

Slow Household Deleveraging*

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Abstract

We use a model of precautionary savings with housing and mortgages to study the effects of a deleveraging shock on consumer spending. We focus on deleveraging caused by a contraction in home values, and compute numerically the partial equilibrium effect of the shock. Our simulations show that household deleveraging is associated to a long and protracted weakness in consumption. These effects appear even if we assume, realistically, that housing wealth is illiquid and mortgage debt is long-term. We show that housing wealth matters for consumption decisions due to an insurance force: consumers know they can sell their house if they get hit by sufficiently negative shocks in the future. We also show that our slow deleveraging mechanism is amplified when incomes are affected by weak aggregate consumption demand through general equilibrium effects.

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1 Introduction

More than a decade after the Great Recession, the debate is still open about the main forces that caused it. [Bernanke \(2018\)](#) distinguishes two main stories. According to the first story, the recession was mostly the consequence of a panic in the financial system and of the ensuing credit crunch. According to the second, the recession was mostly due to weak consumer spending associated to deleveraging in the household sector, as households were dealing with a large accumulated stock of debt and with a widespread contraction in home values. Although the two stories are clearly interconnected and there is abundant evidence that both channels were at work, there is still a lively debate on their relative merits and on their relative quantitative significance.

[Bernanke \(2018\)](#) presents arguments in favor of the financial panic story, emphasizing the fact that the timing of the panic aligns well with the most acute phase of the contraction in real output. [Krugman \(2018\)](#), one of the main proponents of the household deleveraging story, pushes back on the timing argument, by pointing to the fact that although the peak of the financial crisis and the recession are well aligned, financial markets' recovered relatively quickly, while the real economy went through a prolonged period of weak spending, which seems to fit better the deleveraging story.¹

An important input into this debate is the construction of models that capture formally the different pieces of each story and can be used to inspect the mechanism qualitatively and quantitatively. In particular, interest in the household deleveraging story has led to the development of a large literature that puts household debt and household balance sheets at the center of the analysis. In this paper, we selectively review some of this literature, we identify some of the challenges that this approach has faced, and present some new results. In particular, we focus on the dynamic response of household balance sheets and consumption following a simple deleveraging shock due to an unexpected drop in house values. We make our arguments by presenting numerical simulations of a precautionary saving model in the tradition of Bewley-Aiyagari-Huggett with the addition of housing and mortgage debt. Our main point is that the dynamic adjustment of household balance sheets can be very slow and can indeed cause a prolonged period of weakness in consumer demand.

Our first contribution is to address two broad objections that can be raised against the

¹See also related arguments in [Baker \(2018\)](#). Since the debt deleveraging story works over a long time horizon, business-cycle frequency data covering only the Great Recession provides only limited information. Among the most recent empirical work, [Mian et al. \(2017\)](#) look at international evidence over a long time horizon showing that debt accumulation in the household sector tends to be followed by lower GDP growth in the medium run.

deleveraging story.

One objection is that the major component of household debt takes the form of long-term mortgages. When credit conditions or house prices change, this does not affect directly households that keep paying mortgages made in the past. In other words, mortgage contracts are not “marked-to-market” so a large fraction of households are not forced to pay off their debt faster when conditions change. A fact that helps visualize this issue is that the ratio of household debt to household assets actually *increased* at the onset of the crisis (Justiniano et al. (2015)), as the value of homes fell sharply due to the change in house prices, while the stock of household debt only adjusted slowly.²

Another, related objection is that houses are illiquid assets, that are traded infrequently, subject to significant transaction costs. So changes in house values, while they appear to affect dramatically households’ balance sheets on paper, may not have large consequences for consumption decisions if consumers plan to stay in the house where they live.³

Our numerical simulations show that these two critiques have limited bite in precautionary savings models. While the presence of long-term mortgages and illiquid houses dampens a bit the initial effect of a deleveraging shock, the shock still produces a large and protracted weakness in consumer spending. In particular, the main effect of long-term mortgages and illiquidity is to spread out the effect of the shock over time.

The second contribution of this paper is to investigate the reason *why* the households in our model care about their net housing wealth—the value of their house minus their mortgage balance—even though they trade housing and refinance their mortgages infrequently. We present numerical experiments that show that housing wealth serves an insurance purpose. Even though households trade infrequently, they know that if the need arise they can rely on their housing equity as a source of liquidity of last resort. To identify this precautionary effect, we compare the predictions of our model to the predictions of a simple model with Poisson arrival of trading opportunities, in which households trade houses with the same frequency as in our baseline model but are not allowed to choose *when* to trade. In the Poisson model, consumers have less access to their housing wealth when they need it, because trading intensity is not allowed to vary conditional on income shocks. The effect of a house price shock in the Poisson model is much smaller than in the model with endogenous house trading. This suggests that the insurance element plays a

²Relatedly, Adrian and Shin (2010) point out that changes in asset values and changes in leverage tend to be positively related for households, a pattern that distinguishes the household sector from the financial sector.

³See Sinai and Souleles (2005) for an explicit formulation of this argument.

quantitatively significant role in consumption adjustment.⁴

The third contribution of the paper is to explore an added reason why the process of deleveraging can be slow. If weak consumer demand leads to a contraction in output and to lower incomes, the deleveraging process tends to be partly self-defeating: as consumers try to save more, their incomes go down, slowing down their process of rebuilding wealth. This is a dynamic version of Keynes' paradox of thrift. Here we show that this mechanism can amplify the consumption response, especially in the short run, and can substantially lengthen the adjustment process of household balance sheets.

This paper's first contribution is closely related to the analysis in [Berger et al. \(2018\)](#)—where the central message was also that housing wealth effects can be large in precautionary savings models. The main difference is that [Berger et al. \(2018\)](#) mostly focus on the impact response and on the connection with the marginal propensity to consume, while here we focus more on the deleveraging dynamics and study the effects of changing debt maturity. The Poisson experiment and the paradox of thrift analysis are novel.

The paper's numerical experiments are all based on a partial-equilibrium model of consumer choice. This implies that the paper does not answer an important question, which is what are the underlying causes of the boom-bust cycle in house prices. Rather, our objective here is to learn about the transmission mechanism from house prices to consumer spending. Our model with endogenous income makes one step in the direction of general equilibrium, but still without going back to the underlying shocks. Of course, a full general equilibrium exercise would be necessary to verify that the transmission mechanism identified here, together with the underlying forces causing the boom and bust, can provide a credible account of the recession. We leave that to future work.⁵

The model is formulated in continuous time and is solved using a version of the finite-difference approach advocated by [Achdou et al. \(2017\)](#), with the use of a linear complementarity problem (LCP) algorithm to deal with the optimal stopping problem. The details of the algorithm are in the online appendix.

The paper is organized as follows. The literature is discussed in Section 2. In Section 3, we present the model. Section 4 contains our baseline simulation results and comparisons with lower transaction costs and shorter-maturity debt. In Section 5, we present the Poisson model and compare it to the baseline. In Section 6, we present the dynamic paradox of thrift. Section 7 concludes.

⁴The insurance value of housing wealth is a theme that has been explored from a different angle in [Lustig and Van Nieuwerburgh \(2005\)](#), who focus on the effects that it has on risk premia in an asset pricing model.

⁵The next section discusses some papers that analyze different general equilibrium exercises.

2 Related literature

In this section, we review some of the large literature that has developed on household balance sheets and housing wealth, following the Great Recession. The point of this review is to provide some context for the following sections, so we do not aim to be comprehensive and we focus on the construction of models. A wonderful review that focuses more on recent empirical work is [Mian and Sufi \(2018\)](#).

Two early papers that aimed to capture the role of household balance sheets and household deleveraging in the Great Recession are [Eggertsson and Krugman \(2012\)](#) and [Guerrieri and Lorenzoni \(2017\)](#). Both papers use a very simplified representation of household balance sheets, with only a liquid short-term asset/liability, and model a deleveraging shock as coming from a one-time, unexpected tightening in the borrowing limit. The shock can be interpreted either as the effect of the 2007-08 financial crisis, leading to tighter lending standards in bank lending, or as the effect of a drop in house prices that reduced the collateral value and hence borrowing capacity.⁶ [Eggertsson and Krugman \(2012\)](#) work with a two agent model, in which the borrowing agents are always against the borrowing limit, so a contraction in the limit forces constrained agents to delever. [Guerrieri and Lorenzoni \(2017\)](#) consider a model with a continuum of agents and uninsurable idiosyncratic income risk in the tradition of Bewley-Aiyagari-Huggett. A shock to the borrowing limit induces all agents to adjust up their targets for liquid savings, due to a precautionary motive. In fact, a good approximation to the response of consumption to a tightening in the borrowing limit is that the economy responds as if all agents in the economy experienced a loss in liquid wealth equivalent to the dollar amount by which the limit got reduced.⁷ This shows that the consumption response will depend on the marginal propensity to consume. In calibrations with a relatively high marginal propensity to consume, the response can be large.⁸

While these papers highlighted the role of households' deleveraging, a second generation of papers on the topic has recognized that households balance sheets are richer, and, in particular, that it is important to introduce housing wealth and mortgage debt explicitly into the picture.

An early paper that includes housing explicitly in a deleveraging story is [Jones et al.](#)

⁶Incidentally, the possibility of these two interpretations shows the complementarity of the two competing stories mentioned in the introduction.

⁷Proposition 1 in the Online Appendix of [Guerrieri and Lorenzoni \(2017\)](#) proves this result analytically in the special case $r = 0$.

⁸See Section 5.1 of [Guerrieri and Lorenzoni \(2017\)](#).

(2018).⁹ Their model emphasizes less the heterogeneity between borrowers and lenders, and posits a relation between house values and households' access to liquidity, by formulating a money demand model in which housing equity provides liquidity services. The main objective of their exercise is to have a tractable model that can be embedded in a multi-region environment, to map into the evidence in [Mian and Sufi \(2011\)](#).¹⁰

More recent papers have built heterogeneous agent models with housing, enriching the analysis in two dimensions that are relevant to connect to the data: first, houses are illiquid, as trading them entails substantial transaction costs, second, mortgage debt is long term. Papers that embed these features into household balance sheets have focused on different questions. The ones most closely related to this paper are [Greenwald \(2018\)](#), [Berger et al. \(2018\)](#), [Kaplan et al. \(2020\)](#), and [Guren et al. \(2020\)](#). All these papers study heterogeneous agents models, with housing, long-term debt, and collateral constraints, and analyze, from different angles, how adjustments in household balance sheets affect consumption decisions.

[Greenwald \(2018\)](#) enriches a DSGE model with two types of agents, by introducing mortgages that are subject to both a loan-to-value ratio constraint and a payment-to-income constraint, and where debt is prepayable. He argues that household balance sheets amplify the transmission of monetary policy. The idea is that when interest rates decline, the payment-to-income constraint is relaxed, so that the loan-to-value ratio constraint becomes binding for a larger set of households, inducing them to increase their demand in housing to increase the value of their collateral. He also shows that the same mechanism explains why a relaxation of the payment-to-income standards is crucial to explain the recent boom in housing prices.

[Berger et al. \(2018\)](#) use a partial equilibrium approach to show that heterogeneous agents models with incomplete markets and collateral constraints generate sizable consumption elasticities to house price movements that are in line with the data.¹¹ Their main result is that a simple rule-of-thumb formula well approximates the response of consumption to permanent changes in house prices. The simplicity of the formula comes from the fact that in the extreme case of no adjustment costs in combination with Cobb-Douglas preferences,

⁹First circulated as working paper in 2011.

¹⁰Before the crisis, one of the earliest paper to explicitly incorporate housing finance in a two agent general equilibrium model is [Iacoviello \(2005\)](#). One of the main aims of that paper was indeed to produce realistic implications for the relation between house prices and consumption in a monetary general equilibrium model.

¹¹There has been a large body of empirical work finding significant consumption elasticity to house price movement. Among the others, [Mian et al. \(2013\)](#) using US credit card data and the housing supply index from [Saiz \(2010\)](#), find estimates for the non-durable consumption elasticity between 0.13 and 0.26.

the income effect, substitution effect and collateral effect following a permanent change in house prices cancel out, leaving only the wealth effect coming from the change in the valuation of the initial housing endowment. The authors show that this formula is a good approximation for richer environments, once calibrated with recent US micro and macro data. The formula helps to show that the consumption response to house prices critically depends on the joint distribution of household debt and housing values.

[Kaplan et al. \(2020\)](#) build a general equilibrium model with heterogeneous agents and endogenous house prices, and use it to investigate the causes of the US housing boom and bust and the response of consumption. Their first question is what is the nature of the shocks behind house price movements in the 2000s. Their model features three potential shocks: aggregate productivity shocks, shocks to credit conditions, and shifts in beliefs about future housing demand. They argue that shifts in beliefs are the main drivers in movements in house prices in that period. They then show that the boom-bust in house prices can explain half of the corresponding movements in non-durable consumption due to housing wealth effects.

Another related paper is [Garriga and Hedlund \(2020\)](#) who also build a quantitative general equilibrium macro-housing model with heterogeneous agents to study the main drivers of the 2006-2011 housing bust and its spillovers to consumption and credit markets. The distinctive ingredient in their model is the presence of a frictional housing market: directed search for housing generates endogenous liquidity that responds to changes in macroeconomic conditions. They show that the main drivers of the housing crash in their model are tightening lending standards and higher left tail risk from earnings skewness shocks. The combination of the drop in house prices and of the deterioration of housing liquidity damages households' balance sheets, generating a decrease in homeownership, a drop in consumption, a spike in foreclosures.

The main difference between this paper and the general equilibrium analyses in [Kaplan et al. \(2020\)](#) and [Garriga and Hedlund \(2020\)](#) is that we do not endogenize house prices, and push in two different directions: understanding the insurance mechanism behind large wealth effects, and studying the potential for amplification through the general equilibrium response of output through a standard Keynesian channel.

While many of the papers described tend to generate a large consumption elasticity to changes in house prices in line with the empirical evidence, [Guren et al. \(2020\)](#), using both new and existing evidence, make the point that such an elasticity has been pretty stable over time going back to 1980s. This implies that the collapse in consumption in the Great Recession was not due to a change in its sensitivity to house price movements, but to

the large changes in house prices in that period. They also show that these findings are consistent with the behavior of housing wealth elasticity in a partial equilibrium model similar to [Berger et al. \(2018\)](#).

A related group of papers use similar models to focus more specifically on the question whether credit conditions matter for house price movements. In particular, there is still disagreement on the question of to what extent the relaxation and following contraction in credit standards in the 2000s have contributed to the boom and bust in house prices experienced in the US in the same period.

On the one hand, there are papers arguing that credit market conditions are quantitatively not relevant for house prices movements. For example, [Kiyotaki et al. \(2011\)](#) develop a life-cycle model where land, which is limited in supply, and capital are both needed to produce residential and commercial real estate and credit markets are imperfect. They calibrate the model to the US and show that changes in financial constraints have limited effects on house prices. In a similar vein, [Kaplan et al. \(2020\)](#) argue that shifts in credit conditions do not play an important role in explaining the house price boom-bust episode in the 2000s, while shifts in beliefs are the main drivers.

On the other hand, there is a body of work that argues that the relaxation of credit conditions help explain large part of the rise in house prices in early 2000a. [Landvoigt et al. \(2015\)](#) use micro data from the County of San Diego to calibrate a model where houses of different quality are assigned to agents of different income, wealth, and age. Their main finding is that cheaper credit was the main driver of house prices, during the 2000 boom. [Favilukis et al. \(2017\)](#) use a quantitative general equilibrium housing model enriched with business cycle risk and preference heterogeneity for bequests to show that the relaxation in collateral requirement in the early 2000 contributed to large part of the increase in house prices and the consequent reversal in financial liberalization contributed to the housing bust. As already mentioned, [Greenwald \(2018\)](#) focuses on payment-to-income limits and shows that they play an important in recent house price dynamics. [Garriga et al. \(2019\)](#) propose a model with collateralized credit and segmented financial markets and show that the changes in mortgage rates and loan-to-value ratios that matches the data during the boom-bust episode in 1998-2000 can explain between 25 and 45 percent of the house price changes. They also show that if they include shocks to expectations about housing finance conditions, the model substantially improves in explaining house prices. [Garriga and Hedlund \(2018\)](#) arrive to a similar conclusion that the post-2000 decline in mortgage rates is first order in explaining the boom in house prices, but focuses on a different mechanism. In particular, they allow both an extensive and intensive margin of homeownership, by

endogenizing the choice of renting versus owning, and introduce the possibility of debt refinancing and default. More related to our paper, they also focus on the implications of a decline in mortgage rates on consumption and on the fragility of the economy.

Greenwald and Guren (2019) try to reconcile the different results in the literature by arguing that the extent to which credit conditions can drive the 2000s housing boom-bust episode depends on the degree of segmentation in housing markets. In particular, if there is integration between rental and owner-occupied housing and landlords are deep pocketed, when credit conditions become tighter, landlords can step in to buy more houses and house prices are unaffected by the change in credit markets. However, if there is segmentation, changes in credit market can generate large changes in house prices instead than in ownership rates. They then use data to estimate the model and show that US housing markets are pretty segmented so that credit conditions changes can explain a large part (between 28 and 47 per cent) of the house price boom.

3 The model

The model is set in continuous time, with an infinite horizon. The economy is populated by a continuum of consumers with preferences on flows of consumption and housing services, c_t and h_t , represented by the utility function

$$\mathbb{E} \int_0^{\infty} e^{-\rho t} U(c_t, h_t) dt,$$

where

$$U(c, h) = \frac{(c^\alpha h^{1-\alpha})^{1-\sigma}}{1-\sigma}.$$

Each consumer receives a random income flow y_t , that follows a Poisson process with two states, y_1 and y_2 . Income switches from y_1 to y_2 with intensity $\lambda_{y_2|y_1}$ and from y_2 to y_1 with intensity $\lambda_{y_1|y_2}$. The shocks to y_t are idiosyncratic, so aggregate income in the economy is constant and equal to Y .

Consumers hold three type of assets: liquid assets a_t , houses h_t that provide housing services one for one, and mortgage debt d_t .

Liquid asset holdings are adjusted continuously and pay a constant rate of return r . Liquid asset holdings must be non-negative

$$a_t \geq 0,$$

so the only form of borrowing in this economy is mortgage debt.

The housing stock h_t is only adjusted at discrete intervals of time, as trading houses involves a fixed cost. The fixed cost is modeled as a proportional cost of selling a house. Namely, the price of a unit of housing is p , but when a unit of housing is sold the seller receives $(1 - \phi) p$. Holding houses also requires paying a flow of maintenance costs proportional to the value of the house, $\delta p h_t$.¹²

Mortgages are standard fixed rate mortgages. When a consumer takes a new mortgage loan d_t at date t , the mortgage payment is set at

$$m_t = \frac{r_m}{1 - e^{-r_m \tau}} d_t,$$

where τ is the length of the mortgage and r_m is the mortgage interest rate. In all following periods $s > t$, as long as the consumer does not pay off the mortgage, the debt level follows the law of motion

$$\dot{d}_s = r_m d_s - m_t, \quad (1)$$

until the mortgage is fully repaid at $t + \tau$, and from then on $d_s = 0$.

Turning to a recursive description of the problem and to recursive notation, the individual decision problem has five state variables a, d, h, m and y . We now describe how the state variables evolve in periods in which the consumer chooses to adjust, and in periods in which no adjustment occurs.

When a consumer adjusts, the asset positions must satisfy the budget constraint

$$a' + p h' - d' = a + (1 - \phi) p h - d, \quad (2)$$

where a, d, h denote asset positions before adjustment and a', d', h' asset positions after adjustment.

At the moment of adjustment, the debt level d' must satisfy the collateral constraint

$$d' \leq \theta p h', \quad (3)$$

where θ is a constant in $(0, 1)$, that is, debt must be lower than a fraction of the value of the house purchased.

In periods in which no adjustment occurs, the mortgage payment is fixed at m and the

¹²In a model without fixed adjustment costs, assuming consumers have to pay $\delta p h_t$ is equivalent to having the housing stock depreciate at the rate δ . In a model with fixed adjustment costs, the two assumptions are a bit different. Assuming that non-adjusting consumers pay $\delta p h_t$ and keep their housing stock constant simplifies computations, as the state variable h_t does not change for them.

dynamics of a are given by

$$\dot{a} = ra + y - c - m - \delta ph.$$

The dynamics of d are given by (1), which in recursive notation is $\dot{d} = r_m d - m$.

The assumptions made allow us to deal with a relatively realistic mortgage structure, while maintaining computations simple. In particular, when adjustment does not happen the consumer only needs to choose c and \dot{a} and all other state variables evolve mechanically, when adjustment happens the consumer chooses the values of a' , d' , h' , and m' is determined by d' .

Let $V(a, d, h, m, y)$ denote the value function of the consumer. For all states (a, d, h, m, y) at which adjustment does not occur, V satisfies the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} \rho V(a, d, h, m, y) = & \max_c U(c, h) + \partial_a V(a, d, h, m, y)(ra + y - c - m - \delta ph) + \\ & + \partial_d V(a, d, h, m, y)(r_m d - m) + \lambda_{y'|y}(V(a, d, h, m, y') - V(a, d, h, m, y)), \end{aligned} \quad (4)$$

where $\partial_a V$ and $\partial_d V$ denote the partial derivatives of V with respect to a and d . The following condition ensures that the consumer chooses optimally when to adjust

$$V(a, d, h, m, y) \geq J(a - d + (1 - \phi) ph, y), \quad (5)$$

where J is the value after adjusting. Notice that J only depends on the current income y and on net wealth

$$w \equiv a - d + (1 - \phi) ph.$$

The value function J comes from the maximization problem

$$\begin{aligned} J(w, y) = & \max_{a', h', d', m'} V(a', h', d', m', y) \\ \text{s.t. } & a' - d' + ph' = w, \\ & d' \leq \theta ph', \\ & m' = \frac{r_m}{1 - e^{-r_m \tau}} d'. \end{aligned} \quad (6)$$

Imposing that for all states either condition (4) or condition (5) holds with equality, gives an Hamilton-Jacobi-Bellman variational inequality, which is the basis of the solution algorithm in the online appendix .

In the coming sections, we solve the problem above numerically and aggregate over consumers. We first study the economy in steady state, with constant values for r, r_m and p , and derive the cross-sectional distribution of the individual state variables

$$g(a, d, h, m, y).$$

Our model is partial equilibrium, so we compute aggregate values for each variable—denoted by capital letters C_t, A_t, \dots —but we do not look for prices r, r_m, p such that these aggregate variables satisfy some market clearing condition.¹³

After deriving the steady state distribution g_{ss} , we introduce a one time, unexpected, permanent shock to the house price p and derive the endogenous evolution of various aggregate variables. Thanks to the assumption of Cobb-Douglas preferences for consumption and housing, we can prove the following result which will be used to provide some intuition for our numerical results.

Proposition 1. *Following a one time, unexpected, permanent change in p , aggregate consumption C_t and net worth $A_t + (1 - \phi) p_t H_t - D_t$ return to their pre-shock value in the long run.*

The result follows a similar logic as Proposition 1 in Berger et al. (2018), extended to a model with positive adjustment costs $\phi > 0$. However, the result is less powerful, as it does not give a characterization of the short run response. For that, we need to resort to numerical computations, which we do in the following sections.

Relative to the model used in Berger et al. (2018), here we adopt a continuous-time approach and make a few different assumptions, mostly in the direction of simplification. In particular, we have infinitely lived agents, while Berger et al. (2018) use a life-cycle model, and the income process is much simpler. On the other hand, here we explicitly model mortgage contracts that look like standard 30 year mortgages. In Berger et al. (2018) long term debt is only introduced in an extension in Section 5.1 and mortgage contracts are represented in a stylized way. This allows us to focus more on the effect of changing the maturity of the mortgage contract in Section 4.3.

Another simplification here is that we bundle house trading and mortgage refinancing into a single decision: agents can re-optimize their levels of h and d only if they pay the adjustment cost $\phi p h$.

¹³There is a trivial way of turning this into a general equilibrium model, by assuming the consumers live in a small open economy that can lend to the rest of the world at the rate r and borrow from the rest of the world at the rate r_m and assuming there are competitive firms that can transform houses into consumption goods and vice versa at the rate p .

The formulation of the problem in continuous time allows us to use the methods and algorithm proposed in [Ahdou et al. \(2017\)](#). The details are in the online appendix.

4 Slow deleveraging

In this section, we study how consumption and household balance sheets adjust following a permanent shock to house prices.

The crucial feature of the model is that the household problem looks different in periods of adjustment and non adjustment.

When household adjust they liquidate their net housing wealth

$$(1 - \phi) ph - d,$$

and use it to re-optimize. Therefore, when this happens, p matters directly for their consumption decisions.

When household do not adjust, their past housing and mortgage choices determine the fixed payment $m + \delta ph$ they have to make each period. Their problem in these periods is just to take their income net of housing/mortgage payments $y - m - \delta ph$ and choose what portion to consume and what portion to save in liquid assets. A shock to house prices would seem to have little effect on this problem. In fact, the only immediate effect is to reduce the cost of house maintenance cost δph (which we assume proportional to p), thus increasing the household's net resources. However, households are forward looking and are aware that if they decide to adjust in the future, their total wealth would include not just the liquid savings a , but also their net housing wealth. This is the main channel by which a shock to p affects their decisions. In this section, we show that this channel is strong enough to cause a contraction in consumption even though consumers adjust infrequently.

4.1 Parameter choices

To simulate the economy we use the parameters in Table 1. Let us discuss briefly how we choose them and what are some of their implications for the model steady state moments.

The share of non-housing consumption α determines both the share of consumption in housing services and the equilibrium value of the housing stock. The share of consumption in housing services in U.S. national income accounts is pretty stable around 18% in recent

α	0.85	share of non-housing consumption
ρ	0.06	discount factor
σ	2	coeff. of relative risk aversion
δ	0.02	house maintenance cost
ϕ	0.04	transaction cost
θ	0.8	maximum loan to value ratio
τ	30	length of mortgage
y_1, y_2	0.1, 0.2	income levels
$\lambda_{y_2 y_1}, \lambda_{y_1 y_2}$	0.1, 0.1	income process intensity
r	0.03	interest rate

Table 1: Model parameters

years. Matching that number would require setting α at a lower value of 0.82. On the other hand, with $\alpha = 0.85$ we obtain a ratio of steady state value of housing over income, H/Y , equal to 2.7. The median value for homeowners in the Survey of Consumer Finances in 2001 is 2.6, so $\alpha = 0.15$ seems a reasonable choice.¹⁴

We set the same rate $r = r_m$ for liquid assets and mortgages. This simplifies computations, as it implies that consumers always choose the maximum debt level $d = \theta h$ when adjusting. This result holds because if the constraint is slack at the moment of adjustment, consumers always gain by borrowing more and putting the extra money in the liquid asset. The reason is that liquid savings are subject to the non-negativity constraint $a \geq 0$, so increasing their level at the moment of adjustment only gives more room for maneuver in future periods and it has no opportunity cost in terms of forgone interests if $r = r_m$. Of course, this is a strong simplification, but it eliminates a state variable from the problem as m and h are always linearly related under this assumption. Given that r is both the rate on liquid assets and on mortgages, we choose a value of $r = 3\%$. Setting the discount factor $\rho = 0.06$ implies that the median value of the net wealth $a + (1 - \phi)ph - d$ is equal to 1.97 times average income Y , which is a bit on the high side but not unreasonable.¹⁵ Another moment one can look at, to gauge the implications of assuming $\rho = 0.06$, is the marginal propensity to consume, which is equal to 0.139 in steady state. The coefficient of relative risk aversion is set to 2.

For the income process, we choose a symmetric transition process with $\lambda_{y_2|y_1} = \lambda_{y_1|y_2}$. To chose the parameters, we look at their implications for the logarithm of annual income (aggregated over time). With the parameters in Table 1 this variable has a coefficient of autocorrelation of 0.87, and its standard deviation, conditional on the previous year income,

¹⁴See Table 2 in Berger et al. (2018).

¹⁵Table 2 in Berger et al. (2018) shows the same moment in the 2001 SCF is 1.44.

is equal to 0.16. Both numbers are in a reasonable range compared to empirical studies of idiosyncratic income risk.¹⁶

The maintenance cost factor δ is set to 2%. As argued in footnote 12, this parameter is analogous to a depreciation rate and 2% is close to the depreciation rate used in Berger et al. (2018) to match BEA data. The maximum loan to value ratio of 0.8 is conservative as mortgage originations with lower downpayments than 20% are not unusual. As argued above the consumers in our model always choose to set d to its maximum level at adjustment, so a ratio of 0.8 yields a relatively high level of debt to housing D/H , which equals to 0.6 in steady state.

The parameter τ is the length of the mortgage contract, we set it to 30 years and later experiment with shorter maturities.

The parameter ϕ which determines the fixed adjustment cost is set to 0.04. Notice that households must pay the cost $\phi p h$ not only when they trade houses but also when they prepay or refinance their mortgage. Our choice implies that in steady state the rate at which households trade or refinance is 0.088, which implies that in a year a household has a probability of 8.4% to trade or refinance at least once. This number is a bit higher than realistic frequencies of pure house sale transactions and a bit lower than frequencies of sales and refinancing combined.¹⁷ Given that our model does not distinguish the two, this seems a reasonable compromise.

4.2 Simulation results

In Figure 1, we plot the responses of consumption and various measures of consumers' balance sheets to a 10% unexpected reduction in the house price p in the first 50 years following the shock. In particular, in panel (a) we plot the path of consumption C_t in percentage deviation from the steady state; in panel (b) we plot the debt to house value ratio $D_t/(p_t H_t)$ in deviation from its steady state value; in panel (c) we plot the net worth $A_t + (1 - \phi) p_t H_t - D_t$, as a ratio to its steady state value; in panel (d) we plot the fraction of agents who hold on to the mortgage made before the shock.

Consumption drops by a bit more than 1% in the first year after the shock, the drop is larger in the following years, peaks at 1.3% in the third year after the shock and then slowly goes back to steady state. The elasticity on impact is about 0.1, which is in the range

¹⁶The calibration of Berger et al. (2018) targets moments from Floden and Lindé (2001), which give an autocorrelation equal to 0.91 and a standard deviation of innovation equal to 0.21. So the calibration here is a bit more conservative, by targeting a lower level of idiosyncratic risk.

¹⁷See Berger et al. (2018).

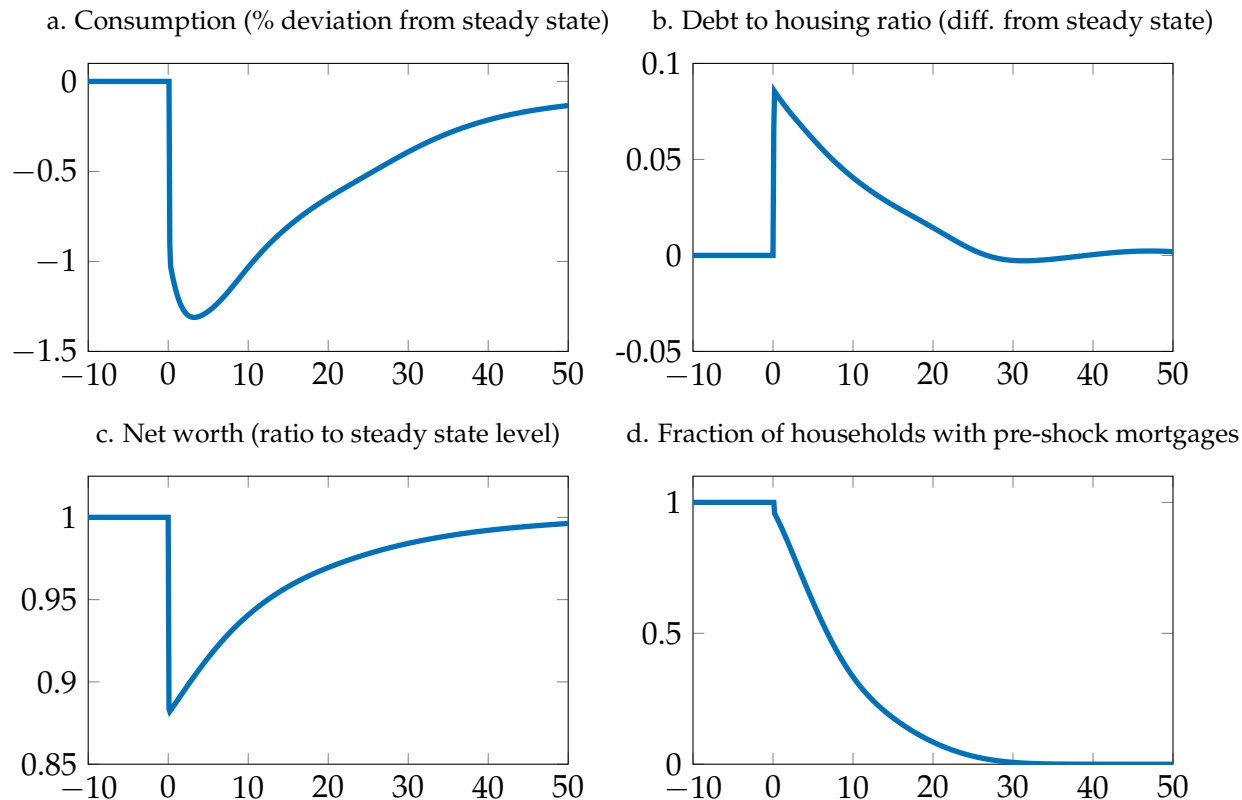


Figure 1: Responses to a 10% reduction in the house price at $t = 0$

of empirically estimated elasticities.¹⁸ But the most striking observation is the very slow speed at which consumption goes back to normal.

Looking at panel (b), notice that household leverage actually increases on impact after the shock, consistently with the observation of [Justiniano et al. \(2015\)](#).¹⁹ The reason is that mortgages are long-term and are infrequently adjusted, so the stock of debt moves slowly, while the value of the housing stock drops immediately as p_t falls. So the consumers in this model are not experiencing any form of “forced deleveraging” as the consumers in [Guerrieri and Lorenzoni \(2017\)](#). Rather, the consumers here are essentially experiencing a large drop in net worth, as shown in panel (c), and they are responding with a large adjustment in consumption.

A way of interpreting the consumption adjustment is that consumers in this class of models, unlike certainty-equivalent permanent-income consumers, aim to rebuild net worth after a shock. Proposition 1 shows that the consumers in the model, in aggregate,

¹⁸A discussion of the empirical literature is in [Berger et al. \(2018\)](#).

¹⁹We plot the ratio $D_t/(p_t H_t)$ for ease of interpretation. Leverage, defined as assets to net worth, is equal to $(A_t + p_t H_t)/(A_t + p_t H_t - D_t)$ and moves in a very similar fashion.

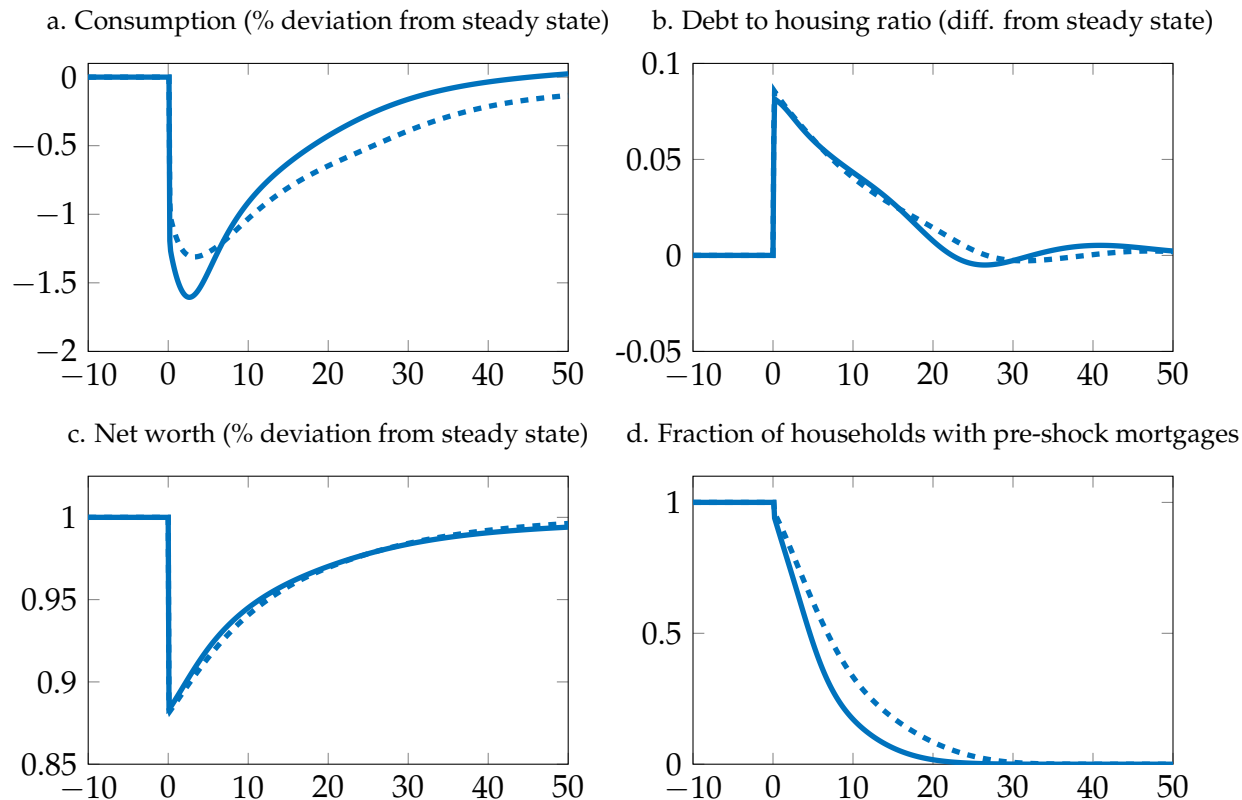


Figure 2: Responses to a house price shock: low adjustment costs
Note: Solid line: low adjustment costs, $\phi = 0.005$. Dashed line: Baseline, $\phi = 0.04$.

want to go back exactly to their pre-shock level of net worth. Since they have lost roughly 11.6% of net worth, as shown in panel (c), their savings have to be larger for a protracted period of time, which requires lower consumption.

Finally, in panel (d) we just confirm that long-run mortgages and adjustment costs are playing an important role, as the fraction of consumers who have not adjusted only falls slowly.

4.3 Changing adjustment costs and debt maturity

We now analyze what happens when we change the parameters ϕ and τ that govern the adjustment costs and debt maturity in our model. Since our aim is to understand how different ingredients contribute to our result, we do not change other parameters. However, once we change ϕ and τ the steady state distribution of asset positions changes. Therefore, to keep the simulations comparable, we change the size of the shock Δp so as to obtain the same reduction in aggregate net worth.

In Figure 2, we compare our baseline simulation (dashed line) to a simulation with a much lower value of transaction cost, equal to $\phi = 0.005$ (solid line). As we see in panel (a), the effect of the shock on consumption is larger on impact and it reverts faster to zero. However, the striking observation is that a very large reduction in transaction costs has an overall small quantitative effect on the consumption response. In other words, even though the housing wealth becomes much more liquid in this calibration, its effect on consumption decisions is similar. In the next section, we provide some explanation for this surprising observation.

Notice that with this value of ϕ the rate at which consumers trade or refinance is 0.1968, more than double than in the baseline. The immediate effect of this is seen in panel (d), where consumers switch to new mortgages more quickly. A bit surprisingly, this does not have much effect on the speed at which consumers are reducing leverage in panel (b). The reduction in consumption seen in panel (a) does speed up the convergence of net worth in panel (c), although the difference in the saving pattern is harder to visualize looking at the dynamics of a stock variable.

We now turn to look at the effect of changing debt maturity. In Figure 3, we compare our baseline to a model where mortgages are repaid faster, with $\tau = 15$. With a lower τ the mortgage balance falls faster over time if agents do not readjust. This implies that reducing τ also has the effect of increasing the frequency of adjustment, because agents who want to maximize their borrowing capacity need to refinance more often. As a consequence, Figure 3 shows that consumption responses are similar as in the low ϕ case of Figure 2: a sharper and more short-lived contraction in consumption. The main difference with the low ϕ case is that with shorter maturities the reduction in leverage is faster as shown in panel (b). The reason is mechanical: when agents do not adjust debt goes down faster with a lower τ , so for non-adjusters there is a faster deleveraging process.

The overall take away from this set of simulations is that the shock to balance sheets has a large and long lasting effect on consumer spending, even in a parametrization with long-maturity mortgages and high fixed costs of adjustment. We now turn to provide an interpretation for this result.

5 The precautionary value of housing wealth

In this section, we aim to provide a better understanding of why consumers care about their housing net wealth $(1 - \phi)ph - d$ even though they only have infrequent access to it.

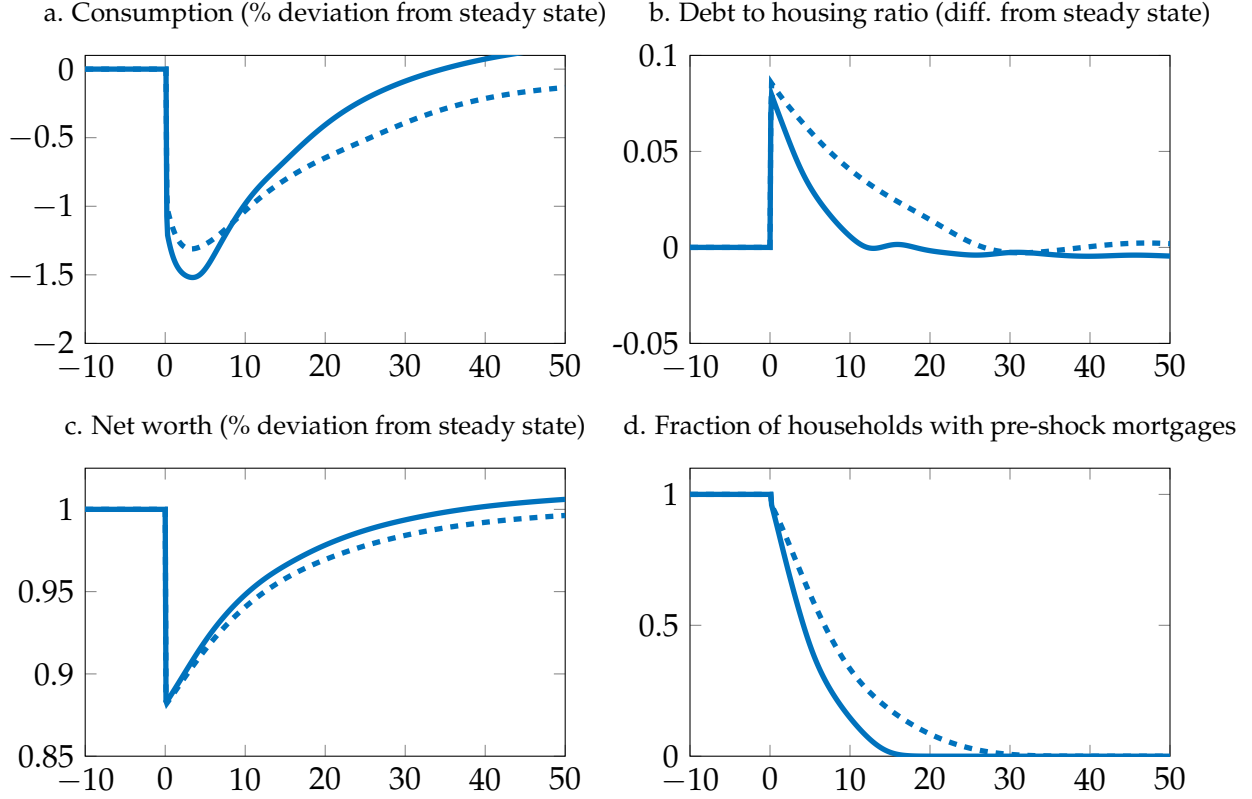


Figure 3: Responses to a house price shock: shorter debt maturity
Note: Solid line: low debt maturity, $\tau = 15$. Dashed line: baseline, $\tau = 30$.

In particular, we explore the idea that housing wealth provides a precautionary buffer that consumers can tap when they are hit by negative income shocks. To do so, we compare the predictions of our baseline model to the predictions of a model in which consumers also trade housing infrequently, but cannot control in what states of the world they trade.

The model is identical to the model of Section 3, except that now agents cannot choose whether or not to adjust at any given point in time. Agents can only adjust at discrete time intervals that arrive with Poisson intensity π . When agents are allowed to adjust they do so at zero cost (so $\phi = 0$).

Given the assumptions just made, the Hamilton-Jacobi-Bellman equation (4) is replaced by the following:

$$\begin{aligned}
 \rho V(a, d, h, m, y) = & \max_c u(c, h) + \partial_a V(a, d, h, m, y)(ra + y - c - m - \delta ph) + \\
 & + \partial_d V(a, d, h, m, y)(r_m d - m) + \lambda_{y'|y}(V(a, d, h, m, y') - V(a, d, h, m, y)) + \\
 & + \pi(J(a + ph - d, y) - V(a, d, h, m, y)), \quad (7)
 \end{aligned}$$

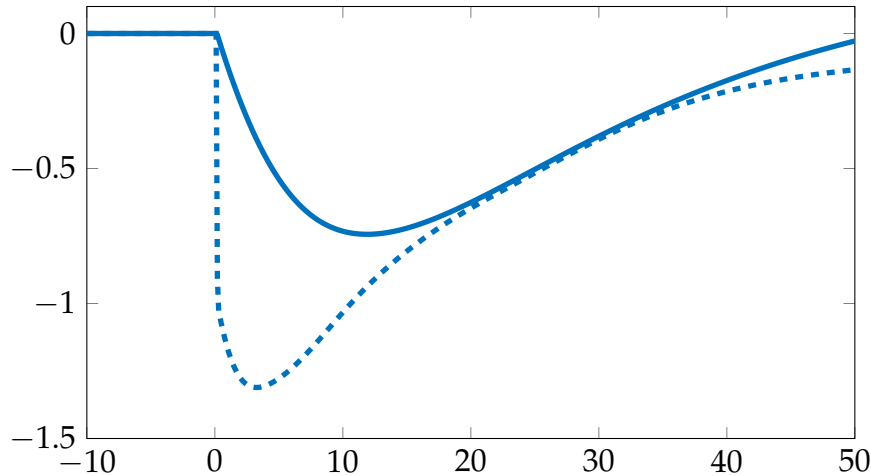


Figure 4: Consumption responses to a house price shock: Poisson adjustment model
Note: Solid line: Poisson model. Dashed line: baseline.

where J is defined by the same optimization problem (6) in Section (3). We are basically replacing a state-dependent adjustment model with a time-dependent adjustment model.²⁰

To simulate the Poisson model we use the same parameters of Section (4), except that we set $\phi = 0$ and set the Poisson arrival rate $\pi = 0.088$ to replicate the frequency of adjustment of the baseline model in steady state.

In the Poisson model housing wealth is a worse form of insurance. To see why, notice that in our baseline model, if a consumer with income y_2 switches to income y_1 , the conditional intensity of adjusting is 0.1580, which is almost double the unconditional intensity of adjusting of 0.088. In the Poisson model, instead, the intensity of adjusting is not allowed to vary conditional on income shocks and stays at 0.088. In other words, consumers have less access to their housing wealth when they need it.²¹

Figure 4 shows the consumption response in the Poisson model (solid lines) compared to the baseline (dashed lines). The response in the Poisson model is smaller and slower, the maximum contraction in consumption in the Poisson model is less than half the maximum contraction in the baseline. This simulation provides support to the view that a reason why

²⁰Notice that we do not need to impose that agents choose the maximum between $J(a + ph - d, y)$ and $V(a, d, h, m, y)$ when hit by the Poisson shock, because adjustment costs are zero, so

$$J(a + ph - d, y) \geq V(a, d, h, m, y)$$

by construction.

²¹Notice that since consumers can adjust when the Poisson shock hits and $\phi = 0$, the Poisson model reduces, but does not fully eliminate the insurance value of housing. It would be interesting to explore alternative experiments in which house trading is completely shut down, but that would require some form of re-calibration to keep the alternative model comparable to the baseline.

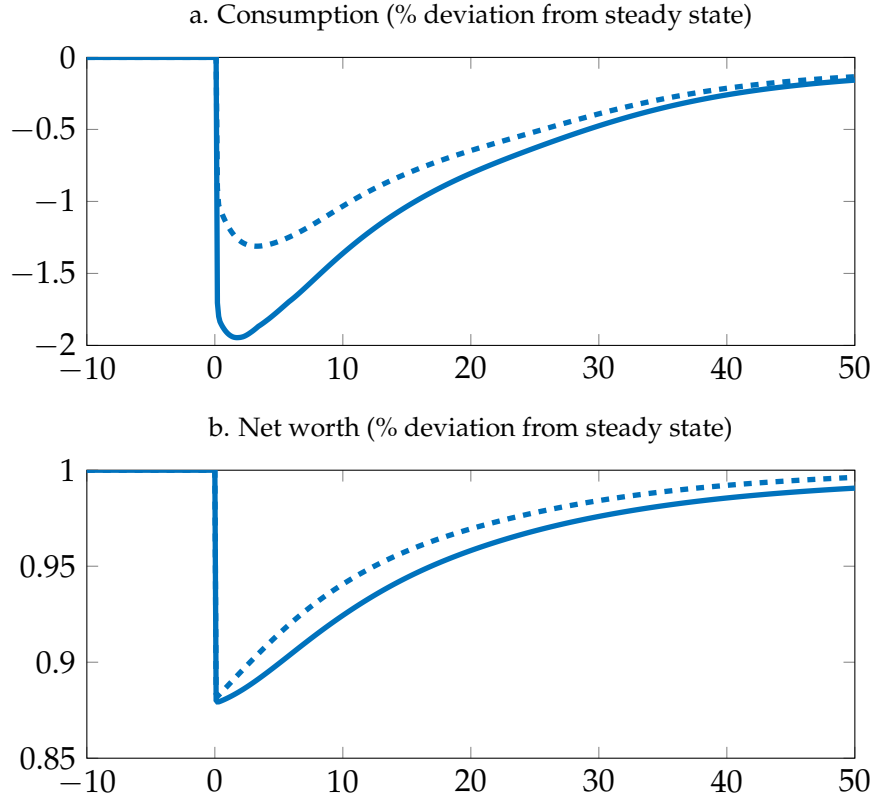


Figure 5: Responses to a house price shock: endogenous income path

consumption is responsive to the value of housing wealth in our baseline is that consumers value the option to tap housing wealth in the event of a negative shock. When that option is less available, the value of housing wealth has a smaller effect on consumer decisions.

6 A dynamic paradox of thrift

Keynes' paradox of thrift argues that an increase in the saving rate may be self-defeating if it causes a contraction in output. In the context of a dynamic model the same logic implies that the household deleveraging process may be slowed down if the attempt of consumers to re-build their net worth causes a reduction in incomes.

Here we examine this force by making the income process endogenous in a very simple way. Namely, we assume that incomes levels vary over time and are equal to

$$y_t = z_t Y_t \tag{8}$$

where z_t is an idiosyncratic income shock and Y_t is aggregate income. The idiosyncratic shock z_t follows the same binary process with Poisson switching we used in our baseline, with two states (z_1, z_2) that are independent of time. For the aggregate Y_t we assume that it is constant at Y_{ss} in steady state. After the house price shock at $t = 0$ the aggregate income path Y_t is endogenous and is equal to

$$Y_t = \left(\frac{C_t}{C_{ss}} \right) e^{-kt} Y_{ss}. \quad (9)$$

This assumption implies that when consumption falls below its steady state level it drives down aggregate output. The presence of the exponent e^{-kt} implies that the effect dies down over time, maybe because of policy interventions. Another interpretation of the term e^{-kt} is that it captures some element of bounded rationality in anticipating the effects of the shock.²² Notice that here we are simply appending an ad-hoc assumption about the aggregate income path to a partial equilibrium model, so the analysis here is only meant to be suggestive of the effects that can play out in a full-fledged new Keynesian model with endogenous output. We still believe this simple exercise provides valuable hints and usefully isolates the feedback mechanism that comes from balance-sheet adjustment, a mechanism that will likely interact with other general equilibrium forces in a fuller model.

To solve the model after the house price shock, we now need to find a fixed point. That is, we look for a path $\{Y_t\}$ such that, given the process for individual incomes (8), consumers optimize, and such that aggregate income satisfies (9). In Figure 5 we show the response of aggregate consumption and net worth in the model with endogenous aggregate output (solid line) and compare it to the baseline with fixed output (dashed line). To compute this path we use the parameters in the baseline and set the parameter k in (9) equal to 0.02.

The presence of endogenous income has two effects: it amplifies the consumption decline, especially in the short run, and it slows down the adjustment of net worth. Notice that the static marginal propensity to consume in the model is not particularly high (it is 0.139 in steady state). So the fact that the short-run consumption response is amplified by almost 60% shows the power of intertemporal effects in this environment (see [Auclert et al. \(2018\)](#)).

²²The effect of $k > 0$ is similar to the “cognitive discounting” parameter in [Gabaix \(2016\)](#). In this interpretation, the paths in Figure 5 show the impact effect at the moment of the shock and the agents’ expectations about the recovery path.

7 Conclusions

In this paper, we discussed some recent advances in building heterogeneous agent models in which the adjustment of household balance sheets is an important determinant of aggregate consumption. We have then presented a model in which trading houses is subject to substantial transaction costs and mortgage contracts are long term, so housing wealth is illiquid. Despite this illiquidity, net housing wealth plays an important role in determining consumption.

Our argument was developed in the context of a partial equilibrium exercise in which house prices are exogenous, and we have focused on the effect of a simple one time, permanent shock to house prices. We hope the mechanisms identified here will provide useful building blocks for general equilibrium models in which the underlying shocks causing house price movements are modeled explicitly and in which other forces can lead households to adjust their balance sheets. The experiment in Section 6 suggest that models with an endogenous determination of aggregate output and keynesian features may provide an important amplifying effect for impulses coming from balance sheet adjustment.

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