411-3: Problem Set 3

Spring 2019

1 Simulating a labor search model

This problem is about simulating a stochastic version of the search model of the labor market seen in class. Suppose that y_t follows a two state Markov process with symmetric transition matrix

$$\begin{bmatrix} 1-\pi & \pi \\ \pi & 1-\pi \end{bmatrix}.$$

a) Choose the values y_l and y_h and the value of π so that: the unconditional average of y_t is 1, the unconditional standard deviation and the coefficient of autocorrelation of $\log y_t$ are, respectively, 0.02 and 0.878.

b) Consider the model seen in class and use the parameters in the lecture notes. Compute the steady state of the non-stochastic economy with $y = 1$ and calibrate h (the constant in the matching function) so that the equilibrium finding rate is 0.83. Calibrate b so that the equilibrium ratio b/w is 0.4. (w.l.o.g. for purposes of this problem, can normalize $\kappa = 1$, why?)

c) Use an iterative method to find a solution to the functional equation (4) (i.e. to find the two values $\Theta(y_l)$ and $\Theta(y_h)$).

d) Compute the standard deviation and quarterly autocorrelation of $\log u, \log v, \log \theta, \log f$. Choose a different value of b (of your choice) and redo the computations. Discuss.

2 Endogenous separation

This problem extends the search model of the labor market covered in class to allow for endogenous separation decisions. We will use the model to analyze the effects of firing costs on unemployment.

Suppose the productivity of each job, $y_{i,t}$, follows an i.i.d. process with a continuous cumulative distribution function $G(.)$. There is no aggregate uncertainty, so $G(.)$ is also the distribution of shocks across firms. At the beginning

of each period, a matched firm-worker pair learns about the realization of the job's productivity $y_{i,t}$ for the current period. Depending on the realization of $y_{i,t}$, they either separate or agree on the wage $w_{i,t}$ by Nash bargaining with the worker's bargaining power given by η (assume wages are renegotiated at the beginning of each period). Separation is costly: when a firm and a worker separate the firm has to pay a cost ξ.

If they separate at the beginning of the period, then the worker spends the period unemployed and gets matched to some other firm with probability $\mu(\theta_t) \equiv m(1, \theta_t)$ (where $\theta_t = v_t/u_t$ is the tightness of the labor market as in the standard setup). The rest of the setup is identical to the one seen in class (a slight difference you will notice, due to the endogenous nature of separation, is that separation can occur on the very first period after a job as been created). Normalize the labor force to 1 and suppose that workers receive unemployment benefits b.

We will study a steady state equilibrium where θ is constant, all jobs with productivity below the cutoff $\hat{y}$ are terminated and the wage for jobs with productivity $y \geq \hat{y}$ is given by the function $w(y)$.

a) Write an expression for the value of a job with current productivity y for the firm, $J(y)$, and the value for the worker, $V(y)$. Combine them to get a recursive expression for the surplus of a job with productivity y, $S(y)$, in terms of y, θ, the cutoff $\hat{y}$, and the value of unemployment U (notice that the outside option of the firm now is not 0, even though there is free entry).

b) Write the free entry condition for the firm.

c) Write an expression for the value of unemployment, U, and using Nash bargaining and the free entry condition, find an expression for U in terms of θ.

d) Argue that $S'(y) = 1$ for $y \geq \hat{y}$ and use this result to obtain a simple expression for the surplus $S(y)$ in terms of $\hat{y}$, also considering what would determine $\hat{y}$. Substitute in the free entry condition derived in (b) and show that you have found a decreasing relation between $\hat{y}$ and θ. Interpret this relation.

e) Substitute the expression for U found in (c) in the expression for $S(y)$ found in (a). Derive an expression for the cutoff $\hat{y}$ in terms of θ. Show that this condition defines an increasing relation between $\hat{y}$ and θ. Interpret this relation.

f) Argue that the equilibrium values of $\hat{y}$ and θ are found at the point where the curves defined in (d) and (e) intersect. (Optional: Discuss conditions under which such a crossing occurs with $\theta > 0$ and $G(\hat{y}) \in (0,1)$).

g) Suppose the firing cost ξ increases. Use a graphical argument to discuss the effects on θ and $\hat{y}$. What are the effects on equilibrium unemployment?

How do they depend on the shape of the distribution G? Give both formal arguments (but no need for closed form derivations) and intuition.

3 Leverage and amplification in Kiyotaki-Moore

Consider the version of the Kiyotaki-Moore model seen in class. Suppose that a_0 is lower than, but near the level $\bar{a}$ that satisfies

$$\beta G'\left(\bar{k} - k^*\right) k^* = (p^* + \bar{a}) k_0 - b_0,$$

so that the transition only lasts one period, i.e., $k_1 < k^*$, $k_2 = k_3 = ... = k^*$.

Use the equilibrium conditions of the model to analyze the effect of a small change da_0 on p_0 and k_1. The idea is to linearize these conditions for $a_0 \to \bar{a}$.

(i) Show that the balance sheet condition for the entrepreneur leads to this condition

$$\frac{d\left(p_0 - \beta p_1\right)}{p_0 - \beta p_1} + \frac{dk_1}{k_1} = \frac{d\left(p_0 + a_0\right)}{p_0 + a_0} \frac{\left(p_0 + a_0\right) k_0}{\left(p_0 + a_0\right) k_0 - b_0}.$$

Argue that a larger leverage, i.e., a larger level of initial debt b_0 over total assets $(p_0 + a_0)k_0$, leads, all else equal to larger effects of a change in $p_0 + a_0$ on investment.

(ii) Now derive endogenously the responses of the downpayment $p_0 - \beta p_1$ and of the asset price p_0 and express dk_1/k_1 in terms of da_0/a_0 and some relevant elasticities. Argue that the downpayment response dampens the effect of a productivity shock and the asset price response amplifies it.

(iii) Now look at the response of the end-of-period leverage ratio

$$\beta p_1/p_0.$$

Show that the response of this ratio to a $da_0 > 0$ shock is negative, so leverage is countercyclical in this model. Argue that this result extends to the case $T > 1$.

4 Fire sales and multiple equilibria

This problem analyzes the possibility of multiple equilibria in a model à la Kiyotaki and Moore (1997), like the one seen in class but with only two period 0 and 1. There is a continuum of entrepreneurs and a continuum of equal measure of consumers. There is a fixed supply of capital $\bar{k}$. Entrepreneurs preferences are described by the utility function $c_0 + c_1$. They enter date 0 with a stock of capital k_0 and an inherited stock of short-term debt b_0 and decide how much to invest for the following period k_1. Entrepreneurial firms produce zero output in period 0 and a units of consumption goods per unit of capital

invested in period 1 (in the notation of the class notes, we are looking at what happens after an extreme temporary "shock" in period 0, such that $a + \Delta a = 0$). Since period 1 is the final period, we simply assume that capital is liquidated in period 1 and yields θ units of consumption goods per unit of capital (that is, $p_1 = \theta$). In period 0, if $b_0 \leq p_0 k_0$ the entrepreneur repays his debt, otherwise he renegotiates the value of the debt down to $p_0 k_0$. Then, he borrows by selling bonds b_1, to be repaid in period 1, and faces the collateral constraint

$$b_1 \leq \theta k_1.$$

Consumers preferences are described by $c_0 + c_1$. Consumers have a large endowment of consumption goods in each period. Each consumer owns a backyard technology described by the concave production function

$$G(\tilde{k}_t) = \tilde{k}_t - \frac{1}{2}\tilde{k}_t^2.$$

Use p_0 to denote the price of capital in period 0. Assume the consumers endowment is large enough that the gross interest rate is always 1 in equilibrium.

(i) State the optimization problem of the entrepreneur and show that if $\max\{\theta, b_0/k_0\} < p_0 < \theta + a$ the entrepreneur problem is well defined, the entrepreneur does not default in period 0, chooses $c_0 = 0$ and the collateral constraint is binding. Derive an expression for the entrepreneur's demand for capital k_1 as a function of p_0, for $p_0 \in (\max\{\theta, b_0/k_0\}, \theta + a)$.

(ii) Show that if $b_0 < \theta k_0$ the entrepreneur's demand for capital is decreasing in p_0, while if $b_0 > \theta k_0$ the entrepreneur's demand for capital is *increasing* in p_0 (always looking at the region $p_0 \in (\max\{\theta, b_0/k_0\}, \theta + a)$). Show that whether the demand is increasing or decreasing depends on the sign of $k_1 - k_0$, i.e., on whether the entrepreneur is a net buyer or a net seller of capital goods. Discuss.

(iii) Suppose $b_0 > \theta k_0$, what happens to the entrepreneur's demand for capital when $\theta < p_0 \leq b_0/k_0$?

(iv) What happens to the entrepreneur's demand for capital when $p_0 \leq \theta$? And when $p_0 \geq \theta + a$?

(v) State the optimization problem of the consumers and derive the first order condition for $\tilde{k}_1$. Write the market clearing condition for the capital market in period 0.

(vi) Suppose $b_0 < \theta k_0$. Depict graphically the equilibrium in the capital market and show that there is a unique equilibrium.

(vii) Suppose $b_0 > \theta k_0$. Depict graphically the equilibrium in the capital market and show that there can be multiple equilibria. In particular, consider the following example

$$k_0 = \bar{k} = 0.9 \quad b_0 = 0.6$$
$$\theta = 0.5 \quad a = 1$$

and show that there is a "good" equilibrium with high p_0, an intermediate (unstable) equilibrium, and a "bad" equilibrium where $p_0 = 0.6$, $k_1 = 0$, entrepreneurs' net worth is zero and they are unable to borrow. Discuss the intuition behind the multiplicity.

(viii) Argue that an appropriate reduction in b_0 eliminates the bad equilibrium.

(ix) Suppose that there are multiple equilibria and the government stands ready to buy assets at the good equilibrium price. How many assets it would have to buy to implement this policy?

(x) Compare the fiscal costs of (viii) and (ix) and use your result to discuss recent policy debates, interpreting (viii) as a plan to inject capital in the banking system and (ix) as a plan to buy toxic securities with government money.

(xi) If you disagree with the conclusions reached in (x), discuss what you think is missing or wrong in the model.