411-3 NOTES: CONSUMPTION 3

GUIDO LORENZONI

1. Monetary policy

1.1. Partial equilibrium.

- Take our infinitely lived consumer, with standard time-separable preferences who receives a deterministic stream of labor income $\{Y_t\}$
- Suppose consumer starts with zero wealth
- The only difference with the simple PIH case discussed in notes 1 is that the real interest rate is time varying and is $\{r_t\}$
- The optimal behavior of this consumer can be derived from the Euler equation

$$U'(C_t) = \beta(1 + r_t) U'(C_{t+1})$$

and the intertemporal budget constraint

$$C_0 - Y_0 + \frac{1}{1 + r_0}(C_1 - Y_1) + \frac{1}{1 + r_0}\frac{1}{1 + r_1}(C_2 - Y_2) + ... = 0$$

- Suppose the real interest rate has been stable at $r^* = 1/\beta - 1$ and, unexpectedly, at time 0, the consumer learns that at some future date $T > 0$ the real interest rate will temporarily be $r_T > r^*$ and then revert back to r^*
- What is the effect of this shock?
- The Euler equation implies that consumption will be constant between periods 0 and T and then will be constant again from $T + 1$ onwards
- So the present value of consumption in the intertemporal budget constraint is

$$\left(1 + \beta + \beta^2 + ... + \beta^{T-1}\right) C_0 + \beta^{T-1}\frac{1}{1 + r_T}(1 + \beta + ...) C_T$$

- Inspecting this expression suggests, correctly, that as $T \to \infty$ we will get

$$C_t = \sum_{j=0}^{\infty} \beta^j Y_{t+j}$$

- Result: an anticipated, interest rate reduction has effects that go to zero as $T \to \infty$

1.2. General equilibrium.

Date: Spring 2019.

- Suppose now that we embed our partial equilibrium model into a new Keynesian model
- We do not need to specify the supply side of our economy if we make two assumptions:
  - we assume the central bank is able to achieve its desired path for the real interest rate $\{r_t\}$
  - we assume the in the long run central bank succesfully aims to keep Y_t at its natural level Y^*, more precisely that $\lim_{t\to\infty} Y_t = Y^*$
- Then take a path r_t with $r_t = r^*$ for all $t \neq T$
- The Euler equations for $t > T$, the market clearing condition

$$C_t = Y_t$$

  and $Y_t \to Y^*$ imply

$$C_t = Y^*$$

  for all $t > T$
- The Euler equation at T gives

$$u'(C_T) = \beta(1 + r_T)\, u'(C_{T+1}) = \beta(1 + r_T)\, u'(Y^*)$$

- The Euler equations for all $t \leq T$ implies

$$u'(C_0) = u'(C_T) = \beta(1 + r_T)\, u'(Y^*)$$

- Result: all consumption levels for $t \leq T$ respond in the same way to an anticipated reduction in the interest rate r_T, irrespective of how long is the horizon of the anticipated intervention T
- This is a surprising result, that shows how powerful is forward guidance in the baseline new Keynesian model
- Various versions of this result have been derived recently (starting with Del Negro, Giannoni, Patterson, 2013) and go under the title “forward guidance puzzle”

2. Monetary policy in heterogeneous agent models

- Consider now a model with heterogeneous agents
- Agents are hit by idiosyncratic shocks
- If Y_t is total output in the economy, agent i receives income $\omega_{it} Y_t$, where ω_{it} is an idiosyncratic shock with

$$\int \omega_{it}\, di = 1$$

  and a continuous distribution on the support $[\underline{\omega}, \overline{\omega}]$
- Assume for simplicity that ω_{it} is an i.i.d. shock

- Assume also that $\phi = 0$ (agents cannot borrow) and there is zero supply of assets from outside the household sector
- That is, there are no government bonds and no claims on firm (see below for micro-foundations of supply side consistent with the latter assumption)
- Let's call this a 0-HANK model, a HANK model with zero household liquidity
- It's useful as it is analytically tractable, in problem set you'll solve a simple HANK with positive liquidity
- This means that in spite of heterogeneity the model, in equilibrium, is very simple as we must have

$$C_{it} = \omega_{it} Y_t$$

- How to find the equilibrium interest rate?
- Euler equation

$$u'(C_{it}) \geq \beta (1 + r_t) E_t u'(C_{it+1}) = \beta (1 + r_t) E_t u'(\omega_{it+1} Y_{t+1})$$

- Since the RHS is same for all i, this equation can be an equality at most for one agent, the one with the highest value of C_{it}
- So r_t must satisfy

$$u'(\overline{\omega} Y_t) = \beta (1 + r_t) E_t u'(\omega_{it+1} Y_{t+1}) \tag{1}$$

- In fact, any r_t smaller than this would work too, but only if we choose r_t that satisfies (1) the model is the limit case of economies with $\phi > 0$ but small (or with small supply of government bonds), so we select equilibria where (1) is satisfied
- In this economy, it looks like the Euler equation only holds for an infinitesimal fraction of consumers, so not much forward looking behavior
- What is the response of output at time 0 to a shock to the interest rate?
- Assume as before that $Y_t \to Y^*$ and $r = r^*$ for all $t \geq 0$ where the natural interest rate is now given by

$$u'(\overline{\omega} Y^*) = \beta (1 + r^*) E[u'(\omega Y^*)]$$

- At date 0 we then have

$$u'(\overline{\omega} Y_0) = \beta (1 + r_0) E[u'(\omega Y^*)]$$

- With CRRA utility

$$\overline{\omega}^{-\gamma} Y_0^{-\gamma} = \beta (1 + r_0) E\left[\omega^{-\gamma}\right] (Y^*)^{-\gamma}$$

- The elasticity of output to a temporary shock to the (real) interest rate is

$$\frac{\frac{dY_0}{Y_0}}{\frac{dr_0}{1+r_0}} = -\frac{1}{\gamma}$$

- And is exactly the same as in a representative agent model!
- Even though there is a zero measure of consumers on their Euler equation!
- What is going on?
- All other consumers have unit MPC

$$C_{i0} = \omega_{i0} Y_0$$

so in aggregate their behavior gives

$$Y_0 = \int C_{i0} di = \int \omega_{i0} di Y_0 = Y_0$$

which imposes no restriction on Y_0

2.1. **A case with a discrete distribution of ω.**

- With a discrete distribution of ω the result is a bit easier to digest
- There is a mass of consumers with $\bar{\omega}$, say $\bar{\mu}$
- For them we have

$$C_{i0} = \xi \left(1 + r_0\right)^{-\frac{1}{\gamma}}$$

(where ξ is a constant term)
- Then total demand is

$$\bar{\mu}\xi \left(1 + r_0\right)^{-\frac{1}{\gamma}} + \left(1 - \bar{\mu}\right) \tilde{\omega} Y_0 = Y_0$$

where

$$\tilde{\omega} = \frac{1}{1 - \bar{\mu}} \int_{\omega < \bar{\omega}} \omega dF\left(\omega\right)$$

is the average ω for all agents with $\omega < \bar{\omega}$
- We then obtain

$$Y_0 = \frac{1}{1 - \left(1 - \bar{\mu}\right) \tilde{\omega}} \bar{\mu}\xi \left(1 + r_0\right)^{-\frac{1}{\gamma}}$$

- Once more, the elasticity of output to r_0 is the same as in a representative agent economy
- Why? Because the constrained agents have MPC=1
- So we can think of decomposing the effect of the policy shock into two pieces
- Changes in the "autonomous" component of consumption

$$\bar{\mu}\xi \left(1 + r_0\right)^{-\frac{1}{\gamma}}$$

- A keynesian multiplier effect

$$\frac{1}{1-(1-\bar{\mu})\,\tilde{\omega}}$$

3. Fiscal policy

- Temporary increase in G in new Keynesian model

$$Y_t = C_t + G_t$$

- Long run: we go back to initial level of Y^*
- Suppose central bank fully accomodates and keeps $r = r^*$
- Then C_t fully determined by Euler equation

$$u'(C_t) = \beta (1 + r^*) U'(C_{t+1})$$

so

$$C_t = Y^*$$

- Multiplier is equal to 1

$$dY_t = dG_t$$

- This is Woodford's simple analytics of the government expenditure multiplier (AEJ Macro 2011)
- What happens in a Aiyagari-Bewley environment?
- Equilibrium from

$$\bar{\mu}\xi (1 + r^*)^{-\frac{1}{\gamma}} + (1-\bar{\mu})\,\tilde{\omega} Y_0 + G = Y_0$$

so

$$dY_0 = \frac{1}{1-(1-\bar{\mu})\,\tilde{\omega}} dG$$

- Very old-fashioned keynesian-cross logic
- If mass of unconstrained agents goes to zero multiplier goes to ∞!
- Summary: if we calibrate NK and simple 0-HANK model to produce the same Y in steady state, the responses of output to a fiscal and monetary policy shock are

	NK	0-HANK
dG	1	$\frac{1}{1-(1-\bar{\mu})\tilde{\omega}}$
$\frac{dr}{1+r}$	$-\frac{1}{\gamma}Y^*$	$-\frac{1}{\gamma}Y^*$

- Take away: in models with household liquidity constraints fiscal policy is more powerful, monetary policy is equally powerful
- Notice that this is a comparison of different models calibrated to obtain the same Y^*
- It is not a comparative static with respect to regime/policy changes that make liquidity constraint more or less relevant (and also affect the equilibrium value of Y^*)