411-3 NOTES: CONSUMPTION 1

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1. Consumption in the recession

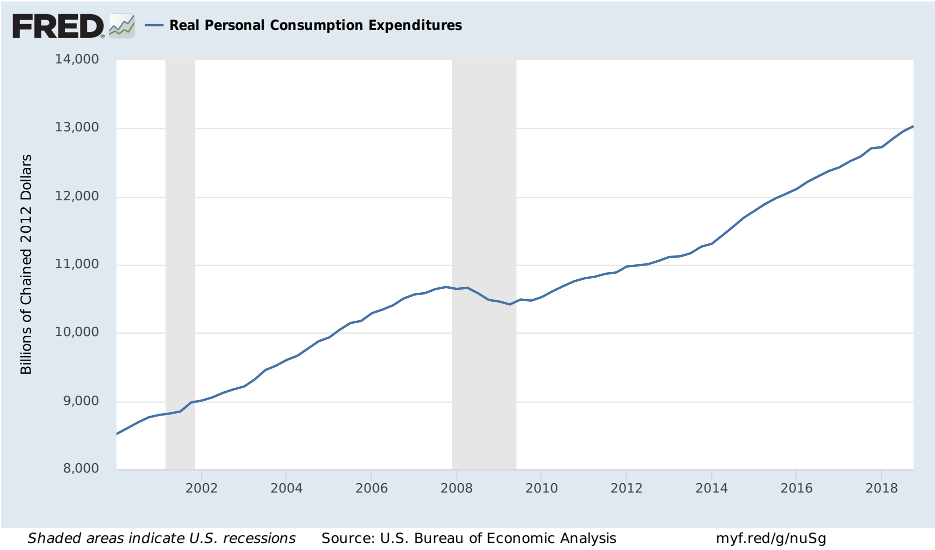

Figure 1. Consumption

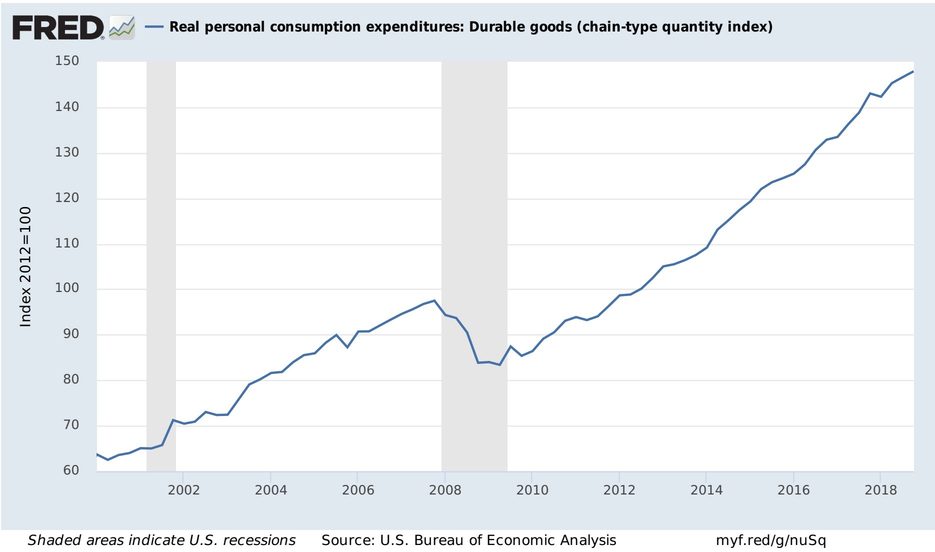

Figure 2. Durable goods

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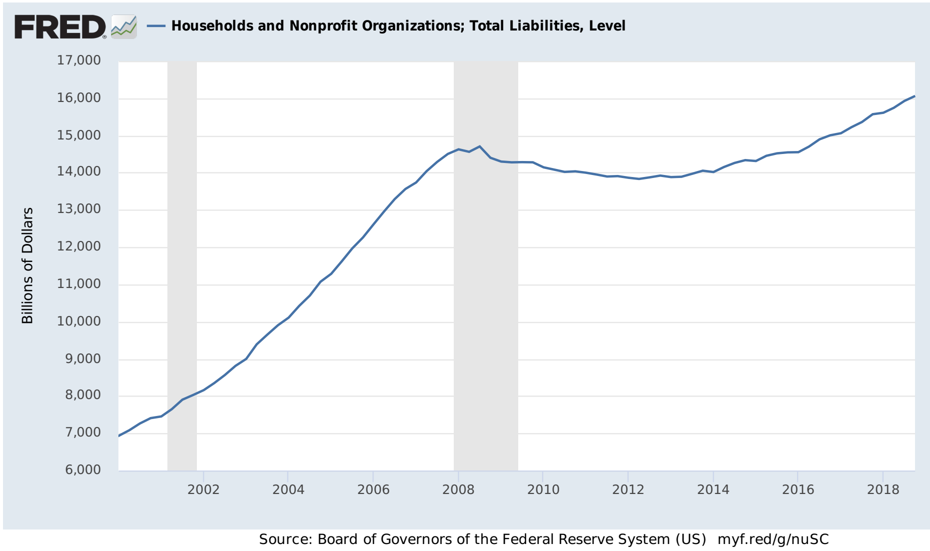

FIGURE 3. Household debt

2. Permanent income hypothesis

- Basic idea: consumption smoothing
- Consumers' objective

$$E\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)$$

- Budget constraint

$$a_{t+1}=\left(1+r\right)a_t+y_t-c_t$$

- Simple case
  - no uncertainty
  - $\beta\left(1+r\right)=1$
- Optimality condition

$$u'\left(c_t\right)=\beta\left(1+r\right)u'\left(c_{t+1}\right)=u'\left(c_{t+1}\right)$$

  so c_t constant
- Intertemporal budget constraint

$$\sum_{j=0}^{\infty}\left(1+r\right)^{-j}\left(c_{t+j}-y_{t+j}\right)=\left(1+r\right)a_t$$

- So we obtain

$$c_t=\frac{r}{1+r}\sum_{j=0}^{\infty}\left(1+r\right)^{-j}y_{t+j}+ra_t$$

- Main insights:

  - consumption depends on expected future income (here with perfect foresight), not just on current income
  - so income process matters
  - marginal propensity to consume out wealth is small (r)
- Can we add uncertainty and get something like this?

$$c_t = \frac{r}{1+r} E_t \sum_{j=0}^{\infty} (1+r)^{-j} y_{t+j} + r a_t$$

- Yes, if we assume quadratic utility, so

$$u'(c_t) = E_t u'(c_{t+1})$$

  becomes

$$c_t = E_t c_{t+1}$$

- Random walk property of consumption (rejected in data)

3. Income fluctuation problem

- Suppose i.i.d. income process y_t
- Utility function $u(.)$ strictly concave, with $\lim_{c \to 0} u'(c) = \infty$
- Borrowing constraint

$$a_t \geq -\phi$$

- Natural borrowing limit

$$\phi = \frac{y_{min}}{r}$$

- Define cash-on-hand

$$z_t = a_t + y_t$$

- Bellman equation

$$V(z) = \max_{a'} u(z - a') + \beta E\left[V\left((1+r) a' + y'\right)\right]$$

- Euler equation

$$u'(c_t) \geq \beta (1+r) E_t \left[u'(c_{t+1})\right]$$

- What happens if $\beta (1+r) = 1$?
- We have

$$u'(c_t) \geq E_t \left[u'(c_{t+1})\right]$$

  so $u'(c_t)$ is a supermartingale and has a limit distribution, but then c_t has a limit distribution and if $c_t < \infty$ we obtain a violation of budget constraints
- Result (Bewley): when $\beta (1+r)$ the optimal solution has $a_t \to \infty$ and $c_t \to \infty$

- Intuition: wealth provides self-insurance, as long as we are away from lower limit, with $\beta(1+r) = 1$ no trade-off between self-insurance and impatience so agents accumulate unbounded wealth
- In general equilibrium supply of assets is "bounded", so to have bounded asset demand the interesting case is $\beta(1+r) < 1$
- We'll see this later in computations
- From now on we assume

$$\beta(1+r) < 1$$

- Properties of the value function
- $V(z)$ is increasing, concave, differentiable (review)
- Properties of consumption and asset accumulation policies
- $c(z)$ is increasing $a'(z)$ is non-decreasing
- Proof: define $\Psi(a') \equiv \beta E[V((1+r)a' + y')]$, then Bellman is just a 2 goods problem with separable utility
- Borrowing constraint is binding iff $z \leq z^*$
- Proof: If

$$u'(z+\phi) > \Psi'(-\phi)$$

  the inequality still holds for any $z' < z$
- An important property: with $\beta(1+r) < 1$ the asset distribution is bounded above
- This property holds if the utility function satisfies

$$\lim_{c\to\infty} -\frac{u''(c)}{u'(c)} = 0 \qquad (1)$$

  that is if risk aversion not important at high levels of wealth, so trade-off now dominated by impatience and consumers stop accumulating wealth
- Proof (sketch)
- Define consumption tomorrow if highest realization of income is realized

$$\bar{c}(z) = c((1+r)a'(z) + y_{max})$$

- Let $z > z^*$ so Euler holds as equality
- Write Euler as

$$u'(c(z)) = \beta R \frac{E[u'(c(z'))]}{u'(\bar{c}(z))} u'(\bar{c}(z))$$

- Suppose that

$$\lim_{z\to\infty} \frac{E[u'(c(z'))]}{u'(\bar{c}(z))} = 1 \qquad (2)$$

  (we'll prove it later)

- Then there is a $\bar{z}$ large enough such that if $z > \bar{z}$

$$u'(c(z)) < u'(\bar{c}(z))$$

and from envelope condition

$$V'(z) < V'((1+r)\,a'(z) + y_{max})$$

- Concavity of V then implies

$$(1+r)\,a'(z) + y_{max} < z$$

so the map

$$(1+r)\,a'(z) + y_{max}$$

crosses the 45 degree line at some z, that's the upper bound for the distribution of z in the long run

- It remains to prove (2), here we need

$$\frac{u'(c-A)}{u'(c)} \to 1$$

for $c \to \infty$

$$1 \leq \frac{u'(c-A)}{u'(c)} = 1 + \int_{c}^{c-A} \frac{u''(\tilde{c})}{u'(c)} d\tilde{c} \leq 1 + \int_{c-A}^{c} \frac{u''(\tilde{c})}{u'(\tilde{c})} d\tilde{c}$$

and under condition (1)

$$\int_{c-A}^{c} \frac{u''(\tilde{c})}{u'(\tilde{c})} d\tilde{c} \to 0$$