## Problem Set 4

## 1 Computing a menu cost model

Consider the partial equilibrium menu cost model seen in class but suppose the process for $z$ is

$$
z^{\prime}=z+\epsilon
$$

where $\epsilon=\Delta>0$ with probability $1 / 2$ and $\epsilon=-\Delta$ with probability $1 / 2$.

1. Start from

$$
V^{(0)}=-x^{2}
$$

and iterate on the value function to find the optimal solution.
2. See what happens to the three values $\underline{x}, x^{*}, \bar{x}$ as you change $\pi$ and as you change $\Delta$.

## 2 Equity injections and welfare

This problem analyzes the welfare effects of a "capital injection" in a model with financial frictions. There are two periods, 0 and 1 . Consumers and entrepreneurs have a linear utility function, $c_{0}+c_{1}$. Consumers have a large endowment of consumption goods in each period and a unit endowment of labor in period 1 , which they sell inelastically on a competitive labor market at the price $w_{1}$. Entrepreneurs have a given endowment of consumption goods $n_{0}=1$ in period 0 , and no capital. Then they borrow $b_{1}$ and invest $k_{1}$. In period 1 they hire workers at the wage $w_{1}$ and produce consumption goods according to the Cobb Douglas production function

$$
y_{1}=k_{1}^{\alpha} l_{1}^{1-\alpha} .
$$

The entrepreneurs face the collateral constraint

$$
b_{1} \leq \lambda\left(y_{1}-w_{1} l_{1}\right),
$$

where $\lambda \in(0,1)$ is a given scalar (think that in period 1 the entrepreneurs can run away with a fraction $(1-\lambda)$ of the firm's profits). Assume the consumers endowment is large enough that the gross interest rate is always 1 in equilibrium.
(i) State the entrepreneurs' problem. Argue that the entrepreneurs will always choose $l_{1}$ to maximize profits in period 1 and that then profits are a linear function of the capital stock $k_{1}$, that is,

$$
y_{1}-w_{1} l_{1}=R\left(w_{1}\right) k_{1} .
$$

Restate the entrepreneurs' problem as a simpler linear problem.
(ii) Argue that if $\lambda R\left(w_{1}\right)<1 \leq R\left(w_{1}\right)$ the entrepreneur's problem is well defined and the entrepreneur's demand for capital is finite. Derive it. What happens if $\lambda R\left(w_{1}\right) \geq 1$ ? What if $R\left(w_{1}\right)<1$ ?
(iii) Show that there is a cutoff $\hat{n}$ such that if $n_{0} \geq \hat{n}$ the entrepreneurs can finance the first-best level of capital $k_{1}=k_{1}^{*}=\alpha^{1 /(1-\alpha)}$, and the collateral constraint is not binding.
(iv) Show that if $n_{0}<\hat{n}$ there is an equilibrium where the entrepreneurs are constrained and the equilibrium value of $k_{1}$ is an increasing function of $n_{0}$.
(v) Suppose the consumers pay a lump-sum $\operatorname{tax} \tau$ in period 0 . The receipts from the tax are transferred directly to the entrepreneurs. Derive an expression for the expected utility of consumers and entrepreneurs as a function of $\tau$.
(vi) Show, analytically or by numerical example, that there is a non-monotone relation between $\tau$ and the expected utility of the consumers. In particular, if $n_{0}$ is sufficiently small, a small positive tax can increase the utility of both consumers and entrepreneurs.

