411-3 Macro

Problem Set 2

1 Forward guidance and precautionary savings

Consider an Aiyagari economy without capital. In each period, households consume c_t or save a_t to maximize discounted utility

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t),$$

where $u(c) = c^{1-\sigma}/(1-\sigma)$ if $\sigma \neq 1$, and $u(c) = \log(c)$ is $\sigma = 1$. The household per-period budget constraint is

$$c_t + a_t \leq (1 + r_{t-1}) a_{t-1} + y_t.$$

The household faces the borrowing constraint

$$a_{t+1} \geq 0.$$

In class we assumed a simple linear relation $y_t = \omega_t Y_t$ between aggregate output and individual labor incomes. In this problem we study what happens when the relation is, in general, non-linear. So we assume that $y_t = f_{n_t}(Y_t)$ where n_t is an idiosyncratic shock, drawn each period independently from $\{1, 2, ...N\}$ with probabilities $\{\pi_1, ..., \pi_N\}$. We assume that $f_n(Y_t) > f_m(Y_t)$ if $n > m$. By construction it must be that

$$\sum_{n=1}^{N}\pi_n f_n(Y_t) = Y_t.$$

Assume there is a zero supply of assets so market clearing in the asset markets requires$A_t = 0$. In the terminology used in class, we are in a 0-HANK environment. Assume the central bank chooses a sequence for the real interest rate $\{r_t\}$.

Market clearing in the goods market requires $C_t = Y_t$, where C_t denotes aggregate consumption.

i. Write the value function problem of an individual household and derive the individual Euler equation.

ii. Use market clearing for consumption and assets to argue that in equilibrium $c_t = f_{n_t}(Y_t)$ for every agent. Show that, if $\{r_t\}$ and $\{Y_t\}$ satisfy

$$u'(f_N(Y_t)) = \beta(1 + r_t)\sum_{n=1}^{N}\pi_n u'(f_n(Y_{t+1})). \tag{1}$$

then the Euler equation of each household is satisfied.

iii. Consider a steady state with no aggregate shocks. Suppose that in steady state $C_t = Y_t = 1$. Show that the steady state natural interest rate satisfies

$$\log(1 + r^*) = -\log\beta - \log\left(\sum \pi_n \left(\frac{f_n(1)}{f_N(1)}\right)^{-\sigma}\right).$$

iv. Define the steady state elasticities of income with respect to aggregate income

$$\varepsilon_n \equiv \frac{f_n'(1)}{f_n(1)}.$$

Log-linearize the Euler equation (1) around the steady state and write it in the form

$$\hat{Y}_t = \alpha \hat{Y}_{t+1} - \eta \hat{r}_{t+1}, \tag{2}$$

where $\hat{Y}_t = d\log Y_t$ and $\hat{r}_t = r_t - r^*$ (that is, log-linearize w.r.t. to Y and just linearize w.r.t. to r). Express the coefficient α as a function of the elasticities ε_n.

iv. Suppose at date 0 the central bank announces a path with $\hat{r}_T > 0$ for some future date T and $\hat{r}_t = 0$ for all $t \neq T$. Assume that $\lim_{t\to\infty} \hat{Y}_t = 0$. Derive the time-0 consumption response using (2).

v. A large empirical literature has found support to the idea that income risk is countercyclical, i.e. that income risk rises in recessions. We can capture this idea in our model by assuming that bottom incomes are more responsive than top incomes to changes in aggregate activity. That is, we can assume $\varepsilon_1 > \varepsilon_2 > ... > \varepsilon_N$. What happens to the effects of the anticipated monetary policy shock $\hat{r}_T$ under this assumption? Is the effect increasing or decreasing in T? Provide an interpretation based on how an increase in output tomorrow alters the incentives of agents to do precautionary savings today.

2 Monetary and fiscal policy: numerical experiments

Consider an Aiyagari economy without capital. In each period, households consume c_t or save a_t to maximize discounted utility

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $u(c) = c^{1-\sigma}/(1-\sigma)$ if $\sigma \neq 1$, and $u(c) = \log(c)$ is $\sigma = 1$. The household per-period budget constraint is

$$c_t + a_t \leq (1 + r_{t-1})\, a_{t-1} + y_t$$

where $y_t = \omega_t Y_t$ (we are back to our linear specification). The household also faces the borrowing constraint

$$a_{t+1} \geq -\phi,$$

for some $\phi > 0$. Assume that ω_t is drawn from a discrete, binary distribution on $\left\{\omega^1, \omega^2\right\}$ with probabilities $\{\pi^1, \pi^2\}$. Remember that you need

$$\sum_{n=1}^{N} \pi^n \omega^n = 1.$$

Assume there is a zero supply of assets so market clearing in the asset markets requires$A_t = 0$ (but since $\phi > 0$ we used in class, we are not in a 0-HANK model). Assume the central bank chooses a sequence for the real interest rate $\{r_t\}$.

As you did in PS1 you will work on a grid for $z = (1 + r_{t-1}) a_{t-1} + y_t$. However, now, due to the fact that the interest rate changes over time, you will have to compute time-varying policies $C_t(z)$.

i. (Steady state) First, let's find a steady state. Let's assume that $Y_t = 1$ in steady state. Use the codes you did for PS1 (with exogenous income) to find $C(z)$. Find the interest rate r that clears the asset market, so $A_t = 0$. Is the goods market also in equilibrium? Why? (You can look at the matlab codes I posted to compute invariant distribution and asset demand, or, even better, you can write your own.)

ii. (Partial equilibrium) Write a code to compute the optimal policy $C_t(z)$ for a sequence of interest rates $\{r_t\}$ that converges to r^*, assuming the income process is just given by $y_t = \omega_t$ (so $Y_t = 1$) . In particular, let's focus on sequences of the form

$$r_t = \rho (r_0 - r^*) + r^*. \tag{3}$$

Hint: assume that for t large enough the policy $C_t(z)$ is equal to the steady state policy and solve backward.

iii. (Disequilibrium) Write a code to compute what happens to the asset distribution if we start from a steady state and then there is an unexpected monetary policy shock at $t = 0$ of the form (3) and we keep aggregate income at $Y_t = 1$. Hint: start from the steady state distribution and use the policies in (ii) to update the distribution from $t = 0$ onwards. Check that for t large enough you converge back to the steady state distribution. In this exercise are you getting $\int c_i di = Y_t = 1$? Are you getting $\int a_i di = 0$? How are excess demand in the good and asset market related?

iv. (Equilibrium) Now we want to check that the goods market is in equilibrium. Here you need to use some updating rule to compute $\{Y_t\}$ and extend the code in (ii) to deal with both time-varying r_t and time-varying Y_t. Again, the idea is to assume that when t is large enough the economy goes back to

$Y_t = 1$ and $r_t = r^*$. Hint: to update Y_t use the goods market equilibrium condition, setting

$$Y_t^{(j)} = Y_t^{(j-1)} + \zeta \left(\int c_i^{(j-1)} di - Y_t^{(j-1)} \right)$$

where j denotes the iteration of your algorithm and $\zeta \in (0, 1)$ controls the speed of convergence.

v. Compare your results to those of a representative agent, new-Keynesian model (with the same σ).