460-1 International Economics

Problem Set 1

1. This problem solves a problem similar to the one in Aguiar and Gopinath (2007). Relative to the certainty equivalent case done in class, here we use CRRA utility, an endowment process that is specified in logs and an interest rate that depends on the country net foreign asset position. The solution approach will be approximation by log linearization.

Consider a small open economy receiving the stochastic endowment $Y_t = \exp y_t$, where y_t is the sum of the permanent component x_t and the transitory component z_t:

$$y_t = x_t + z_t.$$

x_t follows the process

$$\Delta x_t = (1 - \rho_x)\, g + \rho_x \Delta x_{t-1} + \varepsilon_t,$$

z_t follows the process

$$z_t = \rho_z z_{t-1} + \eta_t,$$

and ε_t and η_t are i.i.d. shocks. Define

$$X_t = e^{x_t}.$$

The consumers have preferences

$$E \sum \beta^t U\left(C_t\right)$$

where $U\left(C\right) = C^{1-\gamma}/(1-\gamma)$. Consumer observe x_t and z_t when they are realized. The budget constraint is

$$q_t B_{t+1} = B_t + Y_t - C_t,$$

where q_t is the price of a one period bond and B_t is the country net foreign asset position. We assume the bond price is

$$q_t = \beta e^{-\gamma g} - \psi\left(e^{B_{t+1}/X_t - \bar{b}} - 1\right).$$

The last assumption is basically a trick to get a well defined non stochastic steady state with $B_{t+1}/X_t = \bar{b}$, around which we can log linearize.

(i) Setup the problem of the representative consumer and derive optimality conditions.

(ii) Show that the Euler equation and the budget constraint can be rewritten solely in terms of the exogenous states and the variables $C_t/X_t, Y_t/X_t, B_t/X_{t-1}$.

(iii) Show that there is a non-stochastic steady state for the variables above.

(iv) Define c_t, b_t as deviations of $\ln(C_t/X_t), \ln(B_t/X_{t-1})$ from their stochastic steady state. Log linearize the Euler equation and the budget constraint in terms of these variables.

(v) Choose values for the model parameters (you can look at A-G to get ideas). Solve for the log linear approximate dynamics of the model (Hint: express c_t as function of $\Delta x_t, z_t, b_t$ and use a method of undetermined coefficients.)

(vi) Compute

$$\begin{array}{l} Cov\left[CA_t/Y_t, \Delta y_t\right] \\ Var\left[\Delta c_t\right]/Var\left[\Delta y_t\right] \end{array}$$

for different values of the ratio $\sigma_\epsilon^2/\sigma_\eta^2$ and compare with Figure 4 in Aguiar and Gopinath (2007). Discuss.

2. (Kraay and Ventura, 2000) Consider an economy where consumers can invest in two assets: home capital and foreign capital. Total investment in the two assets is denoted by k_t, k_t^*. Home capital and foreign capital are risky with random linear returns A_t and A_t^*.

Consumers preferences are represented by

$$E\left[\sum_{t=0}^{\infty} \beta^t \ln c_t\right]$$

and the flow budget constraint is

$$k_{t+1} + k_{t+1}^* + c_t = A_t k_t + A_t^* k_t^*.$$

(Relative to what we saw in class there are no bonds here).

Assume that A_t and A_t^* follow Markov processes:

$$\begin{array}{rcl} A_t & = & \left(A_{t-1}\right)^\rho \epsilon_t \\ A_t^* & = & \left(A_{t-1}^*\right)^\rho \epsilon_t^* \end{array}$$

where ϵ_t and ϵ_t^* are i.i.d. log normal disturbances with mean 1. The variances of ϵ_t and ϵ_t^* are equal to σ^2.

Define wealth (with dividends) as

$$w_t = A_t k_t + A_t^* k_t^*$$

(i) Show that the optimal consumption rule is linear and equal to

$$c_t = (1-\beta) w_t$$

(ii) Show that the optimal portfolio share $\theta_t \equiv k_{t+1}/\left(k_{t+1} + k_{t+1}^*\right)$ is time varying and is given by the function

$$\theta_t = h(a_t)$$

where $a_t \equiv A_t/A_t^*$.

(iii) Show that $h(.)$ is a nondecreasing function.

(iv) Use a linear approximation for $u'(c)$ to derive

$$h(a) \approx \frac{a^\rho - 1 + \sigma^2}{(a^\rho + 1)\sigma^2 - (a^\rho - 1)^2}.$$

you might also derive

$$h(a) \approx \frac{1}{2} + \frac{a^\rho - 1}{2\sigma^2}$$

(it depends wether you linearize only w.r.t. ϵ and ϵ^*, or also w.r.t. a.)

Use these relations to argue that the derivative $h'(1)$ is larger for smaller values of σ^2 and for larger values of ρ. Give an economic interpretation.

(v) Suppose the outside world does not invest in the country. Show that the current account can be written as

$$CA_t = (1 - \theta_t)(w_t - c_t) - (1 - \theta_{t-1})(w_{t-1} - c_{t-1}).$$

Derive the usual current account identity for this economy.

(vi) Show that the effect of a productivity shock on CA_t can be decomposed in two components, one due to Δw_t one due to $\Delta\theta_t$. Suppose we are in a situation where $A_{t-1} = A_{t-1}^* = 1$. Show that if h' is small the first component dominates and a domestic productivity shock has a positive effect on the current account surplus, if h' is large the second component dominates and a productivity shock has a negative effect on the current account surplus. Use (iii) and (iv) to interpret this result.

3. (Rogoff, 1992)

Consider a small open economy with a representative consumer with preferences

$$\mathrm{E}\sum \beta^t U(c_t)$$

where $U(c) = c^{1-\gamma}/(1-\gamma)$ and where consumption is the following aggregate of tradable and non-tradable goods

$$c_t = \left(c_t^T\right)^\alpha \left(c_t^N\right)^{1-\alpha}.$$

Production functions of tradables and non-tradables are Cobb-Douglas:

$$\begin{aligned} y_t^T &= A_t^T \left(n_t^T\right)^\alpha, \\ y_t^N &= A_t^N \left(n_t^N\right)^\alpha. \end{aligned}$$

The consumer can only trade a one-period non-state-contingent bond. There is no capital. The world interest rate is fixed at r. Consumers have a unit supply of labor which is fully employed in one of the two sectors:

$$n_t^T + n_t^N = 1.$$

(i) Write the budget constraint and derive the optimality conditions for the consumer and for the firms producing tradables and non-tradables. Use tradables as the numeraire.

(ii) Using consumer optimality derive a relation between the ratio c_t^T/c_t^N and the relative price of non-tradables p_t.

(iii) Using firms' optimality and labor market clearing derive a relation between n_t^T, the relative price of non-tradables and the productivity levels A_t^T and A_t^N.

(iv) Combine (ii) and (iii), the production function for non-tradables and market clearing in non-tradables to find a relation that must hold each period between c_t^T and n_t^T.

Suppose the country has constant productivities $A_t^T = A_t^N = \bar{A}$. At date 0 the country experiences a one-time, unexpected, positive transitory shock to the productivity of tradables so

$$A_t^T = \bar{A} + \rho^t \epsilon$$

with $\epsilon > 0$ and $\rho \in (0, 1)$.

(v) Assume $\gamma = 1$. What is the effect of the productivity shock on consumption of tradables c_t^T at dates $0, 1, 2, ...$?

(vi) What is the effect on the production and consumption of non-tradables (hint: use your result in part (iv))?

(vii) What is the effect on the relative price p_t at dates $0, 1, 2, ...$? Does it depend on ρ? Rogoff claims that "barring shocks to the supply of non-traded goods available for private consumption, the log real exchange rate would follow a random walk, regardless of the serial correlation properties of the shocks to traded goods productivity" (p. 12 of NBER WP 4119). Does your analysis support his claim?