411-3 Macro

Problem Set 1

# 1 Consumption responses under different income processes

Consider a consumer with preferences represented by the utility function

$$\mathbb{E}\sum \beta^t u\left(c_t\right)$$

where $u\left(c\right) = Ac - \frac{B}{2}c^2$ for some $A, B > 0$. The consumer can only trade a risk-free bond and receives an exogenous income $y_t$. The income $y_t$ follows a Markov process, with transition pdf $\pi(y_{t+1}|y_t)$. The consumer faces the budget constraint

$$a_{t+1} + c_t \leq \left(1+r\right)a_t + y_t$$

each period and the no-Ponzi condition $\left(1+r\right)^t a_t \to 0$ for all paths. We assume throughout that

$$\beta\left(1+r\right) = 1.$$

(i) Write the Bellman equation that characterizes the value function (first choose your state variables).

(ii) Derive first order conditions and the envelope condition for $a_t$. Use them to obtain the consumer Euler equation.

Assume that $y_t$ follows the AR1 process

$$y_{t+1} = \bar{y} + \rho\left(y_t - \bar{y}\right) + \epsilon_{t+1},$$

with $\rho \in (0, 1)$.

(iii) Conjecture that the optimal consumption policy takes the linear form

$$c_t = \phi + \chi a_t + \psi y_t.$$

Substituting in the Euler equation find $\phi, \chi, \psi$ as functions of the parameters $r, \bar{y}, \rho$. (Hint: do not substitute for $c_t$, just for $c_{t+1}$.)

Assume now that $y_t$ follows the AR1 process in growth rates

$$y_{t+1} - y_t = \rho\left(y_t - y_{t-1}\right) + \epsilon_{t+1},$$

with $\rho \in (0, 1)$.

(iv) Conjecture that the optimal saving policy takes the linear form

$$c_t = \phi + \chi a_t + \psi y_t + \omega y_{t-1}.$$

Substituting in the Euler equation find $\phi, \chi, \psi, \omega$ as functions of the parameters $r, \rho$.

(v) Show that $\psi < 1$ in the first example and $\psi > 1$ in the second. Provide intuition for why.

## 2 Numerical solution of an income fluctuation problem

Consider the income fluctuation problem studied in class. Write a code (using your favorite program), to analyze the case in which utility is CRRA

$$u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$$

and the income process is i.i.d. with a simple binary distribution that assigns probabilities $\pi, 1-\pi$ to two values $y^h, y^l$ and with a non-negativity constraint for the asset position $a_t \geq 0$ (that is, set $\phi = 0$).

I suggest you use the following algorithm:

**a.** Form a grid for the state variable $z$ and a grid for the decision variable $a'$

**b.** Start from a conjecture for $c(z)$ (e.g., suppose this is the last period of a finite horizon problem).

**c.** For each point on the grid for the decision variable $a'$, starting at $a' = 0$, compute

$$\mathbb{E}u'(c((1+r)a' + y'))$$

- if $a' > 0$ find the $\tilde{c}$ that solves

$$u'(\tilde{c}) = \beta(1+r)\mathbb{E}u'(c((1+r)a' + y'))$$

  and the value

$$\tilde{z} = \tilde{c} + a'$$

- if $a' = 0$ find a collection of pairs $\tilde{c}, \tilde{z}$ that satisfy

$$u'(\tilde{c}) \geq \beta(1+r)\mathbb{E}u'(c((1+r)0 + y'))$$

$$\tilde{z} = \tilde{c} + 0$$

**d.** Use the pairs $(\tilde{c}, \tilde{z})$ derived in the previous step to form a new conjecture for $c(z)$

**e.** Go back to (c) and repeat until convergence

I suggest you use $r = 5\%$, $\beta = 0.9$ and $\gamma = 1$ as starting points, as convergence will be very fast. Then you can experiment with other parameters.

Try the following experiments:

i. Plot the dynamics $z' - z$ as a function of $z$ and of the shock $y'$ and check that your grid covers the ergodic set of the wealth distribution (how?).

ii. See how the consumption function changes if you change the risk aversion between $\gamma = 1/2$ and $\gamma = 2$ (make sure you use a grid that works for both cases!)

iii. See how the consumption function changes if you change the idiosyncratic risk, by varying $\pi$ or the ratio $y^h/y^l$.

iv. See how the saving function $a(z)$ changes if you change the interest rate and make it close to $1/\beta - 1$.

v(optional). Compute the ergodic wealth distribution and average wealth and see how it changes as you change $r$.