

## 411-3 NOTES: FINANCIAL FRICTIONS

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- Financial crisis of 2007-2008, some crucial mechanisms
- At the core of the financial system:
  - Leveraged losses at broker dealers (Greenlaw, Kashyap, Shin)
  - Panic in the repo market (Gorton)
  - Fire sales of assets that become illiquid (MBS)
- To understand these mechanisms we need models of balance sheet of intermediaries and models of why the demand for assets that are relatively small fraction of the market can be downward sloping even if there are a lot of potential buyers/arbitrageurs out there
- To understand panics we need models of short-term financing
- There are broader issues that touch on the economy as a whole. Here the crucial mechanisms are
  - Accumulation of debt in the household sector
  - Growth in subprime finance and securitization
  - House price boom (bubble?)
  - Connection between a crisis at the core of the financial sector and credit availability for households and businesses

### 1. ASSET PRICES AND BALANCE SHEETS

- Kiyotaki and Moore (1997) is a model that can be used to understand the dynamics of the balance sheets of financial institutions and of amplification when fire sales are possible
- Mechanism: balance sheet effects + forward looking prices  $\rightarrow$  amplification
- Two groups of agents: entrepreneurs and consumers
- Both groups have linear preferences

$$\sum \beta^t c_t$$

- Two goods:
  - consumption good,
  - capital in fixed supply  $\bar{k}$ , never depreciates
- Relative price of the capital good  $p_t$
- Entrepreneurs' budget constraint

$$c_t + p_t k_{t+1} - q_t b_{t+1} \leq (a + p_t) k_t - b_t$$

where  $b_t$  is debt

- The right-hand side is their wealth, i.e., their net worth

$$n_t = (a + p_t) k_t - b_t$$

- Collateral constraint

$$b_{t+1} \leq p_{t+1}k_{t+1}$$

- Inalienable human capital of entrepreneurs necessary to produce  $a$  (a form of limited enforcement)
- Assume consumers have an endowment of consumption goods, large enough that  $c_t > 0$  always and  $q_t = \beta$  in the equilibria described below
- Alternative use for capital: concave production function controlled by the consumers

$$y_t = G(k_t^c)$$

this is where we get the downward sloping demand for assets sold by the entrepreneurs

- Market clearing

$$k_t + k_t^c = \bar{k}$$

- Optimality condition for the use of capital in the  $G$  sector (always unconstrained)

$$p_t = \beta [p_{t+1} + G'(k_{t+1}^c)]$$

- Initial conditions:  $k_0$  and  $b_0$
- Suppose initial conditions such that collateral constraint satisfied, i.e., given those initial conditions there is an equilibrium with

$$p_0 k_0 \geq b_0$$

## 2. OPTIMIZATION PROBLEM OF THE ENTREPRENEUR

- We assume for the moment that the price sequence  $\{p_t\}$  is such that the problem is well defined. Along the way we'll find conditions that need to be satisfied for this to be the case
- Value function for entrepreneur with net worth  $n_t$

$$V_t(n_t) = \max_{c_t, k_{t+1}, b_{t+1}} c_t + \beta V_{t+1}((a + p_{t+1})k_{t+1} - b_{t+1})$$

$$\begin{aligned} c_t + p_t k_{t+1} &\leq n_t + \beta b_{t+1} \\ b_{t+1} &\leq p_{t+1} k_{t+1} \end{aligned}$$

- FOC

$$\begin{aligned} 1 &\leq \lambda_t \\ \lambda_t p_t &= \beta (a + p_{t+1}) V'_{t+1} + \mu_t p_{t+1} \\ \lambda_t \beta &= \beta V'_{t+1} + \mu_t \end{aligned}$$

- Envelope

$$V'_t = \lambda_t$$

- From last two conditions we have

$$\lambda_t \geq \lambda_{t+1}$$

- So to check entrepreneurs optimality we need to find a decreasing sequence of  $\lambda_t$
- The sequence needs to converge to  $\lambda_t = 1$  in finite time, otherwise consumption would always be zero which cannot be optimal
- Interesting case is when  $\lambda_0$  starts strictly greater than 1 and we have  $T$  periods with  $\lambda_t > 1$  and  $c_t = 0$  and after that we have  $\lambda_t = 1$

- From FOC and envelope we get

$$\begin{aligned} p_t \lambda_t &= \beta (a + p_{t+1}) \lambda_{t+1} + \mu_t p_{t+1} \\ \mu_t &= \beta \lambda_t - \beta \lambda_{t+1} \end{aligned}$$

which combined give

$$\lambda_t = \beta \frac{a}{p_t - \beta p_{t+1}} \lambda_{t+1}$$

- Therefore we have

$$\lambda_t > \lambda_{t+1}$$

iff

$$\beta a > p_t - \beta p_{t+1}$$

- So we need  $\beta a > p_t - \beta p_{t+1}$  for periods  $t < T$  and  $\beta a = p_t - \beta p_{t+1}$  for  $t \geq T$

### 3. CHARACTERIZING AN EQUILIBRIUM

- Optimality in the  $G$  sector and market clearing imply

$$p_t - \beta p_{t+1} = \beta G'(\bar{k} - k_{t+1})$$

- In the periods  $t \geq T$  combining the last two conditions we have

$$a = G'(\bar{k} - k_{t+1})$$

which implies

$$k_{t+1} = k^*$$

where  $k^*$  satisfies  $a = G'(\bar{k} - k^*)$

- Moreover, ruling out explosive paths for  $p_t$ , we have

$$p_t = p^* = \frac{\beta}{1 - \beta} a$$

- So for  $t \geq T$  capital allocation and price must be constant at  $k^*, p^*$
- Now go back to the periods  $0 < t < T$
- In these periods the collateral constraint is binding and entrepreneurs consume 0
- So the budget constraint for the entrepreneur is

$$p_t k_{t+1} = (a + p_t) k_t - p_t k_t + \beta p_{t+1} k_{t+1} \text{ for } t \leq T$$

which becomes

$$\beta G'(\bar{k} - k_{t+1}) k_{t+1} = a k_t$$

- At  $t = 0$  slightly different (because debt is given)

$$(3.1) \quad \beta G'(\bar{k} - k_1) k_1 = n_0$$

- Proposition: if

$$\beta G'(\bar{k} - k^*) k^* > n_0,$$

there a unique sequence  $\{k_t\}$  that satisfies the following conditions for some  $T > 0$ :

$$k_t \leq k^* \text{ for } t \leq T, \quad k_t = k^* \text{ for } t > T$$

$$\beta G'(\bar{k} - k_1) k_1 = n_0$$

$$\beta G'(\bar{k} - k_{t+1}) k_{t+1} = a k_t \text{ for } 0 < t < T$$

$$\beta G'(\bar{k} - k^*) k^* \leq a k_T$$

- Sketch of argument: any time  $\beta G'(\bar{k} - k_{t+1}) k_{t+1} = ak_t$  and  $k_{t+1} \leq k^*$ , we have

$$\frac{k_{t+1}}{k_t} = \frac{a}{\beta G'(\bar{k} - k_{t+1})} \geq \frac{a}{\beta a} = \frac{1}{\beta}$$

so the sequence grows at a rate bounded below by  $1/\beta > 1$  and so must cross  $k^*$  in finite time

- Now given  $n_0$ :
  - if  $\beta G'(\bar{k} - k^*) k^* \leq n_0$  set all elements of  $\{k_t\}$  equal to  $k^*$ ,
  - if  $\beta G'(\bar{k} - k^*) k^* > n_0$  set the elements of  $\{k_t\}$  as in the previous proposition
  - compute

$$(3.2) \quad p_0 = \sum_{t=0}^{\infty} \beta^{t+1} G'(\bar{k} - k_{t+1})$$

- Result: the construction above defines a non-decreasing function

$$p_0 = P(n_0),$$

the relation is strictly increasing for

$$n_0 < n^* \equiv \beta G'(\bar{k} - k^*) k^*$$

- Result: any equilibrium where the initial net worth of the entrepreneurs is  $n_0$  is uniquely characterized by the sequence  $\{k_t\}$  given above
- Result: if the entrepreneur begins with a balance sheet  $k_0, b_0$  all equilibria of the economy can be found by finding the values of  $n_0$  that solve

$$n_0 = (a + P(n_0)) k_0 - b_0$$

- We can depict this fixed point problem graphically
- Introduce a temporary shock to productivity
- Productivity is

$$a_0 = a + da$$

for first period only

- This temporary shock would have no effect in a frictionless benchmark (which is purely forward looking)
- Here it shifts the balance sheet relation

$$n_0 = (a_0 + p_0) k_0 - b_0$$

without affecting the function  $P(n_0)$

- This increases equilibrium  $p_0$  and has amplified effect
- Backward looking effect of net worth on investment
- Plus amplification due to forward looking element
- Here shock is completely unexpected and there is no investment
- Next: how to add shocks and aggregate investment to the model