\right] \leq \sum \mathbb{E}\left[2^{-n}(l_n^2 + 1) \middle| \omega = 0\right]$$

$$= \sum 2^{-n} + \sum 2^{-n} \mathbb{E}[l_n^2 | \omega = 0] = 1 + \mathbb{E}[l_n^2 | \omega = 0] \quad \text{(which is bounded)}.$$

# Proof

**Kolmogorov Three-Series Theorem (Theorem 5.3.3 in Chung's textbook)**

*Suppose* $\{X_n\}_{n\in\mathbb{N}}$ *are independent random variables. Then* $\sum_n X_n$ *converges a.s. if the following conditions hold for some* $A > 0$

1. $\sum_n \Pr(|X_n| \geq A)$ *converges,*
2. $\sum_n \mathbb{E}[X_n \mathbf{1}\{|X_n| \leq A\}]$ *converges,*
3. $\sum_n Var\left(X_n \mathbf{1}\{|X_n| \leq A\}\right)$ *converges.*

Let $X_n$ be $2^{-n} l_n$ conditional on $\omega = 0$.

$$\sum Var\left(X_n \mathbf{1}\{|X_n| \leq A\} \middle| \omega = 0\right) \leq \sum \mathbb{E}\left[4^{-n} l_n^2 \middle| \omega = 0\right] \leq \mathbb{E}[l_n^2 | \omega = 0].$$

The convergence of all three series uses the fact that $\mathbb{E}[l_n^2 | \omega = 0]$ is bounded.

- Hinges on the existence of continuous density $f_\omega$.

# Stable Interior Beliefs are Likely to Be Wrong

Suppose players' beliefs remain stable at some interior level for a long time, what happens?

**Theorem**

*For every* $[c,d] \subset (1/2,1)$*, there exists* $T \in \mathbb{N}$ *such that if* $a_t \in [c,d]$ *for every* $t \in \{1,2,...,T\}$*, then*

$$\Pr\left(\omega = 0 \middle| (a_1,...,a_T)\right) > \Pr\left(\omega = 1 \middle| (a_1,...,a_T)\right)$$

Why is $\omega = 0$ more likely to be the correct state when agents' belief stablize at an interval above $1/2$?

- Suppose $a_1 \in [c,d] \subset (1/2,1)$. If $\omega = 1$, then $l_2$, $l_3$,...,$l_n$ are likely to be high, which means that $a_t$ will approach 1 in the long run.
- Hence, $a_t \in [c,d]$ for a long time indicates that $\omega$ is likely to be 0.

# Stable Interior Beliefs are Likely to Be Wrong

**Theorem**

*For every $[c,d] \subset (1/2,1)$, there exists $T \in \mathbb{N}$ such that if $a_t \in [c,d]$ for every $t \in \{1,2,...,T\}$, then*

$$\Pr\left(\omega = 0 \middle| (a_1,...,a_T)\right) > \Pr\left(\omega = 1 \middle| (a_1,...,a_T)\right)$$

Let $u \equiv \log \frac{c}{1-c}$ and $v \equiv \log \frac{d}{1-d}$.

- If $\log \frac{a_1}{1-a_1},...,\log \frac{a_t}{1-a_t} \in [u,v]$, then

$$v \geq \log \frac{a_{t+1}}{1-a_{t+1}} = \sum_{\tau=1}^{t} \log \frac{a_\tau}{1-a_\tau} + l_{t+1} \geq tu + l_{t+1}$$

  which means that

$$l_{t+1} \leq v - tu.$$

- When $t$ is large enough, $s_{t+1}$ is a signal in favor of state 0.
- When $l_{t+1} \leq v - tu$ for all $t \leq T$ and $T$ being large enough, the posterior prob of $\omega = 0$ under a rational agent's belief is less than $1/2$.

# Network Among Players

Doubly rich setting, agents are Bayesian, but believe that all their predecessors' actions only reflect their own private signals.

- Agents' belief converges to the wrong state with positive prob.
- Agents' actions are asymptotically inefficient with positive prob.

How general is this finding?

- What if players cannot observe all their predecessors' actions?
- What if players exhibit redundancy neglect, but not as extreme as in the previous model?

# Eyster and Rabin (2014)

- Time $t = 1, 2, \ldots$ One agent arriving in each period.
- State $\omega \in \{0, 1\}$, equally likely.
- Action of agent $t$: $a_t \in [0, 1]$.
- Agent $t$'s payoff is $-(a_t - \omega)^2$, so $a_t \equiv \mathbb{E}[\omega | s_t, a_1, \ldots, a_{t-1}]$.
- Agent $t$'s private signal $s_t \sim G(\cdot|\omega)$, conditionally independent.
  - $\hookrightarrow$ same assumption as before (unbounded, no revealing signal).
- Agent $t$ observes $s_t$ and $\{a_\tau\}_{\tau \in N_t}$ where $N_t \subset \{1, 2, \ldots, t-1\}$.
  - $\hookrightarrow$ Players in $N_t$ are the neighbors of agent $t$.
  - $\hookrightarrow$ The network is deterministic and is common knowledge.
- We write $i \succ j$ if there exist $k(0), \ldots, k(n)$ such that $k(0) = j$, $k(n) = i$, $k(m-1) \in N_{k(m)}$ for every $m \in \{1, 2, \ldots, n\}$.

  Player $i$ can observe player $j$'s action along some path.

# Strategies & Regularity Assumptions on Strategies

Let $l_t$ be the LLR of agent $t$'s private signal and let $\alpha_t \equiv \log \frac{a_t}{1-a_t}$.

Agent $t$'s strategy is $\alpha_t\left(\alpha_1,...,\alpha_{t-1},l_t\right)$, measurable w.r.t $\left(l_t,(\alpha_\tau)_{\tau\in N(t)}\right)$.

## Strictly Increasing Strategies

*Players' strategies are strictly increasing in private signals if for every $t \in \mathbb{N}$ and $(\alpha_1,...,\alpha_{t-1})$, $\alpha_t$ is a strictly increasing function of $l_t$.*

## Boundedly Increasing Strategies

*Players' strategies are boundedly increasing if there exists $K \in \mathbb{R}_+$ such that for every $t \in \mathbb{N}$, $(\alpha_1,...,\alpha_{t-1})$, and $l_t \neq l'_t$, we have*

$$\left|\alpha_t\left(\alpha_1,...,\alpha_{t-1},l_t\right)-\alpha_t\left(\alpha_1,...,\alpha_{t-1},l'_t\right)\right| \leq K|l_t - l'_t|$$

# General Redundancy Neglect Learning Rules

## Redundancy Neglect Strategies

*Players' strategies exhibit redundancy neglect if*

1. *For every $t$ and $j \prec t$, $\alpha_t$ is weakly increasing in $\alpha_j$ regardless of $(\alpha_1, ..., \alpha_{t-1})$ and $l_t$.*
2. *There exist $N \in \mathbb{N}$ and $x > 1$ such that for every player $t \geq N+1$ and $z' > z$, $\alpha_t$ increases by at least $x(z'-z)$ if each of $\alpha_{t-N}, ..., \alpha_{t-1}$ increases from $z$ to $z'$.*

This is a joint condition on the network and players' learning rule.

- Player $t$ observes at least one of their last $N$ predecessors.

The learning rule in the previous model satisfies both requirements.

- $\alpha_t = l_t + \sum_{\tau=1}^{t-1} \alpha_\tau$.

How does rational Bayesian learning violate these two properties?

# General Redundancy Neglect Learning Rules

**Redundancy Neglect Strategies**

*Players' strategies exhibit redundancy neglect if*

1. *For every $t$ and $j \prec t$, $\alpha_t$ is weakly increasing in $\alpha_j$ regardless of $(\alpha_1, \ldots, \alpha_{t-1})$ and $l_t$.*
2. *There exists $N \in \mathbb{N}$ and $x > 1$ such that for every player $t \geq N+1$ and $z' > z$, $\alpha_t$ increases by at least $x(z' - z)$ if each of $\alpha_{t-N}, \ldots, \alpha_{t-1}$ increases from $z$ to $z'$.*

Suppose there are four individuals.

- 2 and 3 can both observe 1, but cannot observe each other.
- 4 observes 1, 2, and 3.

1's action is $l_1$, 2's action is $\alpha_1 + l_2$, 3's action is $\alpha_1 + l_3$.

- 4's optimal action is $l_1 + l_2 + l_3 + l_4$.
- His strategy is $l_4 + \alpha_2 + \alpha_3 - \alpha_1$ (violates requirement 1).

# General Redundancy Neglect Learning Rules

**Redundancy Neglect Strategies**

*Players' strategies exhibit redundancy neglect if*

1. *For every $t$ and $j \prec t$, $\alpha_t$ is weakly increasing in $\alpha_j$ regardless of $(\alpha_1, \ldots, \alpha_{t-1})$ and $l_t$.*
2. *There exists $N \in \mathbb{N}$ and $x > 1$ such that for every player $t \geq N+1$, if each of $\alpha_{t-N}, \ldots, \alpha_{t-1}$ increases by at least $\Delta$, then $\alpha_t$ increases by at least $x\Delta$.*

Suppose every agent can observe all their predecessors.

- Agent $n$'s optimal action $\alpha_n = \alpha_{n-1} + l_n$ (violates requirement 2).

# Result

**Theorem**

*If players' strategies are strictly and boundedly increasing, and exhibit redundancy neglect, then*

- *conditional on* $\omega = 0$, $\alpha_t$ *converges to* $+\infty$ *with positive prob,*
- *conditional on* $\omega = 1$, $\alpha_t$ *converges to* $-\infty$ *with positive prob.*

The proof uses ideas similar to that of their earlier result.

- Since players over-react to earlier players' private signals,

  early players' mistakes are hard to correct,

  so incorrect actions can be taken asymptotically with positive prob.

# When will rational players anti-imitate?

Suppose that players are rational and Bayesian.

- Player $t$ anti-imitates player $j$ if $t \succ j$ and $\alpha_t$ is a strictly decreasing function of $\alpha_j$.

Players $i, j, k, l$ form a shield if

- $j$ and $k$ observe $i$,
- $j$ and $k$ cannot observe each other,
- $l$ observes $i$, $j$, and $k$.

**Theorem**

*Suppose all players are rational. There exists anti-imitation in equilibrium if and only if the network contains a shield.*

Anti-imitation cannot arise under the canonical observation structure.