Equity-efficiency trade-off in quasi-linear environments*

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Abstract

I study a simple equity-efficiency problem: A designer allocates a fixed budget of money to agents differing in privately observed marginal values for money. She can only screen by imposing an "ordeal," that is, by allocating more money to agents who engage in a socially wasteful activity (such as queuing or filling out forms). I show that giving a lump-sum transfer is outperformed by an ordeal mechanism when agents with the lowest money-denominated cost of engaging in the wasteful activity have an expected value for money that exceeds the average value by more than a factor of two.

Keywords: equity-efficiency trade-off, costly screening, allocation of money

1 Introduction

Governments often redistribute by allocating direct cash transfers. A natural concern that arises in these contexts is whether financial aid is received by those most in need. While some basic information about potential recipients may be available to public agencies, many of the relevant characteristics—the detailed financial situation, family circumstances, labor market opportunities—remain unobserved. When these characteristics cannot be easily verified, governments can attempt to improve targeting by requiring applicants to engage in "ordeals"—such as queuing or filling out forms—that may help screen out those who are not in need. For example, Alatas et al. (2016) show that targeting can be improved by imposing the ordeal of traveling to a registration site in the design of the Indonesia's Conditional Cash Transfer program.[1] However, ordeals—by definition—can be quite burdensome, and they decrease the utility of the recipients without offering any direct social benefit. As a result, governments may choose to forgo any screening. For example, the US government

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[1]See Rose (2021) and Zeckhauser (2021) for other practical examples of using ordeals for targeting.

distributed monetary support during the Covid-19 pandemic by mailing checks, imposing only extremely crude eligibility criteria based on income.[2] These contrasting examples motivate the basic question that this paper addresses: When is it optimal to use costly screening to improve the targeting of financial aid?

To answer this question, I consider the following redistribution problem. A designer allocates a fixed amount of money to a population of agents differing in privately observed marginal values for money (that capture designer's redistributive preferences). Absent additional tools, the designer cannot achieve any screening in an incentive-compatible mechanism—she can only offer a lump-sum transfer. To introduce a trade-off, I allow the designer to pay a higher amount of money to agents who publicly "burn" some utility by completing an ordeal—an activity that is costly for the person engaging in it and does not directly benefit anyone. Any such activity entails a pure social waste since, by definition, its direct consequence is a decrease in utility. However, if only some agents find it optimal to complete the ordeal, the designer can potentially achieve a better allocation of money. This leads to an equity-efficiency trade-off.

Assuming that agents' utility is linear in monetary transfers, I show that offering cash for completing the ordeal outperforms the lump-sum transfer mechanism if agents with the lowest money-denominated cost of engaging in the ordeal have an expected value for money that exceeds the average value for money by more than a factor of *two*. The conclusion is robust to allowing the designer to pick an optimal mechanism from the set of all incentive-compatible, individually rational, and budget-balanced mechanisms. I offer a simple graphical intuition for the condition, and explain its connection to quasi-linearity of preferences.

While stylized, the model delivers a simple condition for optimality of using ordeals when allocating financial aid. The condition depends solely on the properties of the joint distribution of costs and values for money and can be tested empirically once values for money are tied to an empirically observable quantity (such as income or wealth). In contexts where governments already rely on observable information to identify those in need of financial aid, the condition—applied to the distribution of costs and values *conditional* on observables—determines whether outcomes can be improved by adding an additional layer of targeting associated with costly screening.[3] For developing countries with scarce data on potential recipients of financial aid, the condition sheds light on the desirability of targeting through ordeals relative to other methods such as community-based targeting or data acquisition that can be costly to implement.[4]

[2]See, for example, Falcettoni and Nygaard (2020) and Nygaard et al. (2020).

[3]For example, see Deshpande and Li (2019) for a discussion of the effects of costly screening on targeting in allocating disability insurance.

[4]See, for example, Banerjee et al. (2021) and Trachtman et al. (2022).

Connections to the literature. The equity-efficiency trade-off lies at the core of public economics. Classical papers, such as Diamond and Mirrlees (1971) or Atkinson and Stiglitz (1976), provided frameworks to evaluate the trade-off in relatively complex environments, where closed-form solutions are generally not available. Weitzman (1977) studied the trade-off in the simpler context of allocating a single good. Nichols and Zeckhauser (1982) were among the first to point out that ordeals might be a part of a second-best design of transfer programs when individuals' characteristics are not perfectly observable to the government. A growing literature on inequality-aware market design studies the equity-efficiency trade-off in allocation problems by modeling the redistributive preferences of the designer with dispersion in marginal values for money. Dworczak ⓡ Kominers ⓡ Akbarpour (2021) (henceforth DKA) studied a two-sided market for a homogenous good, and showed that inefficient rationing is part of an optimal market design when the expected value for money for traders with the lowest rate of substitution exceeds the average value for money by a factor of two or more (the "high-inequality condition"). A number of recent papers obtained an analogous condition (featuring the factor two) in various models of allocating resources to agents with quasi-linear utilities: Kang (2020) and Pai and Strack (2022)—when the allocation of the good generates externalities; Kang and Zheng (2020)—when a good and a bad is allocated, Akbarpour ⓡ al. (2023)—in a setting with heterogeneous qualities of the good, and Kang and Zheng (2023)—when agents endogenously select into buyer and seller roles.

The current paper complements this literature by analyzing the very natural problem of allocating money when the planner has redistributive preferences. While the mathematical model is essentially a reinterpretation of existing models, I believe this interpretation to be useful and economically insightful. First, when money is allocated, the unique (unconstrained) Pareto efficient outcome that can be achieved in the presence of private information is a lump-sum transfer. As a result, there is a sharp trade-off between screening and efficiency: Screening is motivated entirely by planner's redistributive preferences. The clean separation of efficiency and equity allows me to provide a simple intuition for why "two" is the relevant threshold in the high-inequality condition within quasi-linear continuous-type models. Second, as explained earlier, the problem of allocating financial aid is an important challenge for policymakers. The paper provides high-level policy guidance, and sketches a pathway for empirical research to evaluate the optimality of using ordeals in various contexts in which financial aid is allocated.

The idea that ordeals can be useful in screening is well understood, and has roots in the classical analyses of costly signaling (Spence, 1973), contests (Tullock, 1980), and private-value all-pay auctions (Hillman and Riley, 1989). The literature on the design of contests and all-pay auctions has mostly focused on allocative efficiency and maximizing effort as the two leading objectives.[5] To the best of my knowledge, the condition I obtain for the

[5] See, for example, Moldovanu and Sela (2001) and the references therein.

redistributive objective is novel within this literature. I comment on connections to existing results in Section 2.1 after setting up the model.

2 Baseline model

I first study the baseline version of the model and the performance of simple ordeal mechanisms. In Section 4, I demonstrate robustness of the insights to choosing a fully optimal incentive-compatible mechanism. In Section 5, I discuss several extensions of the framework.

A designer has a budget $B > 0$ of money that she allocates to a unit mass of agents. Each agent is characterized by a "marginal value for money" v. The parameter v is interpreted as a social welfare weight—it is the *social* value of giving an agent a dollar, as in Saez and Stantcheva (2016). I assume that the designer knows the distribution of values for money in the population but does not observe individual realizations.

I do not explicitly model observable information about agents that the designer may have access to. Instead, the target population in my model should be interpreted as a subset consisting of all agents with the same observable characteristics. Consequently, the dispersion in values for money captures the *residual* unobserved heterogeneity in agents' need conditional on a given set of observables. For example, the designer may be able to verify whether an agent's income is below or above a threshold; conditional on having low income, however, agents may differ in their wealth, job market opportunities, family situation, or social networks—with v capturing the reduced-form consequence of these unobserved factors for their welfare weight.

If the allocation of money cannot be made contingent on any additional information available to the designer, the only feasible mechanism is to give a lump-sum transfer to all agents. The total value generated for a utilitarian designer is $\mathbb{E}[v] \cdot B$, where $\mathbb{E}[v]$ is the average value for money in the population. Without loss of generality, I normalize that average value to be 1.[6]

Suppose, however, that the designer can ask agents to "burn" utility.[7] Specifically, there is some activity—an "ordeal"—that is costly for the agents, has no intrinsic social value, and is observed by the designer. The designer can choose a difficulty (arduousness) y of the ordeal that I normalize to $[0, 1]$. (The main result is unaffected if $y \in \{0, y_0\}$, that is, when the designer cannot adjust the difficulty of the ordeal.) Each agent has a privately observed cost c of completing the ordeal. Moreover, agents' utilities are quasi-linear: an agent with cost c and value for money v who completes an ordeal with difficulty y and receives a monetary

[6] The model admits a broader interpretation if we think of the budget B as being valued at a some parameter α capturing the "marginal value of public funds." Then, the normalization implies that the values v are measured in units of the opportunity cost of public funds.

[7] The literature uses the phrase "money-burning" to refer to engaging in socially wasteful signaling; I use "utility burning" because agents in my setting could be burning everything but money.

transfer t obtains utility

$$-cy + vt.$$

I normalized the agent's utility so that her private value for money is equal to the social value; this is without loss of generality since agent's preferences are not affected by the choice of units in which utility is measured (the above expression measures the agent's utility in social-utility units). The agent's choices over y and t depend only on the rate of substitution, denoted $k \equiv c/v$. The parameter k is the relative cost of the ordeal for the agent, expressed in monetary units.[8]

Assuming the designer knows the joint distribution of c and v, and is utilitarian, her objective is to maximize

$$\mathbb{E}[-cy(k) + vt(k)]$$

over $y(k)$ and $t(k)$ subject to the budget constraint (the sum of transfers is B) and the incentive-compatibility constraint, stating that an agent with type k chooses $(y(k),\, t(k))$ from the menu $\{(y(k'),\, t(k')\}_{k'}$ offered by the designer. The argument k in $y(k)$ and $t(k)$ indicates that agents differing in the relative cost k may choose different combinations of y and t. I formalize the mechanism-design problem in Section 4.[9]

Any positive level of y is a pure social waste. In particular, if the designer has no redistributive preferences (all v's are equal to 1), then the optimal mechanism is a lump-sum transfer. With redistributive preferences, however, there may be a trade-off between efficiency and equity if choosing positive levels of y for some agents allows the designer to allocate money to those with higher values v.

Throughout, I assume that k has a distribution F with a continuously differentiable density f on $[\underline{k},\, \bar{k}]$, $f(\underline{k}) > 0$, $f'(\underline{k}) < \infty$, with $\underline{k} = 0$. The economically restrictive assumption is that the lower bound of the support of the distribution of costs $\underline{k}$ is 0; I discuss the consequences of relaxing this assumption in Section 5. The role of the remaining assumptions is to ensure that local analysis leads to economically meaningful conclusions.

2.1 What is special about allocating money?

Before analyzing the model, I comment on how it relates to existing models analyzed in the costly-screening and redistributive market design literatures. (A reader not interested in understanding this connection may safely skip this subsection.) In the basic linear-utility

[8] Equivalently, I could have normalized the value for money to 1 for each agent, so that an agent's utility is measured in monetary units: $-ky + t$. Then, v is interpreted as a classical welfare weight, in that the agent's contribution to social welfare is $v(-ky + t)$, which is of course the same as $-cy + vt$.

[9] It is well known that in two-dimensional linear models like the current one the designer cannot do better by trying to screen c and v separately—see Jehiel and Moldovanu (2001), Che et al. (2013), or DKA.

versions of these models, agents' preferences are given by a utility function of the form

$$rx - p,$$

where x is an allocation (e.g., allocation probability), p is a payment (which could be monetary or non-monetary), and r is the agent's private type (the rate of substitution between the allocation and payment). From the point of view of agents' preferences, it does not matter whether the private-type parameter r is placed on x or p. For example, the agent's utility in my model can be written this way as well, with $x \equiv t$, $y \equiv p$, and $r \equiv 1/k$. However, combined with the designer's objective, the parametrization of agents' utilities becomes economically meaningful.

The most common interpretation is that x measures the allocation of a resource, and r is the agent's *value*. If the designer attempts to maximize allocative efficiency, the optimal allocation is *assortatative matching*: Agents with higher values r receive a higher allocation x. If p is interpreted as a monetary payment and the designer does not have redistributive preferences, the first best can be implemented with an appropriate transfer schedule.

The costly-screening literature analyzed (among other less related objectives) the case in which the designer is interested in maximizing allocative efficiency but the variable p is interpreted as "money-burning"—any payment constitutes a social loss. In this case, as shown in various contexts by McAfee and McMillan (1992), Hartline and Roughgarden (2008), Hoppe, Moldovanu, and Sela (2009), Condorelli (2012), and Chakravarty and Kaplan (2013), assortative matching remains optimal if and only if the inverse hazard rate of r is non-decreasing (if it decreasing, the second-best solution is to allocate the good randomly).

The inequality-aware market-design literature takes a different angle by assuming that the designer has redistributive preferences: p is interpreted as money but the designer maximizes the sum of $\lambda(xr - p)$ across all agents, where λ is interpreted as a social welfare weight. When welfare weights are sufficiently negatively correlated with r, assortative matching is suboptimal; instead, inefficient rationing is part of the second-best solution (Condorelli (2013), DKA). Optimality of rationing in such models depends on the properties of the *joint* distribution of r and λ.[10] The equity-efficiency trade-off is relatively complicated: Efficiency requires screening but equity considerations push towards allocating to lower types; additionally, rationing reduces revenue, which decreases the amount of money available for redistribution via lump-sum transfers.

My model considers the case when the payment p is socially wasteful but the designer has a redistributive objective. Values for money are interpreted as social welfare weights as in Saez and Stantcheva (2016). Without redistributive preferences, the unique optimal (and Pareto efficient) mechanism is a lump-sum transfer. This makes the equity-efficiency trade-

[10]When the welfare weight is modeled as a deterministic function of r, optimality of rationing depends on the distribution of r and the functional form of the dependence of λ on r.

off particularly simple: Screening the agent's private information is needed *only* when the designer has redistributive preferences. The presence of social welfare weights implies that—unlike in the classical costly-screening literature—the shape of the distribution of agent's rate of substitution alone is not sufficient to characterize the optimal mechanism.

3 When should ordeals be used?

Consider the simplest possible screening mechanism: The designer offers an additional payment t_0 (on top of the lump-sum transfer) to agents who are willing to engage in some ordeal $y_0 > 0$. Agents with relative cost $k \leq t_0/y_0$ choose this option; the remaining agents decline (and only receive the lump-sum transfer).

Given the joint distribution of (v, c), I introduce the conditional expectation function $\mathbb{E}\left[v|\, \frac{c}{v} = k\right]$. To avoid issues associated with conditional expectations being defined only for almost all k, I assume that this function is continuous. From now on, I use the short-hand notation $\mathbb{E}\left[v|\, k\right]$ interpreted as the expected value for money conditional on the relative cost k. Intuitively, this object will play a key role in the analysis because the mechanism screens agents based on k, while the designer ultimately wants to allocate money based on v. Thus, the function $\mathbb{E}\left[v|\, k\right]$ determines the targeting effectiveness of the mechanism.

By a simple calculation, welfare associated with the simple mechanism is given by

$$\int_0^{t_0/y_0} \mathbb{E}\left[v|\, k\right](t_0 - ky_0) f(k) dk + (B - t_0 F(t_0/y_0)).$$

The first term of the welfare function captures the fact that an agent with type $k \leq t_0/y_0$ receives a payment t_0 but incurs a cost ky_0, thus enjoying net utility equivalent to receiving a monetary payment $t_0 - ky_0$. The designer values that utility at $\mathbb{E}\left[v|\, k\right](t_0 - ky_0)$, since she values a dollar given to an agent with type k at $\mathbb{E}\left[v|\, k\right]$. The second term of the welfare function captures the fact that a total amount $t_0 F(t_0/y_0)$ of money has been paid out in additional compensation, leaving less funds in the budget for lump-sum transfers.

I further choose t_0 so that only a small fraction of agents choose to engage in the ordeal. Specifically, let $t_0 = \epsilon y_0$ for some small $\epsilon > 0$, so that only types $k \leq \epsilon$ accept. Then, the ordeal mechanism, which I will denote by $M(\epsilon)$, outperforms the lump-sum transfer mechanism if and only if

$$\int_0^{\epsilon} \mathbb{E}[v|k](\epsilon - k) f(k) dk > \epsilon F(\epsilon). \tag{1}$$

The condition says that the welfare gain of additional utility enjoyed by types in $[0, \epsilon]$ must exceed the opportunity cost of the required expenditure $\epsilon F(\epsilon)$. (The opportunity cost is equal to the expenditure because I normalized the average value for money to 1.) Observe that the condition will hold for small enough ϵ if the ratio of the left hand side to the right hand side converges to a number strictly larger than 1 in the limit as ϵ goes to zero. We have

$$\lim_{\epsilon\to 0}\frac{\int_0^\epsilon \mathbb{E}[v|k](\epsilon-k)f(k)dk}{\epsilon F(\epsilon)}=\lim_{\epsilon\to 0}\frac{\int_0^\epsilon \mathbb{E}[v|k]f(k)dk}{\epsilon f(\epsilon)+F(\epsilon)}=\frac{\mathbb{E}[v|\underline{k}]}{2}, \tag{2}$$

where I used L'Hôpital's rule twice, and relied on the regularity assumptions on the distribution. We thus obtain the following result.

Proposition 1. *If the expected value for money conditional on the lowest money-denominated cost exceeds the average value for money by more than a factor of two, i.e., if*

$$\mathbb{E}[v|\underline{k}] > 2, \tag{$\star$}$$

then the ordeal mechanism $M(\epsilon)$ strictly outperforms the lump-sum transfer for some $\epsilon > 0$.

For intuition, notice that requiring agents to "burn" utility in exchange for a larger monetary transfer achieves redistribution of money to agents with the lowest relative cost of engaging in the ordeal. Achieving this redistribution is costly: As equation (2) reveals, for each dollar of public funds spent, only 1/2 of the dollar is received by agents in form of a net utility increase; the other 1/2 gets "burned" in the process of screening. Thus, the social value of targeting the monetary transfer must exceed the value of public funds by more than a factor of two for the ordeal mechanism to be socially valuable on the net.[11]

The interpretation of condition ($\star$) is easiest when $\mathbb{E}[v|k]$ is decreasing in k. This is a natural case since $k \equiv c/v$, and thus v and k are typically inversely related. Then, the designer derives the highest expected value from giving money to the agent with the lowest cost $\underline{k}$. The value $\mathbb{E}[v|\underline{k}]$ depends both on the strength of the designer's redistributive preferences (the dispersion in v's), as well as on the targeting effectiveness of the ordeal. Condition ($\star$) states that the targeting effectiveness of the ordeal must be sufficiently high so that the designer is willing to trade off efficiency for equity.

Implicitly, condition ($\star$) depends on observable information available to the designer. For example, if the designer can directly observe agents' socioeconomic characteristics relevant for assessing their value for money v, the residual correlation of v and k is zero, and hence $\mathbb{E}[v|k] = 1$, for any k. Thus, Proposition 1 implies that using ordeals can be optimal only if information available to the designer is relatively imprecise.

The intuition for why "2" appears in condition ($\star$) is related to the intuition offered in DKA but cleaner and simpler, primarily for reasons explained in Section 2.1. I argue that the "2" is in fact a direct consequence of quasi-linearity of preferences in money. To see that,

[11] In the famous leaky-bucket metaphor of Okun (1975), ordeals in the quasi-linear model are associated with a bucket that leaks 50% of the water being transferred. This first part of the intuition is mathematically related to a result from Hoppe et al. (2009) who showed that in the continuous version of their matching model, *half* of the output from assortative matching is wasted through costly signaling.

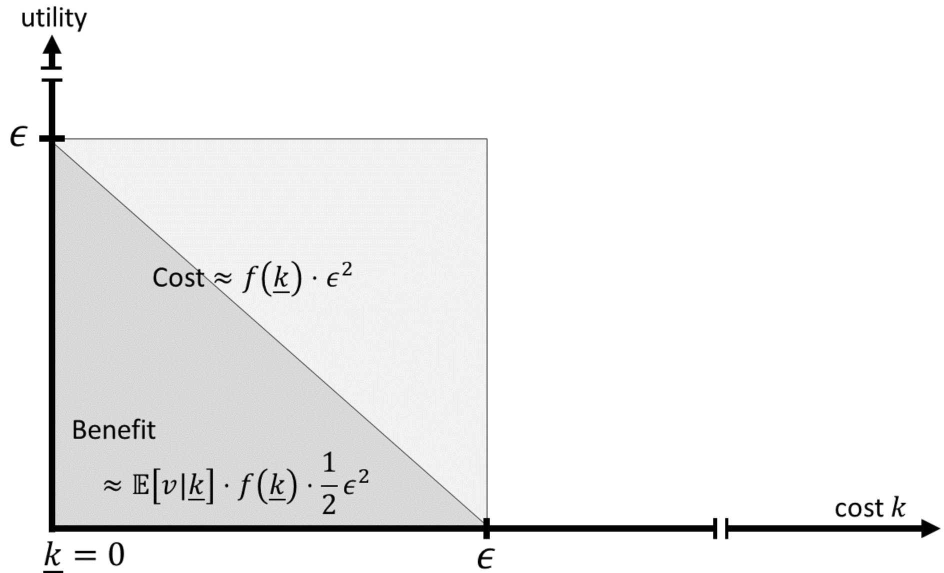

Figure 3.1: The surplus triangle and the equity-effiency trade-off

instead of applying L'Hôpital's rule as in (2), apply the mean value theorem for integrals to the left hand side of (1) to get that, for some $\delta_\epsilon \in [0, \epsilon]$,

$$\int_0^\epsilon \mathbb{E}[v|k](\epsilon - k)f(k)dk = \mathbb{E}[v|\delta_\epsilon]f(\delta_\epsilon)\int_0^\epsilon (\epsilon - k)dk = \mathbb{E}[v|\,\underline{k}]f(\underline{k})\int_0^\epsilon (\epsilon - k)dk + o(\epsilon^2),$$

where $o(\epsilon^2)$ denotes a term that converges to zero faster than ϵ^2 as $\epsilon \to 0$. Intuitively, when ϵ is small, we can ignore the differences in welfare and probability weights applied to the utilities of different agents with costs in $[0, \epsilon]$, and instead apply the same weight $\mathbb{E}[v|\,\underline{k}]f(\underline{k})$ to all of them. Due to quasi-linearity of preferences, the surplus of agents who accept the ordeal can be represented as an isosceles triangle (see Figure 3.1). If the weight is constant, the total surplus is calculated as the area of this triangle. The area is exactly **half** of the area of a square with side ϵ that approximates the associated opportunity cost of public funds (see the light-grey square in Figure 3.1, with $\epsilon F(\epsilon) \approx f(\underline{k})\epsilon^2$ that holds approximately for small ϵ). Thus, to compensate for the surplus lost due to costly screening, the designer must value the area of the triangle more than twice as much as the area of the square, in social utility units. This gives us condition ($\star$).

3.1 Parametric example

I present a parametric example to illustrate condition ($\star$). The parametrization is stylized but shows one possible way towards an empirical test of the condition.[12] The main obstacle is that marginal values for money are not observable. I will assume that they are equal to the

[12] See Allen and Rehbeck (2021) for a discussion of the empirical testability of the high-inequality condition from DKA.

slope of a (common) concave utility function for wealth at agents' respective wealth levels.[13] The underlying assumption is that the designer knows the wealth distribution in the target population but does not perfectly observe any individual's wealth level.

Suppose that each agent has a utility function for wealth w,

$$U(w) = \frac{w^{1-\theta}}{1-\theta},$$

that takes the CRRA form, with coefficient of relative risk aversion $\theta > 0$. Agents differ in their wealth levels w, and thus differ in the marginal values $U'(w)$. Note that condition ($\star$) depends on the shape of the right tail of the distribution of values for money, and hence will depend on the left tail of the distribution of wealth. For this reason, I choose a family of distributions of wealth indexed by how thick the left tail is:

$$G(w) = w^{\beta},$$

for $\beta > 0$ and $w \in [0, 1]$.[14] Under this parametrization, the bottom 1/2 of agents with wealth below w hold $(1/2)^{\frac{1+\beta}{\beta}}$ of the total wealth held by agents with wealth below w, for any w. Thus, lower β corresponds to a thicker left tail. Because

$$\mathbb{P}(U'(w) \leq s) = 1 - \left(\frac{1}{s}\right)^{\frac{\beta}{\theta}},$$

marginal utilities have a Pareto distribution with tail parameter $\alpha \equiv \beta/\theta$. Assuming that $\alpha > 1$ (so that the Pareto distribution has a finite mean) and using the fact that the mean of the Pareto distribution is $\alpha/(\alpha-1)$, we can define

$$v \equiv \frac{\alpha - 1}{\alpha} \cdot U'(w),$$

so that the average marginal value for money is normalized to 1. Finally, suppose that the costs c are uniformly distributed on $[0, 1]$. If c and v are assumed independent, then by direct calculation,[15]

$$\mathbb{E}\left[v|k\right] = \frac{(\alpha-1)^2}{\alpha(\alpha-2)} \frac{1-k^{\alpha-2}}{1-k^{\alpha-1}}.$$

[13] This approach implicitly assumes that the program that the designer is running is small enough that it does not significantly alter the wealth distribution—see Section 5 for an extended discussion of when such an approach is appropriate.

[14] It will be clear that this assumption is only relevant for small enough w; the distribution can be different for high w without affecting the results.

[15] For $\alpha = 2$, the term $(1-k^{\alpha-2})/(\alpha-2)$ is replaced by $-\log(k)$.

In particular,

$$\mathbb{E}[v|\underline{k}] = \begin{cases} \frac{(\alpha-1)^2}{\alpha(\alpha-2)} & \alpha > 2, \\ \infty & \alpha \leq 2. \end{cases}$$

Therefore, condition ($\star$) holds if and only if

$$\frac{\beta}{\theta} < 1 + \sqrt{2}.$$

Intuitively, the designer has stronger redistributive preferences when there are more poor agents (β is lower) or when agents are more risk averse (θ is higher), so that they have a particularly high marginal value for money at low wealth levels. By Proposition 1, when the tail of the Pareto distribution of values for money is thick enough, it becomes optimal to sacrifice efficiency to achieve better redistribution.

While the empirical estimates of θ vary widely depending on the method and context, most studies obtain that θ is weakly greater than 1. A simple empirical property of the left tail of the distribution of wealth is then sufficient for condition ($\star$): The bottom 50% of agents with wealth below some low threshold w should hold no more than $(1/2)^{\frac{2+\sqrt{2}}{1+\sqrt{2}}} \approx 37.5\%$ of wealth in that group. If the designer has access to observable information about the agents (e.g., being above or below an income threshold, or family status), then this property should be tested at the *conditional* distribution of wealth (conditional on a given set of observables).

4 Optimal mechanism

In this section, I connect the result about the simple ordeal mechanism to the question of optimal design. The main message is that the conclusions derived from the analysis of the simple mechanism carry over to the optimal mechanism.

It is well known (see footnote 9) that the optimal mechanism only screens agents based on their relative costs k, and hence the optimization problem for the designer can be written as finding the best direct mechanism of the form:

$$\max_{y(k)\in[0,1],\, t(k)\geq 0} \int_{\underline{k}}^{\bar{k}} \mathbb{E}[v|\, k](-ky(k) + t(k))dF(k), \tag{OBJ}$$

$$-ky(k) + t(k) \geq -ky(k') + t(k'),\ \forall k,\ k', \tag{IC}$$

$$-ky(k) + t(k) \geq 0,\ \forall k, \tag{IR}$$

$$\int_{\underline{k}}^{\bar{k}} t(k)dF(k) = B. \tag{B}$$

By adapting standard arguments (see Appendix A), I can derive the following result.

Proposition 2a. *The optimal mechanism uses an ordeal (y is strictly positive for a positive-measure set of agents) if and only if*

$$\mathbb{E}[V(k)|\, k \le k'] > 0 \text{ for some } k' > 0, \tag{3}$$

where

$$V(k) = \left(\mathbb{E}\left[v|\frac{c}{v} \le k\right] - 1\right)\frac{F(k)}{f(k)} - k. \tag{4}$$

Condition (⋆) implies that $V(k) > 0$ for small enough k; hence, condition (⋆) implies condition (3). Conversely, if $V(k)$ crosses 0 at most once from above at an interior k,[16] *then condition (3) implies that condition (⋆) must hold as a weak inequality.*

The function $V(k)$ expresses the trade-off between efficiency and redistribution. The first term in brackets shows how money gets transferred from an average agent to an agent with cost below k, which is typically a desirable effect for a designer with redistributive preferences. The inverse hazard rate $F(k)/f(k)$ appears because it measures information rents in one-dimensional screening problem in which lower types receive higher utility. The second term captures the inefficiency—it is equal to the cost k due to the normalization of the average value for money to 1. Condition (3) states that we can find a threshold type k' such that the positive redistributive effect exceeds the negative inefficiency effect *on average* for types below k'. The averaging is a consequence of incentive-compatibility, since if type k' finds it optimal to engage in the ordeal, so do all types below k'.

Proposition 2a states that condition (⋆) is not only sufficient for the optimality of some redistribution but also (almost) necessary when the function $V(k)$ crosses 0 at most once from above at an interior k. This regularity condition requires that the positive redistributive effect ($\mathbb{E}\left[v|\frac{c}{v} \le k\right] > 1$) dominates the negative inefficiency effect exactly when k is below some threshold (possibly degenerate). When F is the uniform distribution, $V(k)$ satisfies the regularity condition as long as $\mathbb{E}\left[v|\frac{c}{v} \le k\right]$ is non-increasing, which is a natural case. Assuming that $\mathbb{E}\left[v|\frac{c}{v} \le k\right]$ is non-increasing, the regularity condition rules out the possibility that the positive redistributive effect is weak for low k and strong for high k due to fluctuations in the density $f(k)$.

To understand the connection between Proposition 2a and condition (⋆), observe that (⋆) is equivalent to $V'(\underline{k}) > 0$. When $\underline{k} = 0$, $V(0) = 0$, and condition (⋆) states precisely that the positive redistributive effect dominates the negative inefficiency effect for small enough costs k, which means that condition (3) must hold. Conversely, if $V(k)$ is positive for k small enough, then (⋆) must hold at least as a weak inequality.

The linear-utility model enables an explicit characterization of the optimal mechanism.

[16] Formally, $\{k \in [\underline{k}, \bar{k}] : V(k) \ge 0\}$ is a (potentially degenerate) interval.

Proposition 2b. *When B is large enough, the optimal mechanism offers a single payment p for completing the ordeal $y = 1$, and allocates the remaining budget as a lump-sum transfer. If the optimal mechanism does not include a lump-sum transfer, it offers a payment p for completing the ordeal $y = 1$, and may offer a smaller payment p' for completing an "easier" ordeal $y' < 1$.*

Thus, at least when the budget B is sufficiently large (intuitively, when the budget constraint is non-binding, as was the case in the analysis in Section 3), the simple mechanism $M(\epsilon)$ considered in Section 3 is in fact optimal for an optimally chosen $\epsilon > 0$. Intuitively, the optimal ϵ is a threshold type $k^\star$ such that the positive redistributive effect dominates the negative inefficiency effect precisely for types $k \leq k^\star$. When the budget B is small, so that only agents who complete an ordeal receive a strictly positive amount of money, full optimality may require giving agents a choice between a more difficult ordeal for a larger payment and a simpler ordeal for a smaller payment.[17]

5 Discussion

The role of quasi-linearity of preferences. The quasi-linearity of agents' preferences in monetary transfers is key for condition ($\star$), and in particular for the appearance of "two" as the relevant threshold.[18] It is the linearity of preferences that produces the triangular shape of the surplus function in Figure 3.1. With non-linear preferences in money, the surplus function would have a different shape, and the area representing agents' utility would not be equal to *half* of the area representing costs (the square).

The key observation, however, is that because Proposition 1 relies on the analysis of an arbitrarily small perturbation of the efficient lump-sum transfer mechanism, condition ($\star$) remains *sufficient* for optimality of ordeals even when agents have non-linear utilities. In that case, values for money are interpreted as the slopes of a first-order approximation of the utility function around agents' wealth levels (evaluated at the efficient mechanism). The part of the analysis that no longer holds with non-linear utility is Proposition 2a and 2b. In particular, condition ($\star$) is no longer necessary for optimality of using an ordeal, and the optimal mechanism may become more complex.

This discussion has implications for the practical applicability of my results. The conclusions from the linear model about optimality and *structure* of the ordeal mechanism are credible in cases when income effects are limited; for example, in problems when govern-

[17] The proof reveals that the inclusion of the second option is in general needed to guarantee that the budget constraint can be satisfied with equality when a lump-sum transfer is not given; mathematically, this is a manifestation of a more general property of linear programming with constraints explained elegantly by Doval and Skreta (2018).

[18] The utility function is also assumed to be linear in y, the difficulty of the ordeal, but this is less central; as stated earlier, Proposition 1 remains true even when the difficulty of the ordeal is fixed.

ments allocate one-time aid (e.g., stimulus checks during a crisis). For large programs that determine recipients' long-term income, the model might help determine *whether* ordeals are useful (e.g., whether targeted support is preferred to universal basic income) but not the exact form of the optimal mechanism.

That being said, the simple form of the optimal mechanism from Proposition 2b is consistent with real-life uses of ordeals in welfare programs such as the ones described by Alatas et al. (2016) and Deshpande and Li (2019). While monetary payments allocated in these programs are substantial, this does not necessarily invalidate the quasi-linear model. Quasi-linearity in the current context means primarily that the redistributive preferences of the designer are the same before and after allocating aid. Since beneficiaries of welfare programs tend to remain poor even conditional on receiving aid, quasi-linearity may still be seen as a useful approximation.

The assumption $\underline{k} = 0$. Throughout, I have assumed that there are some agents for whom the marginal cost of the ordeal is arbitrarily small. Suppose instead that the distribution of k is bounded away from zero by $\underline{k} > 0$. Proposition 1 is then false. When $\underline{k} = 0$, equation (2) reveals that the costs and benefits of an ϵ distortion away from a lump-sum transfer are both of order ϵ^2. However, the inefficiency is of order ϵ when $\underline{k} > 0$, and thus it is never optimal to deviate from a lump-sum transfer by offering a small additional payment for a "small" ordeal. The first part of Proposition 2a still holds: A "large" ordeal is optimal if $\mathbb{E}[V(k)|\, k \leq k'] > 0$ for some k', where the phrase "large" is justified since the expectation will be negative for k' close to 0. Indeed, $V(k)$ starts out strictly negative when $\underline{k} > 0$. A positive derivative of $V(k)$ at $\underline{k}$ (which is equivalent to condition ($\star$)) is thus no longer sufficient; however, it is still necessary under mild regularity conditions on V.

For the problem studied by this paper, the condition $\underline{k} = 0$ can be tested empirically. Beyond the context of the current model, the economic meaning of the condition $\underline{k} = 0$ is that there exist perturbations of the efficient mechanism that result in a small (arbitrarily close to zero) per-agent loss in efficiency. This property holds naturally in many other equity-efficiency problems. For example, in goods allocation problems, a small amount of rationing induces a small per-agent loss in efficiency *regardless* of the support of agents' values for the good. This is because rationing results in allocating goods to agents who do not have the highest willingness to pay; the efficiency loss can be kept arbitrarily small if rationing applies to a small interval of agent values, regardless of their absolute magnitude. This explains why papers focusing on redistribution in goods allocation problems, such as DKA, obtained a high-inequality condition analogous to ($\star$) without imposing any restrictions on the support of the distribution of values. In the problem of allocating a good and a bad, Kang and Zheng (2020) assume that there are agents with arbitrarily small disutility from consuming the bad, implying that the first units of the bad can be allocated at an arbitrarily small social loss.

Means testing. In my model, all agents who decide to complete the ordeal receive the associated monetary transfer. In practice, incurring the cost to apply may be followed by a means-testing stage in which the public agency verifies eligibility of the applicant. Alatas et al. (2016) demonstrate that the ordeal of traveling to a registration site appears to be useful in their context primarily because it screens out ineligible agents who have a small (but non-negligible) probability of passing the means test.

This effect can be easily added to the current framework. Suppose that each agent has a third dimension of her type, θ, interpreted as the probability of passing the means test (conditional on applying). All my results go through by defining the "net" value for money $\tilde{v} \equiv \theta v$. Condition ($\star$) becomes

$$\mathbb{E}[\theta v | \underline{k}] > 2\mathbb{E}[\theta v],$$

where $k \equiv c/(\theta v)$. In particular, ordeals become optimal when the correlation between θ and v is sufficiently strong in the left tail of the distribution of costs relative to the average correlation. Strong positive correlation in the left tail of costs is plausible when *(i)* applicants with very low relative cost k are unlikely to have a low θ, and *(ii)* the eligibility criteria used by the agency lead to a high likelihood θ of receiving aid for agents with high values for money v.

Binary eligibility criteria. In their analysis of disability insurance, Deshpande and Li (2019) rely on a binary welfare-weight structure: The designer benefits from allocating a dollar to an agent if and only if that agent is eligible for receiving aid (according to some true, but partially hidden, characteristics). Suppose that in a given group of applicants with the same observables, only a fraction β are eligible. Then, a simple rewriting of condition ($\star$) yields that costly screening should be used for that group if the fraction of eligible agents in the left tail of the distribution of relative costs k is at least 2β. That is, eligible agents must be over-represented by a factor of two in the subgroup of agents with the lowest cost of completing the ordeal for the ordeal to be socially optimal.

"Useful" ordeals. For the purpose of the model, an ordeal was defined as an activity that is a purely wasteful. However, many costly-screening procedures generate concrete benefits. Verifying eligibility for social programs can be burdensome for potential recipients, but it also provides relevant information to the public agency (as in the model of Kleven and Kopczuk, 2011). The requirement to document work search efforts while receiving unemployment benefits helps screen out those who do need support, but it also alleviates the moral-hazard problem. Finally, employment guarantee programs rely on a targeting principle analogous to how ordeals work, but the labor provided by program beneficiaries is used for productive tasks. For example, in one of the largest employment guarantee programs in the world created by India's National Rural Employment Guarantee Act, workers engage in agricultural projects that are designed to benefit local communities (Dréze, 2019).

Since ordeals can be "bundled" with socially useful activities in many different ways, it is difficult to imagine a parsimonious model that would cover a large variety of such possibilities. One benefit of modeling "pure" ordeals is that conditions justifying their use, such as condition ($\star$), remain *sufficient* when the ordeal is associated with some additional social benefit.

Policy implications. While the model is purposefully simplistic and not intended to produce detailed policy implications, its analysis delivers some insights about when and which ordeals should be used.[19] As demonstrated in Section 3.1, under additional parametric assumptions, condition ($\star$) for optimality of ordeals can be tested. A non-parametric test could also be developed under some assumption about how the marginal values of money are determined. Thus, the paper provides a crude but concrete quantitative test of whether targeting monetary transfers through an ordeal is desirable.

Condition ($\star$) also provides some high-level intuition for *which* ordeals might be useful. In order for the condition to hold, y must be less costly to generate for agents with high values for money (i.e., poorer agents). For example, when the ordeal is queuing, agents should not be allowed to pay others to stand in line on their behalf. Similarly, agent's ability to produce y should not depend on characteristics that correlate positively with wealth. If the ordeal is to fill out a complicated form or apply online, more educated agents may be able to complete the task more quickly, and hence find it less costly even if they have a higher opportunity cost of time. Somewhat paradoxically then, certain types of red tape that waste petitioners' time and energy without a clear purpose may actually come closer to being optimal.

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[19] For the question of *which* ordeal to use, see also Yang et al. (2023). For whether ordeals should be used alongside pricing in the context of maximizing revenue, see Yang (2023).

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A Proof of Proposition 2a and 2b

The proof relies on the standard ironing technique.[20] Let u denote the utility of the highest type $\bar{k}$ in the mechanism. The envelope formula yields that in an incentive-compatible individually-rational mechanism, $u \geq 0$ and

$$-ky(k) + t(k) = u + \int_k^{\bar{k}} y(s)ds.$$

The above condition, combined with the requirement that $y(k)$ is non-increasing, is necessary and sufficient for (IC) and (IR). Using integration by parts, I can rewrite the budget constraint (B) as

$$\int_{\underline{k}}^{\bar{k}} \left(k + \frac{F(k)}{f(k)}\right) y(k)dF(k) + u = B. \tag{5}$$

[20]See Myerson (1981), and in the context of optimal redistribution Condorelli (2013) or Akbarpour ⓡ al. (2023), among many others.

Let α be the Lagrange multiplier on the budget constraint (B).[21] Using integration by parts again, I can rewrite the optimal design problem as

$$\max_{y(k)\in[0,1],\, u\geq 0} \int_{\underline{k}}^{\bar{k}} \left[\left(\mathbb{E}\left[v|\frac{c}{v}\leq k\right] - \alpha\right)\frac{F(k)}{f(k)} - \alpha k\right] y(k)dF(k) + (1-\alpha)u \tag{OBJ'}$$

$$y(k) \text{ is non-increasing,} \tag{M}$$

and α must be such that a solution $(y^\star, u^\star)$ to the above problem satisfies the budget constraint (5). Existence of solution requires that $\alpha \geq 1$. I conjecture that $\alpha = 1$, and later discuss how to modify the analysis when the budget constraint (5) does not hold with that conjecture. Under the conjecture, the objective (OBJ') is equal to $\int_{\underline{k}}^{\bar{k}} V(k)y(k)dF(k)$, where V is defined by equation (4).

I define the ironed value function. Let

$$\Psi(t) = -\int_t^1 V(F^{-1}(x))dx,$$

and let coΨ denote the concave closure of Ψ. Then, I can define

$$\overline{V}(k) = (\mathrm{co}\Psi)'(F(k)),$$

as the ironed value function, and the value of the problem of maximizing $\int_{\underline{k}}^{\bar{k}} \overline{V}(k)y(k)dF(k)$ is the same as the original one. Thus, the Lagrangian is maximized at

$$y^\star(k) = \mathbf{1}_{\{\overline{V}(k)\geq 0\}}.$$

This solution is feasible (non-increasing) since the ironed value function is non-increasing. Let $k^\star$ be the largest k such that $\overline{V}(k) = 0$. Then, $y^\star(k) = \mathbf{1}_{\{k\leq k^\star\}}$, and the optimal mechanism is to offer a payment $k^\star$ for the ordeal $y = 1$.

Assuming that the budget constraint is satisfied, a lump-sum transfer mechanism is optimal if and only if $k^\star = 0$, that is, if and only if $\overline{V}(k) \leq 0$ for all k (and otherwise, a simple ordeal mechanism is optimal). This condition is equivalent to co$\Psi(t)$ being a decreasing function, which in turn (given that it is a concave closure of Ψ) is equivalent to $\Psi(0) \geq \Psi(t)$ for all t. Thus, a lump-sum transfer mechanism is optimal if and only if

$$\int_{\underline{k}}^{k} V(k)dF(k) \leq 0, \tag{6}$$

[21] Existence of a Lagrange multiplier follows from a standard constraint qualification.

for all k. Dividing both sides by $F(k)$ allows me to rewrite condition (6) as

$$\mathbb{E}[V(k)|\, k \le k'] \le 0,\, \forall k'.$$

Under the assumption $\underline{k} = 0$, we have $V(0) = 0$. Moreover,

$$V'(0) = \mathbb{E}[v|\, \underline{k}] - 2,$$

so condition ($\star$) implies that $V(k)$ is strictly positive for small k. Thus, if ($\star$) holds, then (6) cannot hold, and using an ordeal mechanism is optimal. When $V(k)$ crosses 0 at most once from above at an interior k, then $\mathbb{E}[V(k)|\, k \le k']$ can be strictly positive for some k' only if $V(k)$ is positive for all k small enough. But then it must be that $V'(0) \ge 0$, which requires that condition ($\star$) holds as a weak inequality.

Under the conjectured value of the Lagrange multiplier $\alpha = 1$, the above solution is valid if the budget constraint can be satisfied by choosing some $u^\star > 0$. The budget constraint holds whenever there exists $u^\star \ge 0$ such that $k^\star F(k^\star) + u^\star = B$, that is, whenever $B \ge k^\star F(k^\star)$. This proves the first part of Proposition 2b.

The above derivation establishes Proposition 2a in the case when B is large enough, so it remains to show that it also holds in the case $B < k^\star F(k^\star)$. Since $k^\star > 0$ in this case, to prove Proposition 2a, I must show that using an ordeal is optimal. Since the budget constraint does not hold with $\alpha = 1$, we must have $\alpha > 1$, and hence it is uniquely optimal to set $u^\star = 0$. But then, since $B > 0$, budget balance requires $y(k)$ to be strictly positive for a positive measure of k, so indeed an ordeal is used in any optimal mechanism.

Finally, I prove the second part of Proposition 2b. When $B < k^\star F(k^\star)$, we must have $\alpha > 1$ and no lump-sum transfer in the optimal mechanism ($u^\star = 0$). Following DKA, problem (OBJ') can be expressed as maximization over a distribution $dy(k)$, and the solution must satisfy a single linear constraint (5). By Doval and Skreta (2018), there exists a distribution with this property that has support of size at most two. This means that the optimal mechanism offers at most two different levels of the ordeal.