The Coherence Side of Rationality

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I. Motivation & Summary

II. Discussion

Object of Study: Coherence of Forecasts

- Consider setting where a reasercher predicts a 1-period-ahead target vector $Z_{t+1} \in \mathbb{R}^k$.
- Let $\hat{Z}_{t+1}$ denote our forecast vector.

$$\text{Is } \hat{Z}_{t+1} \text{ a good forecast? } \begin{cases} \text{(i) Accuracy : } \hat{Z}_{t+1} \text{ and } Z_{t+1} \text{ are close.} \\ \text{(ii) Coherence : } \hat{Z}_{t+1} \text{ is consistent with property of } Z_{t+1}. \end{cases}$$

- Intuitively, coherence means the connection between the elements within the forecast vector is plausible.

- The paper studies coherence of forecast in firm's production problem.

$$Z_{t+1} = (\underbrace{Y_{t+1}}_{\text{output}}, \underbrace{X_{1,t+1}}_{\text{capital}}, \underbrace{X_{2,t+1}}_{\text{labor}})' \text{ and } \hat{Z}_{t+1} = (\mathbb{E}_t[Y_{t+1}], \mathbb{E}_t[X_{1,t+1}], \mathbb{E}_t[X_{2,t+1}])'.$$

- Economic theory tells us $Z_{t+1} = (Y_{t+1}, X_{1,t+1}, X_{2,t+1})$ should satisfy the production function. E.g. $Y_{t+1} = X_{1,t+1}^a X_{2,t+1}^b$ for Cobb-Douglas function.

- The paper declares that $\hat{Z}_{t+1}$ is coherent if it satisfies a "property" implied by the production function and formalizes the "property".

Why is coherence important?

- The paper argues importance of coherence based on the following logic.

A firm has an incoherent forecast $\hat{Z}_{t+1} = (\mathbb{E}_t[Y_{t+1}], \mathbb{E}_t[X_{1,t+1}], \mathbb{E}_t[X_{2,t+1}])$

$\Downarrow$

The firm could end up using sub-optimal combination of capital and labor.

$\Downarrow$

The firm obtains lower profit.

What methods have been used to forecast Z_{t+1}?

- Rules of thumbs
  - (R1) future growth = past growth for each element of Z_{t+1}.

$$\mathbb{E}_t\left[\frac{Y_{t+1}}{Y_t}\right] = \frac{Y_t}{Y_{t-1}}, \mathbb{E}_t\left[\frac{X_{1,t+1}}{X_{1,t}}\right] = \frac{X_{1,t}}{X_{1,t-1}}, \text{ and } \mathbb{E}_t\left[\frac{X_{2,t+1}}{X_{2,t}}\right] = \frac{X_{2,t}}{X_{2,t-1}}$$

  - (R5) regress $X_{1,t} = \alpha + \beta Y_t + \gamma X_{2,t}$;

    choose $\hat{Z}_{t+1}$ s.t. $\mathbb{E}_t[X_{1,t+1}] = \hat{\alpha} + \hat{\beta}\mathbb{E}_t[Y_{t+1}] + \hat{\gamma}\mathbb{E}_t[X_{2,t+1}]$.

- Whether they yield coherent forecasts has not been assessed.

Objectives

In terms of theory, the paper

- derives properties that coherent forecasts must satisfy;
- compares the rules of thumbs in terms of coherence;
- develops a statistical test to detect incoherence.

With data, the paper

- reports that incoherent forecasts are prevalent in survey;
- shows that level of incoherence is negatively correlated with corporate performance, evidence supporting the authors' conjecture

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Strength: Practicality

- The paper derives coherence constraints. If Cobb-Douglas,

$$\mathbb{E}_t \log [Y_{t+1}] = a \cdot \mathbb{E}_t \log [X_{1,t+1}] + b \cdot \mathbb{E}_t \log [X_{2,t+1}] . \tag{1}$$

- The paper provides a guideline to evaluate coherence of a given forecast vector.

- If production is the Cobb-Douglas, and if a practitioner
  1. knows a, b, $\Rightarrow$ plug-in
  2. doesn't know a, b, $\Rightarrow$ use RoT5 – estimate the parameters and plug-in
  3. doesn't know a, b, & faces noisy inputs, $\Rightarrow$ compare to forecast generated by RoT1

Limit: How to Forecast?

- The paper is helpful when we are given with a forecast to assess, but is silent on "how to make a good forecast".
- Ultimately, to make a good forecast, we need to account for "accuracy".
- A natural way to achieve both is to maximize the accuracy under the coherence constraint. E.g.,

$$\min_{\hat{Z}_{t+1}} L(\hat{Z}_{t+1}; \{Z_i\}_{i=1}^t) \text{ s.t. } f(\hat{Z}_{t+1}) = 0$$

where L is some loss function, $\{Z_i\}_{i=1}^t$ is a set of realized data and f is coherence constraint.

Weakness: Simplified Statistical Analysis

- The statistical test can be implemented with one observation; it determines one forecast vector is incoherent.
- Known common production, Structure on price (AR1 with normal error)
  $\Rightarrow$ Neither heterogeneous productions nor specification error
- 73% of incoherent forecasts reported. Rejection could be due to violation of assumptions.
- Modifying the goal, for example testing whether forecasts are incoherent "on average", may require less strict assumptions.
  $\Rightarrow$ The paper may improve its credibility without too much change of the results.