411-3 Notes: Financial frictions

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1 Financial frictions in a stochastic model

- Stochastic model with non state contingent debt, collateral constraints and aggregate investment

- A rich problem with a two dimensional state space

- Related to BGG (but not linearized) and Brunnermeier-Sannikov

- Entrepreneurs risk neutral, with discount factor β

- Lenders risk neutral, with discount factor q

- Capital can be produced now, with an adjustment cost function

- Entrepreneurs only agents that can hold capital

- Instead of the inferior technology, when entrepreneurs are selling capital goods, these are turned back into consumption goods

- Entrepreneur's budget constraint

$$c_t + G\left(k_{t+1}, k_t\right) = A_t F\left(k_t, l_t\right) - w_t l_t - b_t + q b_{t+1}$$

- G is a CRS investment cost function, which includes adjustment costs

$$G(k', k) \equiv k' - (1 - \delta)k + \zeta(k' - (1 - \delta)k)^2/k$$

- Collateral constraint
$$b_{t+1} \leq \theta p_{t+1} k_{t+1}$$
for all realizations of p_{t+1} that have positive probability at t

- More below on the price p_t at which capital can be sold

- A_t, w_t, p_t driven by Markov process s_t

- Crucial assumption: G and F are constant returns to scale

- Then the value function must satisfy

$$V\left(k_t, b_t, s_t\right) = v\left(b_t/k_t, s_t\right) k_t$$

  for some function v

- Bellman equation

$$v\left(\tilde{b}_t, s_t\right) k_t = \max_{c_t, l_t, \tilde{b}_{t+1}, k_{t+1}} c_t + \beta E_t\left[v\left(\tilde{b}_{t+1}, s_{t+1}\right)\right] k_{t+1}$$

  subject to

$$c_t + G\left(k_{t+1}, k_t\right) = A_t F\left(k_t, l_t\right) - w_t l_t - \tilde{b}_t k_t + q \tilde{b}_{t+1} k_{t+1}$$

  and

$$\tilde{b}_{t+1} \leq \theta \underline{p}_{t+1|t}$$

- Optimality for k_{t+1} yields

$$\beta E_t\left[v\left(\tilde{b}_{t+1}, s_{t+1}\right)\right] + q \lambda_t \tilde{b}_{t+1} = \lambda_t G_1\left(k_{t+1}, k_t\right)$$

- If it's optimal to consume $\lambda_t = 1$, in this case

$$G_1\left(k_{t+1}, k_t\right) = \beta E_t\left[v\left(\tilde{b}_{t+1}, s_{t+1}\right)\right] + q \tilde{b}_{t+1}$$

- The LHS is marginal Q the RHS is average Q (Abel 1982 and Hayashi 1982)

- If the non-negativity of consumption is never binding this model yields standard Q theory predictions: asset price over capital stock is a sufficient statistic for the investment rate k_{t+1}/k_t

- In general we can have $\lambda_t > 1$ which implies marginal Q smaller than average Q: firms have an incentive to issue more claims to finance investment, but entrepreneurs cannot buy these claims, since they are at $c_t = 0$

- If $\lambda_t > 1$ it means that either the collateral constraint is binding today or it will be binding in the future

- Optimality condition with respect to $\tilde{b}_{t+1}$ is

$$\lambda_t q k_{t+1} + \beta E_t\left[\frac{\partial v\left(\tilde{b}_{t+1}, s_{t+1}\right)}{\partial \tilde{b}}\right] k_{t+1} - \mu_t = 0$$

  and using envelope condition

$$q \lambda_t = \beta E_t\left[\lambda_{t+1}\right] + \mu_t/k_{t+1}$$

- Envelope condition for k_t is

$$v\left(\tilde{b}_t, s_t\right) = \lambda_t \left[A_t F_k\left(k_t, l_t\right) - G_2\left(k_{t+1}, k_t\right) - \tilde{b}_t\right]$$

- Combining with optimality for k_{t+1}

$$\lambda_t = \frac{\beta E_t\left[v\left(\tilde{b}_{t+1}, s_{t+1}\right)\right]}{G_1\left(k_{t+1}, k_t\right) - q\tilde{b}_{t+1}} = \frac{\beta E_t\left[\lambda_{t+1}\left[A_{t+1} F_{k,t+1} - G_{2,t+1} - \tilde{b}_{t+1}\right]\right]}{G_{1,t} - q\tilde{b}_{t+1}}$$

$$\tag{1}$$

- Suppose now entrepreneurs can trade used capital from other entrepreneurs, before employing the adjustment cost technology

- Then to reach capital k_{t+1} they will choose to minimize total cost of achieving it

$$\min_{\hat{k}_t} G\left(k_{t+1}, \hat{k}_t\right) + p_t\left(\hat{k}_t - k_t\right)$$

- Representative entrepreneur, so no trade and $\hat{k}_t = k_t$ in equilibrium

- First order condition

$$p_t = -G_2\left(k_{t+1}, k_t\right)$$

gives us the price of capital that appears in the collateral constraint

- Then the optimality condition can be rewritten as

$$E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{A_{t+1} F_{k,t+1} + p_{t+1} - \tilde{b}_{t+1}}{G_{1,t} - q\tilde{b}_{t+1}}\right] = 1$$

- This is an asset pricing equation where

$$\beta\frac{\lambda_{t+1}}{\lambda_t}$$

is the stochastic discount factor of the entrepreneurs and

$$\frac{A_{t+1} F_{k,t+1} + p_{t+1} - \tilde{b}_{t+1}}{G_{1,t} - q\tilde{b}_{t+1}}$$

is the levered return on entrepreneurial capital

- We can also rewrite optimality for borrowing ratio as an asset pricing equation

$$1 = E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{q}\right] + \frac{\mu_t}{q\lambda_t}\frac{1}{k_{t+1}}$$

which implies

$$E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{q}\right] \leq 1$$

here the expected return on bonds, discounted with the discount factor $\beta\lambda_{t+1}/\lambda_t$ can be < 1 if the collateral constraint is binding

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- Rewrite (1) as

$$E_t \left[\beta \lambda_{t+1} \left[A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1} - \tilde{b}_{t+1} \right] \right] = \lambda_t \left(G_{1,t} - q \tilde{b}_{t+1} \right)$$

and then as

$$\begin{aligned}
E_t \left[\beta \lambda_{t+1} \left[A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1} \right] \right] &= \lambda_t G_{1,t} - \left(\lambda_t q - E_t \left[\beta \lambda_{t+1} \right] \right) \tilde{b}_{t+1} \\
&= \lambda_t G_{1,t} - \mu_t \tilde{b}_{t+1}
\end{aligned}$$

so

$$E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1}}{G_{1,t}} \right] \leq 1$$

- Agents are willing to accept a lower return on capital, since holding capital helps to relax the collateral constraint

- Using the same condition and $q \lambda_t \geq E_t \left[\beta \lambda_{t+1} \right]$ we also get that if

$$\lambda_{t+1} \text{ and } A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1} - \tilde{b}_{t+1}$$

are negatively correlated we have

$$q \lambda_t E_t \left[A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1} - \tilde{b}_{t+1} \right] \geq E_t \left[\beta \lambda_{t+1} \left[A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1} - \tilde{b}_{t+1} \right] \right] = \lambda_t \left(G_{1,t} - \right.$$

which imply

$$E_t \left[\frac{A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1}}{G_{1,t}} \right] \geq \frac{1}{q}$$

so the expected rate of return on capital is greater than the risk free interest rate

- New possibility: the collateral constraint can be slack even though the rate of return on entrepreneurial capital is greater than $1/q$

- Rewrite condition as

$$E_t \left[\beta \lambda_{t+1} \left[A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1} - \tilde{b}_{t+1} \right] \right] = \lambda_t \left(G_{1,t} - q \tilde{b}_{t+1} \right)$$

- If constraint is slack $\mu_t = 0$ and this becomes

$$E_t \left[\beta \lambda_{t+1} \left[A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1} \right] \right] = \lambda_t G_{1,t}$$

or

$$E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{A_{t+1} F_k \left(k_{t+1}, l_{t+1} \right) + p_{t+1}}{G_{1,t}} \right] = 1$$

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- If there are no shocks we have

$$\beta \frac{\lambda_{t+1}}{\lambda_t} = q$$

  and

$$\frac{A_{t+1} F_k (k_{t+1}, l_{t+1}) + p_{t+1}}{G_{1,t}} = \frac{1}{q}$$

  so collateral constraint can be slack only if investment is efficient at date t

- With risk, rate of return on entrepreneurial capital is correlated with λ_{t+1}

- Temporary productivity shocks generate negative correlation: high return on entrepreneurial wealth, high net worth, economy closer to efficient investment, lower return on entrepreneurial capital

- Then

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{A_{t+1} F_k (k_{t+1}, l_{t+1}) + p_{t+1}}{G_{1,t}} \right] < E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] E_t \left[\frac{A_{t+1} F_k (k_{t+1}, l_{t+1}) + p_{t+1}}{G_{1,t}} \right]$$

  so

$$E_t \left[\frac{A_{t+1} F_k (k_{t+1}, l_{t+1}) + p_{t+1}}{G_{1,t}} \right] > \frac{1}{q}$$

- This is a form of precautionary behavior: entrepreneurs are avoiding excess leverage because they anticipate states of the world in which the rate of return on their wealth will be higher than today (high λ_{t+1}/λ_t)

- Notice that entrepreneurs are risk neutral so "precautionary behavior" is really driven by general equilibrium forces

2 Empirical

- The investment model with adjustment costs leads to some empirical predictions

- First one was the following:

  - if there are no financial frictions, F and G are CRS and $G'' > 0$, then Tobin's Q is a sufficient statistic for the investment rate I_t/K_t (see above Abel (1982) and Hayashi (1982))

- Fazzari-Hubbard-Petersen (1988) focus on this prediction, show it is rejected in the data as adding cash flow to a regression of I/K on Tobin's Q, cash flow's coefficient is significantly different from zero

- However, this is really testing all the assumptions of Hayashi's theorem
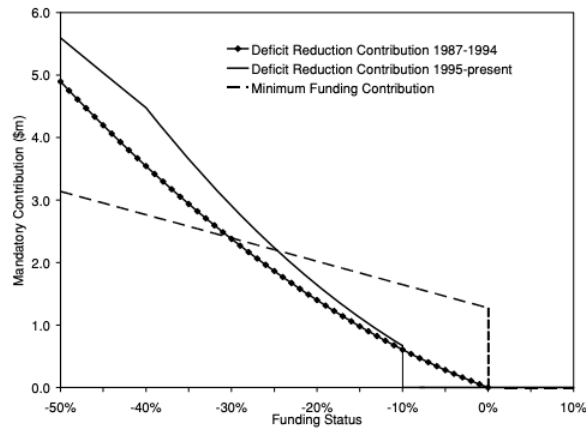
Figure 2. Mandatory pension contributions. A firm's required pension contribution is the maximum of two components: The minimum funding contribution (MFC) and the deficit reduction contribution (DRC). The graph shows mandatory contributions in dollar terms for a firm with sample mean characteristics (liabilities of $37.3m and "normal cost" of $1.3m). The DRC as a percentage of firm funding is given by $min\{0.30,[0.30 - 0.25 * (funding\ status - 0.35)]\}$ for 1987 to 1994 and $min\{0.30,[0.30 - 0.40 * (funding\ status - 0.60)]\}$ for 1995 and later. The minimum funding contribution is defined as the "normal cost" plus 10% of the ERISA underfunding. The "normal cost" differs on a firm-by-firm basis depending on the accounting cost method and the rate of liability accrual.

Figure 1:

- If we want clean evidence that financing frictions matter the simplest experiment would be to give a pure transfer to some firms and see if their investment plans change

- Absent financial frictions investment plans should be independent of cash available

- Researchers have been looking for quasi-experiments with these features

- One of the cleanest recent examples is Rauh (2006)

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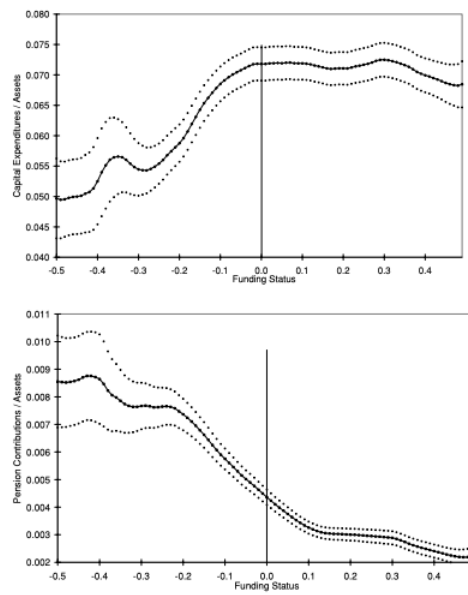
Figure 5. Kernel regressions of capital expenditures and pension contributions on funding status. Kernel regression estimation is performed on pooled data using the Epanechnikov kernel. The funding status is aggregated to the firm level. The top graph shows the relationship between funding status and pension contributions. The bottom graph shows the relationship between funding status and capital expenditures. The error bounds are 95% confidence intervals (±1.96 standard deviations). The bandwidth of 0.1 is validated using a cross-validation algorithm that minimizes the sum of squared residuals (Härdle (1990), p. 159). The error bounds are pointwise confidence intervals, calculated using an algorithm that is based on the variance of the estimate (Härdle (1990), p. 100).

Figure 2:

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3 Computing a recursive equilibrium

- Suppose $F(k, l) = k$ and $A_t = a_t$ that is an i.i.d. shock

- Model can be analyzed with single state variable

$$s_t \equiv a_t - \tilde{b}_t$$

- Recursive equilibrium is given by

$$\lambda(s), x(s), b(s)$$

  where

$$x_t = \frac{k_{t+1}}{k_t},$$

  the three functions must satisfy three sets of conditions for all $s > \underline{s}$, where $\underline{s}$ is a lower bound to be determined

- Recursive condition for λ

$$\lambda(s) = \beta \frac{E\left[\lambda(a' - b(s))\left[a' - b(s) - G_2(x(a' - b(s)), 1)\right]\right]}{G_1(x(s), 1) - qb(s)},$$

- Condition for $x(s)$ that

$$s + qb(s)x(s) \geq G(x(s), 1)$$

  with strict equality if $\lambda(s) > 1$

- Condition for the borrowing ratio $b(s)$

$$q\lambda(s) \geq \beta E\left[\lambda(a' - b(s))\right]$$

  and

$$b(s) \leq -\theta \min_{a'} G_2(x(a' - b(s)), 1)$$

  with complementary slackness

- Equilibrium can be computed recursively

- As initial condition think of finite horizon problem, set $\lambda = 1$ in the final period and G_2 to some fixed value

- Code stoch_KM.m computes equilibrium using following algorithm

- Iteration, endogenous gridpoint method, find $\hat{b}$ that satisfies

$$b = -\theta \min_{a'} G_2(x(a' - b), 1),$$

- Choose candidate pairs (b, λ) as follows

- Set $b = \hat{b}$ and let

$$\hat{\lambda} = \max\{\frac{\beta}{q}E\left[\lambda\left(a' - b\right)\right], 1\},$$

  then choose any λ in $[\hat{\lambda}, \infty)$

- Set $b < \hat{b}$ and if

$$\frac{\beta}{q}E\left[\lambda\left(a' - b\right)\right] < 1$$

  discard, otherwise set

$$\lambda = \frac{\beta}{q}E\left[\lambda\left(a' - b\right)\right]$$

- For each pair (b, λ) find x that solves

$$\lambda\left[G_1\left(x, 1\right) - qb\right] = \beta E\left[\lambda\left(a' - b\right)\left[a' - b - G_2\left(x\left(a' - b\right), 1\right)\right]\right],$$

  or

$$\lambda\left[G_1\left(0, 1\right) - qb\right] \geq \beta E\left[\lambda\left(a' - b\right)\left[a' - b - G_2\left(x\left(a' - b\right), 1\right)\right]\right],$$

  if $\lambda = 1$ this is the optimal solution for all s that satisfy

$$s \geq G\left(x, 1\right) - qbx,$$

  if $\lambda > 1$ this is the optimal solution for

$$s = G\left(x, 1\right) - qbx$$

- The lower bound for s is

$$\underline{s} = \min_{x \geq 0} G\left(x, 1\right) - q\hat{b}x$$

  (which arises when $\lambda \to \infty$)

- Functional form used for G is

$$G\left(k', k\right) = k' - k + \frac{\xi}{2}\frac{\left(k' - k\right)^2}{k}$$

  or

$$G\left(x, 1\right) = x - 1 + \frac{\xi}{2}\left(x - 1\right)^2$$

  so derivatives are

$$G_1 = 1 + \xi\left(x - 1\right)$$

  and

$$G_2 = -1 - \xi\left(x - 1\right) - \frac{\xi}{2}\left(x - 1\right)^2$$

- Then this equation

$$\lambda\left[G_1\left(x,1\right)-qb\right]=\beta E\left[\lambda\left(a'-b\right)\left[a'-b-G_2\left(x\left(a'-b\right),1\right)\right]\right],$$

  becomes

$$x=1+\frac{1}{\xi}\left\{\frac{\beta E\left[\lambda\left(a'-b\right)\left[a'-b-G_2\left(x\left(a'-b\right),1\right)\right]\right]}{\lambda}+qb-1\right\}$$

- Frictionless benchmark

$$G_1\left(x,1\right)=qE\left[a-G_2\left(x,1\right)\right]$$

  investment constant with x solving

$$1+\xi\left(x-1\right)=q\left[Ea+1+\xi\left(x-1\right)+\frac{\xi}{2}\left(x-1\right)^2\right]$$

- Assume that

$$r<Ea<r+\frac{\xi}{2}r^2$$

  where $r=1/q-1$ to ensure that a solution to the frictionless problem exists and is bounded

- Choose solution with $x<1+r$ to satisfy transversality condition