Supplementary Appendix
Reputation Effects under Interdependent Values

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I discuss extensions of the product choice game introduced by the end of section 2.1 of the paper. A long-lived patient seller (player 1) interacts with *a continuum of buyers* (or player 2s). The buyers are potentially long-lived, i.e., interacting with the seller in multiple periods. A buyer is active in period t if he has the ability to buy (i.e., choose T) in period t. I assume that the measure of *active buyers* is constant in every period, which is normalized to 1. In every period, the seller chooses between H and L, and every active buyer chooses between T and N. Other aspects of the model remain the same except for the specification of public histories and an arbitrarily small fraction of active buyers being *non-strategic*:

1. The public history in period t consists of the seller's action and the fraction of buyers that play T in period s, for every $s \in \{0, 1, ..., t-1\}$. Importantly, each individual buyer's action *cannot be observed* by the seller or other buyers. Since each buyer's action has negligible impact on the aggregate distribution over buyers' actions, it also has negligible impact on other players' future play.

2. A small fraction $\varepsilon > 0$ of active buyers are non-strategic and mechanically play T in every period. A fraction $1-\varepsilon$ of buyers are strategic and maximize their payoffs. As will be clear later, each buyer's discount factor and the number of periods that she is active are irrelevant for my results.

Strategic-type seller and *strategic-type* buyers' stage-game payoffs are given according to the matrices in section 2.1. In particular, the strategic seller's stage-game payoff in period t depends on the persistent state θ, his action in period t, and the *aggregate distribution* over the buyers' actions in period t. For example, if in period t, the seller plays $a_{1,t}$, a fraction λ of buyers choose T, and a fraction of $1-\lambda$ of buyers choose N, then the strategic seller's stage-game payoff is $\lambda u_1(\theta, a_{1,t}, T) + (1-\lambda) u_1(\theta, a_{1,t}, N)$.

Compared to the baseline model in section 2, I replace *one short-lived buyer* in every period with *a continuum of buyers*, who are potentially long-lived. This addresses the concern that in practice, some buyers demand items from the seller in multiple periods. In the current setup, a buyer plays her *myopic best reply* based on her belief about the state and the seller's current period action even when she is long-lived. This is because first, each individual buyer's action *cannot* influence the aggregate distribution over buyers' actions, and therefore, has no influence over the seller's and other buyers' future actions. Second, each buyer's action in period t does not affect what she learns in that period, i.e., she has no incentive to experiment. This requires that:

1. Each buyer *cannot* observe her stage-game payoff. This is the case when the state θ captures the *credence quality* of the product, which is defined in Darby and Karni (1973) as attributes of a product that affect consumers' willingness to pay but consumers cannot observe even after consuming the product. As argued in section **??** in context of the US toy industry, consumers cannot directly observe the safety of toys (e.g., lead content in the paint), and therefore, cannot observe their own payoffs.

2. For every $t > s$, each active buyer in period t can observe the seller's action in period s no matter whether she buys in period s or not. This is the case in the toy industry when buyers who bought in period s can post information about the product's design (which is affected by the seller's effort) online, so that other buyers can learn about the seller's effort in period s by reading his review.

I also perturb the game using an ε fraction of mechanical buyers who automatically choose T. This addresses the concern that in practice, future buyers cannot learn the seller's action in period t if no buyer buys the seller's product in period t. It justifies my modeling assumption that each buyer can observe the seller's action in all previous periods. The qualitative features of my results remain robust under this perturbation. My results are also robust to alternative specifications of mechanical-type buyers' strategies, as long as a positive fraction of buyers choose to purchase at every history.

In the current setting, one can apply the same arguments in the proofs of Theorems 2 and 3 to show that:

1. If the prior probability of state θ_h is more than $1/2$, then in every equilibrium, a sufficiently patient seller secures payoff 1 in state θ_h, and secures payoff $1-\eta$ in state θ_l.

2. If the prior probability of state θ_h is less than $1/2$, then in every equilibrium,

   - Type θ_h seller's equilibrium payoff is $\eta + \frac{\varepsilon}{1-\varepsilon}$, and he plays H at every on-path history.
   - Type θ_l seller's equilibrium payoff is 2ε, and he plays H at every on-path history with positive probability, and he plays L at every on-path history with complementary probability.

   These equilibrium payoffs converge to η and 0 respectively as the fraction of mechanical-type buyers ε vanishes to 0. The mixing probability of type θ_l is such that after observing H in period 0, future buyers' posterior belief attaches probability $1/2$ to the state being θ_h.

In addition, when the prior probability of state θ_h is more than $1/2$, there exist many equilibria in which playing L in some periods is strictly optimal for the strategic seller, that is, the seller has a strict incentive to play H in some periods and to play L in other periods. This contrasts to the scenario in which the probability of state θ_h is less than $1/2$, in which the seller's actions are perfectly correlated over time. As a result, the testable prediction delivered by the end of section 3.3 remains valid in the current setting, i.e., the intertemporal correlation of the seller's observable effort increases after a negative shock on consumers' beliefs about the company's quality θ.