Decomposition and Interpretation of Treatment Effects in Settings with Delayed Outcomes

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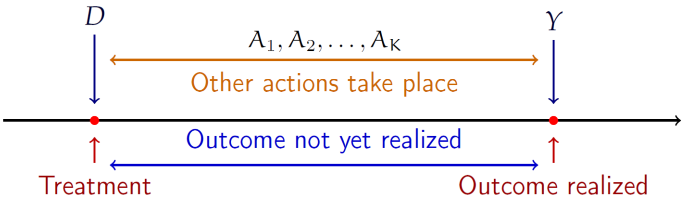
Introduction

- We consider an analyst who is interested in the average causal effect of a binary treatment D on an outcome Y.
- Y's realization is delayed, i.e., there is a time gap between D and Y.
- This allows other actions, $A_1, A_2, \ldots, A_K$, to occur. These actions may be influenced by D and may affect Y.

– *Questions*: What is the interpretation of popular regression-based estimands? Do they estimate average causal effects?



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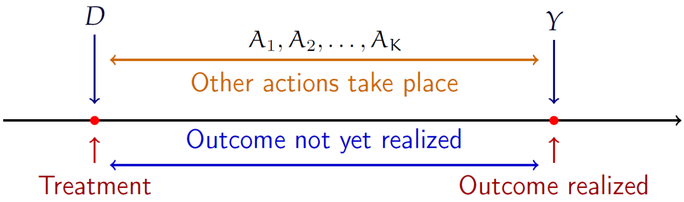

Examples

1) **Beaman et al. (13, AER) - Field experiment in Mali**
   - D: free fertilizer to female rice farmers on May 2020,
   - Y: output or profits on December 2020,
   - A: herbicides, hired labor, etc.

2) **Covid clinical trials by Moderna**
   - D: Covid vaccine in month 1 (vs. month 5),
   - Y: Covid infection in months 1-4,
   - A: weak masks in public, avoid large gatherings, etc.

3) **Akhtari et al. (21) - Selection on Observables in AirBnB**
   - AirBnB customers consider several decision on their platform.
   - D: booking a property,
   - Y: profits in the long run,
   - A: cancellations, leaving a review, etc.

Contributions

- We study 5 regression-based methods to estimate avg. causal effect of D on Y.
  - We do not assume linearity of the potential outcomes.
- Estimands are decomposed into *direct* & *indirect* effects (also *selection* effects).
  - *Direct*: effect of D on Y *holding* A *constant*.
  - *Indirect*: effect of D on Y *via* A.
- We use these decompositions to understand when these estimands have the desired interpretation. Preview of main findings:
  1) Popular reg.-based methods have undesirable properties in general.
  2) We provide reg.-based methods that avoid these issues.
- Our paper does *not* contribute on:
  - Identification of direct, indirect, or total effects.
  - Discussion on relative merits of total vs. direct causal effects.

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Related literature

1) **Mediation analysis in social sciences**:

- Baron & Kenny (86), Pearl (01), Robins (03), Imai et al. (10), etc...
- Their interest in on the mediators, their decomposition is based on the *natural* effects, and their analysis uses more restrictive assumptions.
- The actions (i.e., their mediators) are a nuisance to us, and our decomposition is based on the *controlled* effects.

2) **Total vs. partial/direct treatment effects in economics**:

- Manski (97), Heckman (00), Rosenzweig & Wolpin (00), etc...
- We discuss how regression-based estimands capture one or the other.

3) **Regression-based estimands without assuming linearity**:

- Angrist (98), Goodman-Bacon (21), de Chaisemartin & D'Haultfuille (20), Goldsmith-Pinkham et al. (22), Zhao & Ding (22), etc...
- *Mechanical connection*, but we differ on questions & main lessons.

Overview

- Setup and definitions
- Decomposition of regression-based estimands
  1) Short regression
  2) Long regression
  3) Long regression with interactions
  4) Strata fixed effects regression
  5) Saturated regression
- Conclusions

Setup

- **Variables**: binary treatment D, outcome Y, and actions $A = (A_1, \ldots, A_K)$.
- **Actions**: $A = (A_1, \ldots, A_K)$ is discrete, taking values in

$$\mathcal{A} \equiv \{a = (a_1, \ldots, a_K) : a_j \in \mathcal{A}_j, \;\; j = 1, \ldots, K\},$$
$$\mathcal{A}_j \equiv \{0, 1, \ldots, \bar{a}_j\} \quad \text{for} \quad \bar{a}_j \geq 1.$$

- **Potential outcomes**: $Y(d, a)$ with expected value

$$\mu(d, a) \equiv E[Y(d, a)].$$

- **Pooled potential outcomes:**

$$Y(d) = \sum_{a \in \mathcal{A}} Y(d, a) \, I\{A = a\}.$$

- **Observed outcome:**

$$Y = \sum_{(d,a) \in \mathcal{D} \times \mathcal{A}} Y(d, a) \, I\{(D, A) = (d, a)\}.$$

Causal effects

- We are interested in *ceteris paribus* effect of D on Y.

Definition: Average partial causal effect

The **average partial causal effect** of D on Y is

$$\mu(1,a) - \mu(0,a) \qquad \text{for } a \in \mathcal{A}.$$

Definition: Average direct causal effect (DCE)

An **average direct causal effect** of D on Y is *any* convex combination of average partial causal effects, i.e.,

$$\Delta = \sum_{a \in \mathcal{A}} \omega(a)\, (\mu(1,a) - \mu(0,a)),$$

where $\omega(a) \geq 0$ for all $a \in \mathcal{A}$ and $\sum_{a \in \mathcal{A}} \omega(a) = 1$.

Assumptions

- We rely on the following assumptions:

Assumption: Weak CI

$D \perp Y(d) \mid X$ for all $d \in \{0,1\}$.

- **Weak CI**: natural starting point, but insufficient to id. *ceteris paribus* effects.

Assumption: Strong CI

$(D,A) \perp Y(d,a) \mid X$ for all $(d,a) \in \{0,1\} \times \mathcal{A}$.

- **Strong CI:** sufficient for identification of $\mu(d,a)$, and any parameter of interest, but insufficient to deliver a desirable interpretation of popular estimands.

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  1) Short regression: Y on D
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  3) Long regression with interactions: Y on $D + A + AD$
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  5) Saturated regression: Y on $D\ \text{strata}(A) + \text{strata}(A)$
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Short regression: decomposition

short regression: $Y \;=\; \Delta_{\text{short}} D \;+\; \alpha \;+\; \varepsilon.$

$$\Delta_{\text{short}} \;=\; E[Y \mid D=1] \;-\; E[Y \mid D=0].$$

- Δ_{short} can be decomposed into three terms:

$$\Delta_{\text{short}} \;=\; \Delta^{s}_{\text{dce}} + \Delta^{s}_{\text{ind}} + \Delta^{s}_{\text{sel}},$$

with

$$\Delta^{s}_{\text{dce}} \;\equiv\; \sum_{a \in \mathcal{A}} \pi_1(a)\; E[Y(1,a) - Y(0,a) | D=1, A=a]$$

$$\Delta^{s}_{\text{ind}} \;\equiv\; \sum_{a \in \mathcal{A}} (\pi_1(a) - \pi_0(a))\; (E[Y(0,a)|D=0, A=a] - E[Y(0,0)|D=0, A=0])$$

$$\Delta^{s}_{\text{sel}} \;\equiv\; \sum_{a \in \mathcal{A}} \pi_1(a)\; (E[Y(0,a)|D=1, A=a] - E[Y(0,a)|D=0, A=a])$$

and $\pi_d(a) \;\equiv\; P\{A=a \mid D=d\}.$

Short regression: comments

$$\Delta_{\text{short}} \;=\; \Delta^{s}_{\text{dce}} + \Delta^{s}_{\text{ind}} + \Delta^{s}_{\text{sel}}.$$

1) Decomposition does not invoke Weak CI or Strong CI.

2) Under *Weak CI*: Δ_{short} captures the *total effect*.

$$\Delta_{\text{short}} \;=\; E[Y(1) - Y(0)] \;=\; E[Y(1, A(1)) - Y(0, A(0))].$$

3) Under *Weak CI*: Δ_{short} is *not a DCE*.

- Δ_{short} includes a DCE, Δ^{s}_{dce}, but also includes the sum of:
  - Δ^{s}_{ind}: combines selection and indirect effects.
  - Δ^{s}_{sel}: a pure selection effect.

Short regression under Strong CI

- Under *Strong CI*, the decomposition simplifies as follows:

$$\Delta_{\text{short}} = \Delta^{\text{s}}_{\text{dce}} + \Delta^{\text{s}}_{\text{ind}} + \cancel{\Delta^{\text{s}}_{\text{sel}}}.$$

with

$$\begin{aligned} \Delta^{\text{s}}_{\text{dce}} &= \sum_{a \in \mathcal{A}} \pi_1(a) \, (\mu(1,a) - \mu(0,a)) \\ \Delta^{\text{s}}_{\text{ind}} &= \sum_{a \in \mathcal{A}} (\pi_1(a) - \pi_0(a)) \, (\mu(0,a) - \mu(0,0)). \end{aligned}$$

- Strong CI yields $\Delta^{\text{s}}_{\text{sel}} = 0$ and isolates the indirect effect in $\Delta^{\text{s}}_{\text{ind}}$.
  - Strong CI does not restrict how D affects A: $(\pi_1(a) - \pi_0(a)) \neq 0$.
  - Strong CI does not restrict how A affects Y: $\mu(0,a) - \mu(0,0) \neq 0$

Short regression under Strong CI: comments

Under *Strong CI*, $\Delta_{\text{short}} = \Delta^{s}_{\text{dce}} + \Delta^{s}_{\text{ind}}.$

1) Δ_{short} (*still*) captures a *total effect*.

2) Δ_{short} is (*still*) *not a DCE* because of the *indirect effect* in Δ^{s}_{ind}.

- It's possible to have $\mu(1,a) - \mu(0,a) > 0$ for all $a \in \mathcal{A}$ and $\Delta_{\text{short}} < 0$. This will happen if $\Delta^{s}_{\text{ind}} < -\Delta^{s}_{\text{dce}} < 0$.

- i.e., the indirect effect may have *opposite sign* and *dominate the DCE*.

- **From here on**: we focus on analysis under Strong CI (See paper for Weak CI).

Short regression under Strong CI: comments

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Overview

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- Decomposition of regression-based estimands **(ignore X)**
  1) Short regression: Y on D $\quad$ ($=$ *total effect*)
  2) Long regression: Y on $D + A$

$$Y \; = \; \Delta_{\text{long}} D \; + \; \sum_{j=1}^{K} \theta_j A_j \; + \; \alpha \; + \; \varepsilon.$$

  3) Long regression with interactions: Y on $D + A + AD$
  4) Strata fixed effects regression: Y on $D + \text{strata}(A)$
  5) Saturated regression: Y on $D \, \text{strata}(A) + \text{strata}(A)$
- Conclusions

Long Regression

Theorem: Long regression

- Assume Strong CI and that $cov(D,A)$ is PD. Then,

$$\Delta_{\text{long}} = \Delta^{l}_{\text{dce}} + \Delta^{l}_{\text{ind}},$$

  where

$$\Delta^{l}_{\text{dce}} \equiv \sum_{a\in\mathcal{A}} \omega^{l}_{\text{dce}}(a) \, (\mu(1,a) - \mu(0,a)) \quad \text{with} \quad \sum_{a\in\mathcal{A}} \omega^{l}_{\text{dce}}(a) = 1,$$
$$\Delta^{l}_{\text{ind}} \equiv \sum_{a\in\mathcal{A}} \omega^{l}_{\text{ind}}(a) \, (\mu(0,a) - \mu(0,0)) \quad \text{with} \quad \sum_{a\in\mathcal{A}} \omega^{l}_{\text{ind}}(a) = 0.$$

- Furthermore, the following statements are equivalent:

  a) A are mutually exclusive binary variables, i.e., $\mathcal{A}_j = \{0,1\}$ for $j = 1,\dots,K$ and $A_j A_l = 0$ for all $j,l = 1,\dots,K$ with $j \neq l$.

  b) For *any* distribution of (A,D), $\omega^{l}_{\text{dce}}(a) \geq 0$ for all $a \in \mathcal{A}$.

  c) For *any* distribution of (A,D), $\omega^{l}_{\text{ind}}(a) = 0$ for all $a \in \mathcal{A}$.

Long regression: comments

$$\Delta_{\text{long}} \;=\; \Delta^{\text{l}}_{\text{dce}} + \Delta^{\text{l}}_{\text{ind}}.$$

1) If A are not mutually exclusive binary vars., Δ_{long} is not a *DCE*.

   (i) The indirect effect may have *opposite sign* and *dominate the DCE*. ($=$ *Short*).

   (ii) Even without an indirect effect, $\omega^{\text{l}}_{\text{dce}}(a) < 0$ for some $a \in \mathcal{A}$ ($\neq$ *Short*).

2) (ii) raises a red flag regarding the adjustment of A in a linear regression:

   - In particular, it implies that Δ_{long} is neither a DCE nor a total effect (next slide).

   $\Rightarrow$ Adjusting for A makes things worse.

Long regression: comments (ctd.)

- Consider **Example 1:** $\mathcal{A} = \{0,1,2\}$, i.e., non-binary.

– Let $P\{D=1\} = 0.5$, $\{A|D=1\} \sim \text{Bi}(2, 0.3)$, & $\{A|D=0\} \sim \text{Bi}(2, 0.9)$.

$$\Rightarrow \quad \omega^{l}_{dce} = [-0.1, 0.76, 0.34] \quad \text{and} \quad \omega^{l}_{ind} = [-0.14, 0.28, -0.14].$$

– Then, suppose that $\mu(1,0) > 0$ and $\mu(d,a) = 0$ for all $(d,a) \neq (1,0)$.

a) $\Delta_{long} = \Delta^{l}_{dce} < 0$ despite $\mu(1,a) - \mu(0,a) \geq 0$ for all $a \in \mathcal{A}$.

b) $\Delta^{l}_{ind} = 0$ as $\mu(0,a) - \mu(0,0) = 0$ for all $a \in \mathcal{A}$.

$\Rightarrow$ Δ_{long} is neither a DCE nor a total effect (c.f. $\Delta_{short} = \Delta^{s}_{dce} > 0$).

- We also have **Example 2**: $\mathcal{A} = \{0,1\}^2$ but not mutually exclusive.

Long regression is popular

- The long regression is extensively used in mediation literature:
  - **Baron & Kenny (86)**: largely established the use of these regressions in mediation analysis, has over 115,000 citations.
  - **Glynn (12)** discusses the popularity of the long regression in mediation and social sciences, and writes "examples are too numerous to cite."
  - **Imai et al. (10)**: recommends inference with a long regression under stronger assumptions + scalar A + linear model for $\mu(d,a)$.
- It has also been used in economics, even recently:
  - **Heckman, Pinto, & Savelyev (13, AER)**: use inference with a long regression. They assume linear model for $\mu(d,a)$.
  - **Fagereng, Mogstad, & Ronning (21, JPE)** also assume linear model for $\mu(d,a)$. They use long regression and interpret Δ_{long} as a DCE.
- Our results: these conclusions rely on linear model for $\mu(d,a)$.

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$$Y \;=\; \Delta_{\text{inter}} D \;+\; \alpha + \sum_{j=1}^{K} \theta_j \, A_j \;+\; \sum_{j=1}^{K} \lambda_j \, A_j \, D \;+\; \varepsilon.$$

  4) Strata fixed effects regression: Y on $D + \text{strata}(A)$
  5) Saturated regression: Y on D $\text{strata}(A) + \text{strata}(A)$
- Conclusions

Long regression with interactions

Theorem: Long regression with interactions

- Assume Strong CI and that $cov(D, A, AD)$ is PD. Then,

$$\Delta_{\text{inter}} = \Delta^{\text{i}}_{\text{dce}} + \Delta^{\text{i}}_{\text{ind}},$$

where

$$\Delta^{\text{i}}_{\text{dce}} \equiv \sum_{a \in \mathcal{A}} \omega^{\text{i}}_{\text{dce}}(a)(\mu(1,a) - \mu(0,a)) \quad \text{with} \quad \sum_{a \in \mathcal{A}} \omega^{\text{i}}_{\text{dce}}(a) = 1,$$
$$\Delta^{\text{i}}_{\text{ind}} \equiv \sum_{a \in \mathcal{A}} \omega^{\text{i}}_{\text{ind}}(a)(\mu(0,a) - \mu(0,0)) \quad \text{with} \quad \sum_{a \in \mathcal{A}} \omega^{\text{i}}_{\text{ind}}(a) = 0.$$

- Furthermore, the following statements are equivalent:

  a) A are mutually exclusive binary variables, i.e., $\mathcal{A}_j = \{0,1\}$ for $j = 1, \ldots, K$ and $A_j A_l = 0$ for all $j, l = 1, \ldots, K$ with $j \neq l$.

  b) For *any* distribution of (A, D), $\omega^{\text{i}}_{\text{dce}}(a) \geq 0$ for all $a \in \mathcal{A}$.

  c) For *any* distribution of (A, D), $\omega^{\text{i}}_{\text{ind}}(a) = 0$ for all $a \in \mathcal{A}$.

Long regression with interactions: comments

1) Δ_{inter} and Δ_{long} share problems: Unless A are mutually exclusive binary vars., Δ_{inter} is neither a DCE nor a total effect.

2) What about alternative estimands? For example:

$$\Delta_{\text{inter}} + \sum_{j=1}^{K} \lambda_j E[A_j] \quad \text{or} \quad \Delta_{\text{inter}} + \sum_{j=1}^{K} \lambda_j a_j.$$

$\Rightarrow$ We obtain an alternative decomposition, but with analogous problems.

3) This regression is also very popular. In particular, extensively used in mediation; advocated by Judd & Kenny (81), Kraemer et al. (02,08).

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$$Y \;=\; \Delta_{\text{sfe}}\, D \;+\; \sum_{a \in \mathcal{A}} \theta_a\, I\{A = a\} \;+\; \varepsilon.$$

  5) Saturated regression: Y on $D\ \text{strata}(A) + \text{strata}(A)$

- Conclusions

SFE regression

Theorem: SFE regression

- Assume Strong CI, and $P\{A = a\} > 0$ and $\pi_d(a) \in (0,1)$ for all $a \in \mathcal{A}$. Then,

$$\Delta_{\text{sfe}} = \sum_{a \in \mathcal{A}} \omega_{\text{sfe}}(a) \, (\mu(1,a) - \mu(0,a)),$$

  where

$$\omega_{\text{sfe}}(a) \equiv \frac{\pi_1(a)\pi_0(a)}{\sum_{\tilde{a} \in \mathcal{A}} \pi_1(\tilde{a})\pi_0(\tilde{a})}.$$

- Note that $\omega_{\text{sfe}}(a) \geq 0$ and $\sum_{a \in \mathcal{A}} \omega_{\text{sfe}}(a) = 1$.

SFE regression: comments

$$\Delta_{\text{sfe}} = \sum_{a \in \mathcal{A}} \omega_{\text{sfe}}(a) \, (\mu(1,a) - \mu(0,a)).$$

1) SFE regression automatically implements the main lesson from long regression:

   Δ_{long} identifies a DCE $\iff$ actions are mutually exclusive binary variables.

2) SFE regression gets a DCE without full saturation.

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  5) Saturated regression: Y on $D\ \text{strata}(A) + \text{strata}(A)$

$$Y \;=\; \sum_{a\in\mathcal{A}} \Delta_{\text{sat}}(a)\; D\; I\{A=a\} \;+\; \sum_{a\in\mathcal{A}} \gamma_a\; I\{A=a\} \;+\; \varepsilon,$$

Under Strong CI, $\quad \Delta_{\text{sat}}(a) \;=\; \mu(1,a) - \mu(0,a) \quad$ for all $a \in \mathcal{A}$.

- Conclusions

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  1) Short regression: Y on D (= *total effect*)

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  4) Strata fixed effects regression: Y on $D+\text{strata}(A)$ (= *DCE*)

  5) Saturated regression: Y on $D\ \text{strata}(A)+\text{strata}(A)$ (= *everything*)

$$Y = \sum_{a\in\mathcal{A}} \Delta_{\text{sat}}(a)\, D\, I\{A=a\} + \sum_{a\in\mathcal{A}} \gamma_a\, I\{A=a\} + \varepsilon,$$

Under Strong CI, $\Delta_{\text{sat}}(a) = \mu(1,a)-\mu(0,a)$ for all $a\in\mathcal{A}$.

- Conclusions

Conclusions

- We consider analyst interested in the avg. causal effect of binary treatment D on a "delayed" outcome Y. The delay implies that other actions A occur.
- We study regression-based estimands to capture avg. causal effect of D on Y.
- We decompose estimands into direct & indirect effects, and study conditions under which they have desirable interpretations:
  - **Short regression**: total effect.
  - **Long regression (with or without interactions)**: problematic in general, unless A are mutually exclusive binary variables.
  - **SFE regression**: direct causal effect.
  - **SAT regression**: everything.

Thanks!

Comparison with Imai et al. (10)

- Imai et al. (10) formalizes regression-based analysis for mediation analysis, initially proposed by Barron & Kenny (86).

- They assume:

  a) scalar A (though not necessarily binary),

  b) linear model for $\mu(d,a)$, i.e., $\mu(d,a) = \kappa_1 + \kappa_2 d + \kappa_3' a$.

  c) sequential ignorability, which implies Strong CI.

Assumption: Sequential Ignorability (SI)

- $(Y(\tilde{d},a), A(d)) \perp D \mid X$ for all $(\tilde{d},d,a) \in \{0,1\} \times \{0,1\} \times \mathcal{A}$,
- $Y(\tilde{d},a) \perp A(d) \mid (D=d,X)$ for all $(\tilde{d},d,a) \in \{0,1\} \times \{0,1\} \times \mathcal{A}$.

- Under these conditions, Imai et al. (10) use Δ_{long} to identify a DCE.

Comparison with Imai et al. (10) (ctd.)

- Their analysis imposes strong assumptions:

  a) Sequential Ignorability, more restrictive than Strong CI,

  b) Scalar A, which we don't require.

  c) Linear model for $\mu(d,a) = \kappa_1 + \kappa_2 d + \kappa_3' a$, which we don't require.

- Under these conditions, they show that

$$
\begin{aligned}
\bar{\zeta}(d) &\equiv E[Y(1, A(d))] - E[Y(0, A(d))] \\
&\stackrel{\text{(SI)}}{=} \sum_{a \in \mathcal{A}} \pi_d(a)\ (\mu(1,a) - \mu(0,a)) \\
&\stackrel{\text{(linear)}}{=} \sum_{a \in \mathcal{A}} \pi_d(a)\ \kappa_2 \ = \ \kappa_2.
\end{aligned}
$$

- The argument and the DCE interpretation break down with non-linear $\mu(d,a)$.

Back

Long regression: Example 2

- **Example 2**: $\mathcal{A} = \{0,1\}^2$ & $P\{A_1 = A_2 = 1\} > 0$, i.e., not mutually excl.

– $P\{D=1\} = 0.5$, $\{A_j | D=0\} \sim \text{Be}(0.1)$, $\{A_j | D=1\} \sim \text{Be}(0.7)$ for $j = 1,2$.

$\Rightarrow$ $\omega^{\text{l}}_{\text{dce}} = [0.34, 0.38, 0.48, -0.1]$ and $\omega^{\text{l}}_{\text{ind}} = [-0.14, 0.14, 0.14, -0.14]$.

– Then, suppose that $\mu(1,3) > 0$ and $\mu(d,a) = 0$ for all $(d,a) \neq (1,3)$.

a) $\Delta_{\text{long}} = \Delta^{\text{l}}_{\text{dce}} < 0$ despite $\mu(1,a) - \mu(0,a) \geq 0$ for all $a \in \mathcal{A}$.

b) $\Delta^{\text{l}}_{\text{ind}} = 0$ as $\mu(0,a) - \mu(0,0) = 0$ for all $a \in \mathcal{A}$.

$\Rightarrow$ Δ_{long} is neither a DCE nor a total effect (c.f. $\Delta_{\text{short}} = \Delta^{\text{s}}_{\text{dce}} > 0$) Back