NORTHWESTERN RTG DYNAMICS SUMMER SCHOOL 2023

1. Schedule

6/20 Tuesday.

8:30-9:00	Coffee/Tea
9:00-10:00	Lee-1
10:30-11:30	Burns-1
11:30-14:00	Lunch
14:00-15:00	Erchenko-1
15:30-16:30	Slutsky-1

6/21 Wednesday.

8:30-9:00	Coffee/Tea
9:00-10:00	Slutsky-2
10:30-11:30	Problem session
11:30-14:00	Lunch
14:00-15:00	Erchenko-2
15:30-16:30	Burns-2

6/22 Thursday.

8:30-9:00	Coffee/Tea
9:00-10:00	Khalil-1
10:30-11:30	Vinhage-1
11:30-14:00	Lunch
14:00-15:00	Erchenko-3
15:30-16:30	Lee-2
17:00-	Social events*

6/23 Friday.

8:30-9:00	Coffee/Tea
9:00-10:00	Khalil-2
10:30-11:30	Problem session
11:30-14:00	Lunch
14:00-15:00	Burns-3
15:30-16:30	Vinhage-2

6/26 Monday.

8:30-9:00	Coffee/Tea
9:00-10:00	Miller-1
10:30-11:30	Vinhage-3
11:30-14:00	Lunch
14:00-14:50	Wieser - Research talk
14:50-15:40	Sanchez - Research talk
15:40-16:05	Break
16:05-16:55	Call - Research talk
16:55-17:45	Marshall - Research talk

*Social events will be held outdoors near Lunt Hall.

6/27 Tuesday.

8:30-9:00	Coffee/Tea
9:00-10:00	Mohammadi-1
10:30-11:30	Problem session
11:30-14:00	Lunch
14:00-15:00	Miller-2
15:00-15:30	Break
15:30-16:30	Fraczyk-1
16:40-17:30	Bersudsky - Research talk

6/28 Wednesday.

8:30-9:00	Coffee/Tea
9:00-10:00	Mohammadi-2
10:30-11:30	Wei-1
11:30-13:50	Lunch
13:50-14:50	Miller-3
14:50-15:20	Break
15:20-16:10	Slutsky - Research talk
16:10-17:00	Lee - Research talk
17:00-	Social events*

6/29 Thursday.

8:30-9:00	Coffee/Tea
9:00-10:00	Mohammadi-3
10:30-11:30	Problem session
11:30-14:00	Lunch
14:00-15:00	Wei-2
15:00-15:30	Break
15:30-16:30	Fraczyk-2
16:40-17:30	Wilkens - Research talk

6/30 Friday.

8:30-9:00	Coffee/Tea
9:00-10:00	Fraczyk-3
10:30-11:30	Wei-3
11:30-14:00	Farewell

2. Title and Abstract

2.1. Minicourse.

Alena Erchenko. Title: Flexibility principles in dynamics and beyond
Abstract: Flexibility and rigidity questions are natural to formulate in many settings. In particular, classical smooth dynamical systems have been systematically studied through the prism of flexibility and rigidity over the past several years. While rigidity in dynamics has been investigated since the 1980s, flexibility questions were presented as a substantial research direction in the field more recently by A. Katok. Roughly speaking, the aim is to study natural classes of smooth dynamical systems and to find constructive tools to freely manipulate dynamical data inside a fixed class. During the minicourse, we will discuss the formulation of flexibility questions, introduce techniques that have recently been used to establish flexibility results, and pose open questions.

Mikolaj Fraczyk. Title: Stationary random subgroups
Abstract: I will discuss the concept of stationary random subgroups and their applications to the study of discrete subgroups of semi simple Lie groups. The mini course will contain the proof od Nevo=Zimmers structure theorem for stationary actions of higher rank groups and the proof that every confined discrete subgroup of a simple higher rank group is a lattice (where confined means that there is a bounded set U such that any conjugate of the group contains a non-trivial element of U).

Nick Miller. Title: Arithmeticity, superrigidity, and totally geodesic submanifolds
Abstract: In the 1970s seminal work of Margulis showed that higher rank lattices have superrigid representations, which in particular implies that all such lattices are arithmetic. Since then Gromov–Piatetski-Shapiro and Deligne–Mostow have shown that a similar superrigidity theorem cannot hold for all lattices in the isometry group of real or complex hyperbolic space, i.e., in the rank 1 setting. However in recent work joint with Bader, Fisher, and Stover we show that one can prove certain superrigidity/arithmeticity theorems provided the associated manifold satisfies the geometric condition that it contains infinitely many (maximal) totally geodesic submanifolds. Specifically, we show that this latter criteria forces a hyperbolic manifold to be arithmetic. The goal of this mini-course will be first to recount the work of Margulis on superrigidity of higher rank lattices and then to go on to discuss the proof of the aforementioned theorem. This will include a discussion of the connections between homogeneous dynamics and geodesic submanifolds, their interaction with superrigidity, and an introduction to techniques introduced by Bader and Furman for studying algebraic representations of ergodic actions. If time permits, we will also discuss the analogous theorem for complex hyperbolic manifolds and the key differences from the real hyperbolic setting.

Amir Mohammadi. Title: Effective equidistribution in unipotent dynamics
Abstract: A celebrated result of Ratner states that if G is a (linear) Lie group, Γ a lattice in G and if u_t is a one-parameter unipotent subgroup of G, then for any $x \in G/\Gamma$ the orbit $u_t.x$ is equidistributed in a periodic orbit of some subgroup $L < G$, and moreover that the orbit of x under u_t is contained in this periodic L orbit. A key motivation behind Ratner's equidistribution theorem for one-parameter unipotent flows has been to establish Raghunathan's conjecture regarding the possible orbit closures of groups generated by one-parameter unipotent groups; using the equidistribution theorem Ratner proved that if G and Γ are as above, and if $H < G$ is generated by one parameter unipotent groups then for any $x \in G/\Gamma$ one has that $\overline{H.x} = L.x$ where $H < L < G$ and $L.x$ is periodic. Important special cases of Raghunathan's conjecture were proven earlier by Margulis and by Dani and Margulis by a different, more direct, approach. These results have had many beautiful and unexpected applications in number theory, geometry, and other areas. A key challenge has been finitary aspects of the analysis. In these lectures, we will discuss (recent and less recent) progress toward this problem.

Kurt Vinhage. Title: Kakutani Equivalence of Flows and Beyond
Abstract: Kakutani equivalence plays an important role as a natural classification of flows up to time change. We will discuss the definition of Kakutani equivalence for flows, and find natural generalizations to group actions. Analogies to the isomorphism relation will be established, in particular an entropy/complexity theory (the Kakutani invariant), and with it a class of minimal complexity (the loosely Kronecker systems). We will discuss techniques and theorems around these topics, especially as they relate to homogeneous flows and actions. This course will be related D. Wei's course.

Daren Wei. Title: Time change rigidity for unipotent flows
Abstract: Two flows are said to be Kakutani equivalent if one is isomorphic to the other after time change, or equivalently if there are Poincare sections for the flows so that the respective induced maps are isomorphic to each other. Ratner showed that if $G = \mathrm{SL}(2, \mathbb{R})$ and Γ is a lattice in G, and if u_t is a one parameter unipotent subgroup in G then the u_t action on G/Γ equipped with Haar measure is loosely Bernoulli, i.e. Kakutani equivalent to a circle rotation. Thus any two such systems $(\mathrm{SL}(2, \mathbb{R})/\Gamma_i, u_t, m_i)$ are Kakutani equivalent to each other. On the other hand, Ratner showed that if $G = \mathrm{SL}(2, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R})$ and Γ is a reducible lattice, and u_t is the diagonally embedded one parameter unipotent subgroup in G, then $(G/\Gamma, u_t, m)$ is not loosely Bernoulli. We show that in fact in this case and many other situations one cannot have Kakutani equivalence between such systems unless they are actually isomorphic. This is a joint work with Elon Lindenstrauss.

2.2. **Preparatory Courses.**

Keith Burns. Title: Some examples in dynamics
Abstract: The lectures will survey some of the basic examples in dynamical systems.

Osama Khalil. Title: Introduction to homogeneous dynamics
Abstract: We will discuss basic objects in the study of unipotent flows on quotients of Lie groups by their lattices. The goal is to introduce Ratner's fundamental theorems and the focus will be on definitions, examples, and motivations. This will serve as an introduction to several of the mini-courses during the summer school.

Homin Lee. Title: Introduction to higher rank lattices
In these lectures we will introduce lattices in higher rank semisimple Lie groups. We start with the definition and examples of the main objects, higher rank Lie group and lattices. Then we will focus on 2 main theorems, Margulis' arithmeticity theorem and Margulis' superrigidity theorem. This will be a starting point for Nick Miller's lectures in the following week.

Raz Slutsky. Title: Introduction to Invariant Random Subgroups
In these two lectures, we will introduce $Sub(G)$, the space of closed subgroups of a given group, and the concept of an Invariant Random Subgroup (IRS), with a special emphasis on simple Lie groups and their discrete subgroups. We will then see how the interplay between dynamics, geometry and group theory entailed in these notions has proven to be extremely useful in the last decade. This will serve as a starting point for the talks by Mikolaj Fraczyk in the second week.

2.3. **Research talks.**

Michael Bersudsky. Title: Limiting distribution of dense lattice orbits in a space of discrete subgroups of the Euclidean space
Abstract: It is common that a lattice subgroup Γ of a Lie group G has dense orbits in a homogeneous space G/H. Then, it is natural to ask about the limiting distribution of the dense orbits with respect to a filtration given by increasing "balls" in Γ. The problem was solved in great generality for many different types of subgroups H, but an interesting case which wasn't addressed so far is in the setting that H has infinitely many non-trivial connected components. I will talk about my joint work with Hao Xing where we solve a non-trivial case of this problem. Specifically, in our case G/H describes the space of rank-m discrete subgroups of covolume one of $\mathbb{R}^{m+1}$, where every orbit of a every lattice subgroup is dense. I will describe our main results and give an overview of the main methods.

Ben Call. Title: Unique Equilibrium States for Geodesic Flows on Translation Surfaces
Abstract: Equilibrium states are invariant measures which maximize the measure-theoretic pressure of a given dynamical system equipped with a potential. In the case of negatively curved manifolds, equilibrium states for the geodesic flow and Holder continuous potentials are unique and have the Bernoulli property, meaning that in one sense, they are the "most random" they can be. In this talk I will give a brief overview of some techniques that show uniqueness and mixing properties of equilibrium states with weaker assumptions on the geometry of the setting. Then I will discuss results establishing uniqueness and Bernoullicity of many equilibrium states for the geodesic flow on flat surfaces with cone singularities, which is joint work with Dave Constantine, Alena Erchenko, Noelle Sawyer, and Grace Work.

Homin Lee. Title: Smooth actions on manifolds by higher rank lattices
Abstract: We discuss smooth actions on manifolds by higher-rank lattices. We mainly focus on lattices in $\mathrm{SL}(n, \mathbb{R})$ (n is at least 3). Recently, Brown-Fisher-Hurtado and Brown-Rodriguez Hertz-Wang showed that if the manifold has dimension at most (n-1), the action is either isometric or projective. In both cases, we don't have chaotic dynamics from the action (zero entropy). We focus on the case when one element of the action acts with positive (topological) entropy. These dynamical properties (positive entropy element) significantly constrain the action. Especially, we deduce that if there is a smooth action with a positive entropy element on a closed n-manifold by a lattice in $\mathrm{SL}(n, \mathbb{R})$ (n is at least 3) then the lattice should be commensurable with $\mathrm{SL}(n, \mathbb{Z})$. This is the work in progress with Aaron Brown.

James Marshall Reber. Title: A positive proportion Livshits theorem
Abstract: Given a transitive Anosov system on a closed connected Riemannian manifold, the Livshits theorem states that a Holder function is a coboundary if all of its periods vanish. In this talk, I will explain how a finer statistical understanding of the distribution of these periods can be used to show that a function is a coboundary if all of its periods vanish on a set of positive asymptotic upper density. I will also discuss applications of this to various rigidity results. This is joint work with Caleb Dilsavor.

Anthony Sanchez. Title: Effective equidistribution of large dimensional measures on affine invariant submanifolds
Abstract: The unstable foliation, that locally is given by changing horizontal components of period coordinates, plays an important role in study of translation surfaces, including their deformation theory and in the understanding of horocycle invariant measures.

In this article we show that measures of large dimension equidistribute in affine invariant manifolds and give an effective rate. An analogous result in the setting of homogeneous dynamics is crucially used in the effective equidistribution results of Lindenstrauss-Mohammadi and Lindenstrauss–Mohammadi–Wang.

Raz Slutsky. Title: The space of traces of certain discrete groups
Abstract: A trace on a group is a positive-definite conjugation-invariant function on it. These functions play an important role in harmonic analysis of discrete groups, and their study found many exciting connections to rigidity, stability, and dynamics in the past couple of decades. In this talk, I will explain these connections and focus on the topological structure of the space of traces. We will see the distinct behaviour of free groups vs. higher-rank lattices, a non-commutative version of the Glasner-Weiss Theorem, and more. This is based on joint works with Arie Levit, Joav Orovitz and Itamar Vigdorovich.

Andreas Wieser. Title: Equidistribution of subspaces and their shapes
Abstract: Given a k-dimensional rational subspace of n-dimensional Euclidean space, one may associate to it two shapes: the shape of the integer lattice in the subspace and in the orthogonal complement. Moreover, one may measure the arithmetic complexity of a rational subspace by its discriminant which is the square of the covolume of the integer lattice in the subspace. Following work of Maass, Roelcke, and Schmidt it is conjectured that the set of triples consisting of a subspace L and the two shapes is equidistributed in the appropriate product space when L varies with fixed discriminant D and D goes to infinity. In this talk, we first recall known results towards this conjecture and then discuss new work joint with Aka, Einsiedler, Luethi, and Michel in which an effective variant of the conjecture is established for most dimensions. The proof uses a bootstrapping technique based on effective mixing as well as a discrepancy trick and will be discussed in a model case.

Amanda Wilkens. Title: Poisson systems and fixed price in higher rank
Abstract: Let G be a group and X a standard probability space such that the action of G on X is measure preserving and essentially free. This action partitions X into orbits. Levitt defined the cost of an orbit equivalence relation for countable groups in 1995; the cost measures how much information is needed to construct the relation. The theory of cost, further developed by Gaboriau, is rich, with applications to various fields and many remaining open questions.

In particular, one question of Gaboriau asks whether all countable groups have fixed price. A group has fixed price if each of its measure preserving, essentially free actions share the same cost. The question can also be asked more generally for locally compact, second countable groups. We answer it positively in the case where G is a higher rank semisimple real Lie group. This has several immediate corollaries we will mention. We'll sketch the proof, which is geometric and relies on a peculiar Poisson-Voronoi tessellation. No prior knowledge on cost, Lie groups, or Poisson-Voronoi tessellations will be assumed. This is joint work with Mikolaj Fraczyk and Sam Mellick.