Menu costs

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1 Partial equilibrium

- Want to minimize distance for optimal relative price z (in log terms)
- z follows random walk
$$z' = z + \epsilon$$
- Loss function
$$(p_i - p - z)^2$$
- Fixed cost ϕ of changing p_i
- State variable
$$x = p + z - p_i$$
- Law of motion if price not adjusted
$$x' = p' + z' - p_i = x + \pi + \epsilon'$$
where π inflation
- Let $V(x)$ be the value if no adjustment occurs
- Bellman equation
$$V(x) = -x^2 + \beta E \max\{V(x + \pi + \epsilon'), V^* - \phi\}$$
where $V^* = \max_x V(x)$
- The function V is quasi-concave and bounded. We can apply contraction mapping theorem to that function space
- There exists two values $\underline{x}, \overline{x}$ such that it is optimal to keep price unchanged if $x \in (\underline{x}, \overline{x})$ and is optimal to set $x = x^*$ otherwise

2 General equilibrium

- Suppose money (or nominal demand) follows geometric random walk (with drift)

$$M_t = M_{t-1} e^{\epsilon_t}$$

with binary shock

$$\epsilon_t = \begin{cases} \Delta & \text{with prob. } q \\ 0 & \text{with prob.} 1-q \end{cases}$$

- Output

$$C_t = Y_t = \frac{M_t}{P_t}$$

- Preferences

$$\log C_t - N_t$$

so wages

$$\frac{1}{C_t} \frac{W_t}{P_t} = 1$$

- Production function

$$Y_{it} = A N_{it}$$

- Firms maximize profits

$$\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{1}{A} \frac{W_t}{P_t} Y_{i,t}$$

facing demand function

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\sigma} \frac{M_t}{P_t}$$

- So profits are

$$\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{1}{A} \frac{W_t}{P_t} Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{1-\sigma} C_t - \frac{1}{A} \left(\frac{P_{i,t}}{P_t} \right)^{-\sigma} C_t^2$$

- Conjecture, real consumption is constant and unaffected by money shocks

$$C = \frac{M_t}{P_t}$$

so

$$\left(\frac{P_{i,t}}{P_t} \right)^{1-\sigma} C - \frac{1}{A} \left(\frac{P_{i,t}}{P_t} \right)^{-\sigma} C^2$$

- Then analyze problem with individual state variable
$$X = \frac{P_{it}}{P_t}$$
define
$$\Pi\left(X\right) = CX^{1-\sigma} - A^{-1}C^2\left(X\right)^{-\sigma}$$

- Bellman equation
$$V\left(X\right) = \Pi\left(X\right) + \beta E \max\{V\left(X'\right), V^* - \phi\}$$

- Optimal policy
$$P_{it} = X^* M_t$$
if
$$P_{it} > \bar{X} M_t$$
or
$$P_{it} < \underline{X} M_t$$
and
$$P_{it} = P_{it-1}$$
otherwise

- Equilibrium uniform distribution of prices
$$P_{it}/M_t = X^*$$
$$P_{it}/M_t = X^* e^{-\Delta}$$
$$...$$
$$P_{it}/M_t = X^* e^{-(L-1)\Delta}$$
each with probability $1/L$

- To find L just look for smallest integer such that
$$X^* e^{-L\Delta} < \underline{X}$$

- Aggregating
$$\frac{1}{C} = \frac{P_t}{M_t} = \left\{\int \left(\frac{P_{it}}{M_t}\right)^{1-\sigma} di\right\}^{\frac{1}{1-\sigma}} = \left\{\frac{1}{L}\sum_{l=0}^{L-1}\left(e^{-\Delta l}\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$$
from this equation, given L and Δ we get C

- Fixed point problem in C
- To complete equilibrium, check that distribution is uniform

- Yes, because any time there is a money shock $1/L$ prices would go from

$$P_{it-1}/M_{t-1} = X^* e^{-(L-1)\Delta}$$

to

$$\frac{P_{it-1}}{M_t} = \frac{P_{it-1}}{M_{t-1}} \frac{M_{t-1}}{M_t} = X^* e^{-L\Delta} < \underline{X}$$

so they jump to X^*, all other prices stay unchanged