

Note on corporate tax algebra

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Krugman and Mankiw use two different taxes to discuss the effects of a corporate tax cut. This leads to some confusion. Mankiw uses a tax on capital compensation that raises $t \cdot MPK$ per unit of capital. Krugman uses a tax that raises τ per unit of capital. Mankiw's tax is more realistic. But Krugman's tax leads to easier interpretation, as we shall see.

Let's start with the specific tax of Krugman. Using the fiction of thinking of workers as owners of the firm who rent capital at the rate $r + \tau$, write the wage bill

$$w = \max_k \{f(k) - (r + \tau)k\},$$

so, by an envelope argument,

$$dw = k \cdot d\tau.$$

If we look at the short run revenue effect of the tax, i.e., the effect with fixed k , and call it $[dx]_{\bar{k}, \tau}$, we get

$$[dx]_{\bar{k}, \tau} = k \cdot d\tau,$$

and we get the ratio

$$\frac{dw}{[dx]_{\bar{k}, \tau}} = -1.$$

This is intuitive and in line with envelope logic: if workers, as the firm's owners, pay a dollar more per unit of capital, their wage bill goes down by k , which is the increased tax revenue with fixed k .

Turning to Mankiw's tax, let's first find the t tax equivalent to the τ tax in the long run by setting

$$tf'(k) = \tau.$$

Differentiating both sides gives

$$d\tau = f'(k) dt + tf''(k) \frac{dk}{d\tau} d\tau.$$

Using $dk/d\tau = 1/f''(k)$, and manipulating gives

$$dt = \frac{1-t}{f'(k)} d\tau. \tag{1}$$

If we now look at the short run effect of the t tax, we realize that the two taxes are different. The short run effect is

$$[dx]_{\bar{k},t} = f'(k) k \cdot dt$$

and substituting (1) gives

$$[dx]_{\bar{k},t} = (1 - t) k \cdot d\tau = (1 - t) [dx]_{\bar{k},\tau}.$$

So the short run effect is smaller with a t tax, even though the two taxes are equivalent in the long run. The reason for the discrepancy is that the t tax leads to a higher effective tax in the long run because, as the capital stock is reduced, the MPK goes up and the tax applies to a higher MPK . This makes the tax per unit of capital $t \cdot MPK$ increasing over time, while the τ tax is, obviously, constant over time.

If we take the ratio of the long-run wage change to the short-run revenue increase we get

$$\frac{dw}{[dx]_{\bar{k},t}} = -\frac{1}{1 - t},$$

which is Mankiw's formula.

The interpretation of Mankiw's formula coming from this analysis is that a t tax has an increasing time profile: the effective tax rate is higher in the long run than in the short run. Therefore, the tax shows smaller revenue effects in the short run, which accounts for the high ratio of long-run wage reduction to short-run revenue gain. From the formula, we are not learning that the tax has an especially powerful effects on wages, independently of the curvature of the production function. We are just learning that the initial revenue increase is small relative to the long-run revenue increase because the tax rate is increasing over time.

The fact that the ratio of the short-run to the long-run revenue effects is exactly $1 - t$ comes from the fact that before the tax increase (and in the short run) we have

$$(1 - t) MPK = r,$$

while in the long run we have

$$(1 - t - dt) (MPK + dMPK) = r,$$

from which we get

$$\frac{dMPK}{MPK} = \frac{1 - t}{1 - t - dt} - 1 = \frac{1}{1 - t} dt.$$

Once we look at long run effects and compute what has been dubbed the dynamic Furman's ratio, the discrepancy between the two approaches disappears—by construction—and both taxes give

$$\frac{dw}{dx} = \frac{1}{1 + \frac{f'(k)}{f''(k)k}}.$$