# Discussion of "Monetary Policy Analysis When Planning Horizons Are Finite" by Mike Woodford

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This paper fits in a fast growing literature that attempts to introduce forms of bounded rationality in macroeconomic models. There are a variety of ways in which bounded rationality can be introduced, depending on how we describe the agents' limited ability to process information, to form forecasts, and to compute optimal plans. The paper I am discussing (Woodford, 2018) captures bounded rationality by giving agents a finite planning horizon and exploring in depth a variety of consequences of this modeling assumption. The paper provides a nice motivation for the exercise, by connecting the macro literature to existing work in artificial intelligence.

In this discussion I want to make two points, one on the role of general equilibrium effects and one on difference between finite lives and finite planning horizons.

There is one dimension of bounded rationality which appears in different forms in a variety of models: the limited capacity of agents to think through general equilibrium effects in their environment. My first point is that this limited capacity for general equilibrium thinking also plays an important role in Woodford (2018). To make this point let me use a simple example of the so called "forward guidance puzzle" (Del Negro et al., 2012), inspired by Farhi and Werning (2017).

Take an infinitely lived consumer, with standard time-separable preferences, who receives a deterministic stream of labor income $\{Y_t\}$ and has access to a single bond that pays the real interest rate $r_t$. The optimal behavior of this consumer can be derived from the Euler equation

$$U'(C_t) = \beta (1 + r_t) U'(C_{t+1})$$

and the intertemporal budget constraint

$$C_t - Y_t + \frac{1}{1+r_t} (C_{t+1} - Y_{t+1}) + \frac{1}{1+r_t} \frac{1}{1+r_{t+1}} (C_{t+2} - Y_{t+2}) + ... = 0.$$

Assume that the consumer starts with zero wealth. Suppose the real interest rate has been stable

at $r_t = r^* = 1/\beta - 1$ and, unexpectedly, at time $t$, the consumer learns that the central bank, at some future date $t + J$ will temporarily raise the interest rate to $r_{t+J} > r^*$. What is the effect of this shock?

The Euler equation implies that consumption will be constant between periods $t$ and $t + J - 1$ and then will be constant again from $t + J$ onwards. So the present value of consumption in the intertemporal budget constraint will take the form

$$\left(1 + \beta + \beta^2 + ... + \beta^{J-1}\right) C_t + \beta^{J-1} \left(1 + \frac{1}{1 - \beta} \frac{1}{1 + r_T}\right) C_{t+T}.$$

Inspecting this expression suggests, correctly, that as $J \to \infty$ we will get

$$C_t = \sum_{j=0}^{\infty} \beta^j Y_{t+j}.$$

From this simple derivations it seems that, as the time of the intervention is moved farther in the future the intervention will have no effect on consumer spending today. So it would appear that there is no forward guidance puzzle after all! What is missing from the argument here is that this was purely a partial equilibrium exercise, taking as giving the path of income $\{Y_t\}$.

Suppose now that we build a simple representative agent economy around the consumer above. Suppose output is produced linearly using only labor. To introduce an extreme form of nominal rigidity, suppose wages and prices are fixed so output and labor income is simply determined by demand in the goods market

$$Y_{t+j} = C_{t+j}.$$

Assume now that the representative consumer realizes that all other consumers are behaving identically and is aware of the goods market clearing condition above. To solve for the equilibrium of this economy we can now ignore the intertemporal budget constraint—which is trivially satisfied since $C_t = Y_t$ for all $t$—and only use the Euler equation. As a terminal condition, we use the assumption that after $t + J$ the central bank leads the economy to the natural level of output $Y^*$.[1] Now repeated use of the Euler equation gives

$$U'(C_t) = U'(C_{t+1}) = ... = U'(C_{t+J}) = \beta \left(1 + r_{t+J}\right) U'(Y^*).$$

This gives us a simple version of the forward guidance puzzle: the effect of increasing $r_{t+J}$ on consumption $C_t$ is the same, irrespective of how far in the future the shock is expected to occur.

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[1]Here, I am implicitly assuming labor supply is inelastic at some level $\bar{N}$, so $Y^* = \bar{N}$. This would cause some problems if I wanted to experiment with a reduction in $r_{t+J}$ and insisted that wages be rigid both downward and upward. Of course there are many ways around this issue e.g., having only downward wage rigidity, introducing an initial shock that makes the economy start below full employment, or introducing full blown sticky prices and wages.

One can get even stronger forms of the puzzle by adding details of the new Keynesian model, like a Phillips curve and a central bank policy formulated in terms of nominal rates, so effects on future inflation will affect the real interest rate. Going in that direction one can find examples in which the effect on $C_t$ increases without bounds as $J$ increases. But for my point here, this simple version is enough. Comparing the partial equilibrium response and the general equilibrium response highlights that it is crucial that consumers forms expectations about future income levels by being fully aware that other agents are responding like themselves and that their responses are affecting equilibrium incomes.[2]

One of the most direct ways of capturing the idea that agents have limited grasp of the general equilibrium forces that an external shock sets in motion is the idea of $k$-level rationality.[3] García-Schmidt and Woodford (2015) first proposed to use a version of $k$-level rationality in monetary models. The present paper offers a different approach to bounded rationality, but what I want to point out here is that limits to general equilibrium thinking are still very much central to the argument in the present paper. To see that, let me begin with the value function used in the finite horizon plans of Woodford (2018). Still using my simple deterministic example, the value function for an individual saver with labor income $\{Y_{t+j}\}$ and access to a single bond can be defined as follows[4]

$$V_t(B_t) = \max \sum_{j=0}^{\infty} \beta^j U(C_{t+j}) \qquad s.t. \sum_{j=0}^{\infty} Q_{t+j|t}(C_{t+j} - Y_{t+j}) = B_t.$$

Assume now that the interest rate and income are fluctuating around the values $r^*$ and $Y^*$. Then the approximate value function that the agents in Woodford (2018) will use is not the one defined above, but one that abstracts from the fluctuations in interest rates and income, that is, one that uses the intertemporal budget constraint

$$\sum_{j=0}^{\infty} \beta^j (C_{t+j} - Y^*) = B_t.$$

This approximate value function is

$$V(B_t) = \frac{1}{1-\beta} U((1-\beta)B_t + Y^*). \tag{0.1}$$

To compute the response at date $t$ of an agent with planning horizon $j$ we need to proceed recursively. Start from an agent at date $t+j-1$ and an horizon of 1 and look at his response. Then use this response to compute the general equilibrium expectations of an agent at $t+j-2$ with a

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[2]Marios. Maybe something on idea that a characteristic of a demand shock is that that agents will respond more than what they think others will (Lorenzoni, 2009).

[3]See Crawford et al. (2013).

[4]I use the notation $Q_{t+j|t} = \frac{1}{1+r_t} \frac{1}{1+r_{t+1}} \cdots \frac{1}{1+r_{t+j}}$.

planning horizon of 2 periods, and so on and so forth until we reach $t$. Proceeding in this way, it is easy to show that the agents in Woodford (2018) will not respond at all to an anticipated shock at $t+J$, if their planning horizon is smaller than $J$. What I want to remark is that this result relies on the fact that even though the agents in Woodford (2018) are allowed to use full general equilibrium reasoning *within* their planning horizon, the use of the value function (0.1), by replacing the present value of future labor income with the fixed value $Y^*$ is essentially stopping general equilibrium reasoning beyond the planning horizon and that plays an important role. It is useful to add that limits to general equilibrium thinking feature in a variety of approaches that have been used to address the forward guidance puzzle in the recent literature: in Gabaix (2016) by introducing a myopia parameter in agents' forecasting equations, in Angeletos and Lian (ming) and Wiederholt et al. (2014) by keeping rational expectations but ruling out common knowledge, in Farhi and Werning (2017) by using a combination of heterogeneity and $k$-level rationality.

The second point I want to make is to answer the naive question: if the idea is to give agents a limited future horizon, isn't enough to give agents finite lives? The answer is no, and again the crucial issue is not how far agents live, but how far in the future they think about general equilibrium effects. To see why, consider a simple overlapping generations economy, with two-period lived agents who receive labor income only when young. Agents save by holding a Lucas tree, in unit supply, with price $P_t$ and dividend $D_t$. The preferences of agents born at time $t$ are represented by the utility function

$$\ln\left(C_{1t}\right) + \beta \ln\left(C_{2t+1}\right).$$

Their budget constraints are

$$C_{1t} = W_t N_t - P_t S_t,$$

$$C_{2t+1} = \left(P_{t+1} + D_{t+1}\right) S_t,$$

where $S_t$ are holdings of the tree. Again, I assume a linear production function that uses only labor and make the extreme assumption of fixed wages and prices (with prices normalized to 1). Let me assume that dividends are proportional to total output produced by labor, which I denote by $Y_t$. Namely, set $D_t = \delta Y_t$. Optimal behavior of young agents imply that they consume

$$C_{1t} = \frac{1}{1+\beta} W_t N_t.$$

Old agents consume the value of the tree they sell

$$C_{2t} = P_t + D_t.$$

Good market clearing takes the form

$$C_{1t} + C_{2t} = Y_t + D_t = (1 + \delta) Y_t.$$

Suppose labor productivity is 1 and labor supply is 1. A stationary equilibrium with full employment has $W_t = 1$ and $Y_t = N_t = 1$. The goods market clearing condition at full employment then requires the asset price to satisfy

$$P_t = P^* \equiv \frac{\beta}{1 + \beta},$$

and no arbitrage between bonds and trees require the interest rate to be $r = r^*$, where

$$P^* = \frac{P^* + \delta}{1 + r^*}.$$

Consider an economy where the central bank is keeping the real rate at $r = r^*$ and consider the effect of a one time, anticipated increase in the interest rate at $t + J$. Repeatedly using the no arbitrage condition between bonds and trees implies that

$$P_t = \frac{1 + r^*}{1 + r_{t+J}} P^*.$$

Assuming complete nominal price and wage rigidity as in the example above, good market clearing now gives

$$\frac{1}{1 + \beta} Y_t + P_t = Y_t,$$

which then implies

$$Y_t = \frac{1 + r^*}{1 + r_{t+J}}.$$

Once again we have a basic form of the forward guidance puzzle: the effect of an intervention is the same even if the intervention happens very far in the future. Even though agents live only two period, they correctly forecast the effect of future policy on asset prices in the future, and these determine asset prices today, thus affecting real activity.

If some agents, at some future date, uses the approximate value function

$$V\left(S_{t+j}\right) = \beta \ln \left( \left( Q^* + D^* \right) S_{t+j} \right),$$

or if agents at $t$ form expectations on the behavior of these agents attributing them this value function, then the chain of general equilibrium reasoning is broken and we move in the direction of Woodford (2018). So finite lives are different from finite planning horizons. Agents with finite

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lives can still look beyond their lifetime to make forecast about future economic variables. Only when they start making simplifications in thinking about these future outcomes, we start to see the type of effects that Woodford (2018) and other papers are after.

This simple OLG example allows me to make one more point. This example has two predictions, one is that the asset price $Q_t$ will be very sensitive to forecasts about future central bank actions, the second is that consumption and real activity will also be very sensitive. In the model, these two predictions are strictly intertwined. However, the first prediction sounds much more palatable than the second. In other words, it seems like it could make sense to build models where financial market prices are determined by agents that look relatively far in the general equilibrium consequences of some shock, while other, real variables are determined by decision makers that use less general equilibrium thinking. Some combination along these lines seems a promising avenue for future research in this area.

Before concluding, I want to add a comment on Section 4 of the paper in which Woodford combines finite horizon planning and some form of learning. Finite horizon planning seems a good way of capturing what happens when agents are used to a given environment and are forced to think about the effects of an "unusual" shock, so they have to form conjectures about other agents' behavior by thinking through the optimization problems these other agents are solving. If this reasoning is boundedly rational, these agents will make systematic mistakes. Eventually one imagines that agents will learn from these mistakes and adapt. Section 4 of the paper captures this adaptation by introducing a form of learning and applying it to the following problem.

Consider a central bank that, instead of following a Taylor rule or some other responsive rule, just keeps the nominal interest rate fixed at some level below the natural interest rate of the economy. A model with finite planning horizons—or a model with other forms of bounded rationality like those listed above—will still be able to derive unique predictions about the behavior of this economy. The reason is that these models can basically be solved going backwards starting at some future date and expectations are always uniqueley derived. And the model predictions will be that inflation is stable at some level above the reference steady state used to conduct the exercise.

The question is: is this a desirable property of this class of models? Woodford (2018) argues that it is not. If the central bank follows a nominal interest rate peg below the natural rate, we expect inflation to be systematically higher than what people expect, and we expect agents to start adjusting their inflation expectations upward. The logic of the accelerationist hypothesis suggests that, absent any reaction of the central bank, we will see inflation increasing without bounds. Adding learning to the model, Woodford (2018) shows that an interest rate peg does indeed lead to explosive inflation dynamics.

I see that part of the paper as providing a very useful general warning. Introducing bounded rationality in macro models is a desirable development, but we want to avoid making so many

assumptions on the way in which agents simplify their optimization problems, that we end up ruling out pathological outcomes under any possible policy regime. Identifying policy regimes that lead to various forms of instability is an important task of macro modeling. Combining bounded rationality and learning seems a good way to make progress in that direction.

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