PORTFOLIO CHOICE AND OPTIMAL MONETARY POLICY

GUIDO LORENZONI

- A simple model of portfolio choice based on "Monetary Policy, Capital Controls and International Portfolios," Fanelli 2017
- Two periods $t-1$ and t
- Tradable and non-tradable goods
$$c_t = \left(c_t^T\right)^{\omega} \left(c_t^N\right)^{1-\omega}$$
- Preferences
$$\log c_t - \frac{\psi_t}{1+\phi} n_t^{1+\phi}$$
- Shocks to labor supply ψ_t
- Endowment of tradables
$$y_t^T$$
- Production of non-tradables
$$y_t^N = n_t$$
- Budget constraint at $t-1$, country can only take long and short positions in domestic and foreign bonds that pay nominal returns R and R^*, respectively
$$B_t^* + B_t = 0$$
- Budget constraint at t
$$P_t^T c_t^T + P_t^N c_t^N = P_t^T y_t^T + W_t n_t + R^* \mathcal{E}_t B_t^* + R B_t$$
- Optimal portfolio in economy with no capital controls gives
$$E_{t-1}\left[\frac{1}{P_t^T} u'\left(c_t\right)\left(\frac{R}{\mathcal{E}_t} - R^*\right)\right] = 0$$
- Foreign investors are risk neutral so R must satisfy
$$E_{t-1}\left[\frac{R}{\mathcal{E}_t} - R^*\right] = 0$$
or
$$R = \frac{R^*}{E_{t-1}\left[\frac{1}{\mathcal{E}_t}\right]}$$
- Price of tradables pinned down by the world price (normalized to 1) so
$$P_t^T = \mathcal{E}_t$$
- Price of NT pinned down by optimality of firms
$$P_t^N = W_t$$
- Rigid wages W_t (can be modeled with labor service monopolists)

Date: January 2019.

- Relative price of non-tradables

$$p_t \equiv \frac{P_t^N}{P_t^T} = \frac{W_t}{\mathcal{E}_t}$$

- Demand for non-tradables conditional on tradable demand

$$\frac{\omega}{1-\omega}\frac{c_t^N}{c_t^T} = \frac{P_t^T}{P_t^N}$$

which gives

$$c_t^N = \frac{1-\omega}{\omega}\frac{c_t^T}{p_t}$$

- First best, with complete markets

$$c_t^T = E_{t-1}\left[y_t^T\right]$$

constant across states of the world
- Substituting budget constraint at $t-1$ we get

$$P_t^T c_t^T + P_t^N c_t^N = P_t^T y_t^T + W_t n_t + \left(R - R^* \mathcal{E}_t\right) B_t$$

which, using market clearing in NT, becomes

$$c_t^T = y_t^T + \left(\frac{R}{\mathcal{E}_t} - R^*\right) B_t$$

- To keep c_t^T constant we need $\mathcal{E}$ to go down when y_t^T goes down
- That is, appreciate domestic currency in bad states to provide insurance
- First best allocation of N satisfies

$$(1-\omega)\frac{1}{c_t^N} = \psi_t n_t^{\phi}$$

$$1-\omega = \psi_t n_t^{1+\phi}$$

so we need n_t to move with ψ

$$\frac{\omega}{1-\omega}\frac{c_t^N}{c_t^T} = \frac{\mathcal{E}_t}{W_t}$$

exchange rate must depreciate when ψ_t is lower so we produce and consume more non-tradables
- Result: if agents can trade complete set of state contingent claims at $t-1$, then optimal monetary policy achieves the first best
- If agents can only trade the 2 bonds denominated in the 2 currencies, in general, we have incomplete markets
- With incomplete markets optimal monetary policy needs to trade-off insurance vs optimal allocation
- Rewrite objective of monetary policy as

$$E_{t-1}\left[\omega \log c_t^T + (1-\omega)\log\left(c_t^N\right) - \frac{1}{1+\phi}\psi_t\left(c_t^N\right)^{1+\phi}\right]$$

and let's solve the relaxed planner problem in which the only constraints faced by the planner are

$$c_t^T = y_t^T + \left(\frac{\frac{1}{\mathcal{E}_t}}{E_{t-1}\frac{1}{\mathcal{E}_t}} - 1\right) R^* B_t$$

and

$$\frac{\omega}{1-\omega}\frac{c_t^N}{c_t^T}=\frac{\mathcal{E}_t}{W_t}$$

- Since W_t is pre-set at $t-1$ we can write

$$\frac{\frac{1}{\mathcal{E}_t}}{E_{t-1}\frac{1}{\mathcal{E}_t}}=\frac{p_t}{Ep_t}$$

- Also, use the notation $b=R^*B_t$, drop time subscripts and make explicit dependence on state of the world s to get the problem in the following form
- Choose $z(s), b$ to maximize

$$\sum\pi(s)\left[\omega\log c^T(s)+(1-\omega)\log\left(c^N(s)\right)-\frac{1}{1+\phi}\psi(s)\left(c^N(s)\right)^{1+\phi}\right]$$

subject to

$$c^T(s)=y^T(s)+\left(\frac{p(s)}{\sum\pi(\tilde{s})p(\tilde{s})}-1\right)b$$

$$\omega c^N(s)p(s)=(1-\omega)c^T(s)$$

- Optimality

$$\frac{\omega}{c^T(s)}=\lambda(s)-(1-\omega)\mu(s)$$

$$\frac{1-\omega}{c^N(s)}-\psi(s)\left(c^N(s)\right)^{\phi}=\omega\mu(s)p(s)$$

$$\lambda(s)\frac{b}{\sum\pi(\tilde{s})p(\tilde{s})}-\sum\pi(s)\frac{\lambda(s)p(s)}{\left(\sum\pi(\tilde{s})p(\tilde{s})\right)^2}b=\omega\mu(s)c^N(s)$$

$$E\lambda(s)\left(\frac{p(s)}{\sum\pi(\tilde{s})p(\tilde{s})}-1\right)=0$$

- Rearranging we can write

$$\mu(s)=\frac{1}{\omega}\frac{1}{p(s)}\left[\frac{1-\omega}{c^N(s)}-\psi(s)\left(c^N(s)\right)^{\phi}\right]$$

$$\lambda(s)=\frac{\omega}{c^T(s)}+\frac{1-\omega}{\omega}\frac{1}{p(s)}\left[\frac{1-\omega}{c^N(s)}-\psi(s)\left(c^N(s)\right)^{\phi}\right]$$

- We can omit dependence on s from now on
- Optimality for b yields, after rearranging,

$$E[\lambda z]=E[\lambda]E[z]$$

and optimality for z yields

$$\mu=\frac{1}{\omega}\frac{1}{c^N}\left[\lambda-\frac{E[\lambda z]}{E[z]}\right]\frac{b}{E[z]}=\frac{1}{\omega}\frac{1}{c^N}\left(\lambda-E[\lambda]\right)\frac{b}{E[z]}$$

- So we end up with three conditions

$$\lambda=\frac{\omega}{c^T}+\frac{1-\omega}{\omega}\frac{1}{z}\left[\frac{1-\omega}{c^N}-\psi\left(c^N\right)^{\phi}\right]$$

$$\left(\lambda-E[\lambda]\right)\frac{b}{E[z]}=\frac{c^N}{z}\left[\frac{1-\omega}{c^N}-\psi\left(c^N\right)^{\phi}\right]$$

$$E\left[\left(\lambda-E[\lambda]\right)z\right]=0$$

- Marginal value of tradable resources is
$$\lambda = \frac{\omega}{c^T} + \frac{1-\omega}{\omega}\frac{1}{z}\left[\frac{1-\omega}{c^N} - \psi\left(c^N\right)^{\phi}\right]$$
which captures direct effect plus an "aggregate demand externality" term which reflects the fact that when consumption of traded goods increases, for a given real exchange rate p, the consumption and production of NT also increases, which may be good or bad, from a social welfare perspective, depending on the sign of the efficiency (output-gap) wedge
- Notice that a weighted average of μ is zero because
$$E\left[c^N\mu\right] = \frac{1}{\omega}E\left(\lambda - E\left[\lambda\right]\right)\frac{b}{E\left[z\right]} = 0$$
this reflects the fact that we can always scale p by a constant factor, helping to fix the allocation of NT on average, and keeping the allocation of T unchanged, as the latter only depends on $p/E\left[p\right]$

Exercise 1. Choose a discrete set of states S and choose a distribution $\pi\left(s\right)$ and functions $y^T\left(s\right)$ and $\psi\left(s\right)$. Find $b, p\left(s\right)$ tha maximize the planner allocation. You can try a direct maximization algorithm that just sets
$$c^T\left(s\right) = y^T\left(s\right) + \left(\frac{p\left(s\right)}{\sum\pi\left(\tilde{s}\right)p\left(\tilde{s}\right)} - 1\right)b$$
$$c^N\left(s\right) = \frac{1-\omega}{\omega}\frac{c^T\left(s\right)}{p\left(s\right)}$$
and maximizes the planner's objective, or try to use the optimality conditions derived above (or use one method to check the other!)