MPCs, MPEs and Multipliers:
A Trilemma for New Keynesian Models

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Abstract

We establish an impossibility result for New Keynesian models with a frictionless labor market: these models cannot simultaneously match plausible estimates of marginal propensities to consume (MPCs), marginal propensities to earn (MPEs), and fiscal multipliers. A HANK model with sticky wages provides a solution to this trilemma.

Introduction

In recent decades, macroeconomics has experienced a micro-moment revolution. As the field has recognized the importance of heterogeneity for macroeconomic outcomes, it has increasingly used estimates from disaggregated data to discipline its models.

One moment that epitomizes this revolution is the marginal propensity to consume (MPC): the amount by which consumption increases in response to a one-time transitory increase in income. Recent research has established that MPCs are extremely important for the macroeconomic effects of shocks and policy, both in partial equilibrium (e.g. Kaplan and Violante 2014, Auclert 2019, Berger, Guerrieri, Lorenzoni and Vavra 2018) and in general equilibrium (e.g. Kaplan, Moll and Violante 2018, Auclert, Rognlie and Straub 2018). Moreover, there exists a widespread consensus on the average level of MPCs in the data. We summarize this consensus as:

Fact 1. *Average MPCs are high in the data, around 0.25 quarterly and 0.5 annually.*

As is widely recognized in the literature, Fact 1 is important because it disqualifies almost all standard representative-agent models, which are based on the permanent-income hypothesis and therefore generate low average MPCs. Instead, Fact 1 favors models with heterogeneous agents and incomplete markets.

In standard macroeconomic models, agents have two immediate margins of adjustment when they receive a one-time income shock: they can consume more or work less (the rest is saved). The MPC measures how much they increase their consumption. The *marginal propensity to earn*, or MPE, measures how much they reduce their earned income. While the MPE has received somewhat less attention in the macroeconomic literature,[1] it is the labor market equivalent of the MPC. As such, it is in principle equally important for macroeconomic adjustment, and constitutes an equally natural target for calibrating models with heterogeneous agents. Moreover, there also exists a literature estimating MPEs in micro data. We summarize the evidence on the average level of MPEs in that literature as:

Fact 2. *Average MPEs are small in the data, around 0 to 0.04 annually.*

The combination of Facts 1 and 2 poses a challenge for heterogeneous agent models with a flexible labor supply choice. Under separable preferences, unless utility has very different curvatures in consumption and labor, agents that are free to adjust their labor supply should adjust to income transfers on the consumption and the labor supply margins by comparable amounts. If average MPCs are high, then average MPEs should be high as well.

The standard solution to this challenge is to consider alternative preference specifications that dampen wealth effects on labor supply, such as GHH preferences (Greenwood, Hercowitz and Huffman 1988). As we prove formally in this paper, any preference specification that lowers wealth effects must also increase the degree of complementarity between consumption and labor in preferences. While there is empirical support for some complementarity between consumption and labor (e.g. Aguiar and Hurst 2013), in this paper we show that too much complementarity poses a critical third challenge for business cycle models.

To understand this challenge, keep in mind that the micro-moment revolution should not eclipse the long tradition of disciplining macro models with macro moments. In this paper, we will focus on one particular conditional macro moment: the fiscal multiplier, i.e. the effect of government spending on aggregate output. There is a long empirical tradition of estimating the fiscal multiplier in aggregate time-series data, and of using it to gauge the plausibility of business cycle models.[2] We summarize the consensus evidence in that literature as:

Fact 3. *Fiscal multipliers lie in a moderate range in the data. Under accommodative monetary policy, both impact and cumulative multipliers are between 0.6 and 2.*

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[1] A large literature in labor focuses on the static MPE, which is different from the dynamic MPE that is of interest for macro models. See our discussion in section 1.

[2] For example, following Galí, López-Salido and Vallés (2007), the positive response of consumption to government purchases in structural VARs has motivated a very large literature searching for models that can deliver this feature. Monetary policy shocks provide alternative conditional moments to discipline business cycle models.
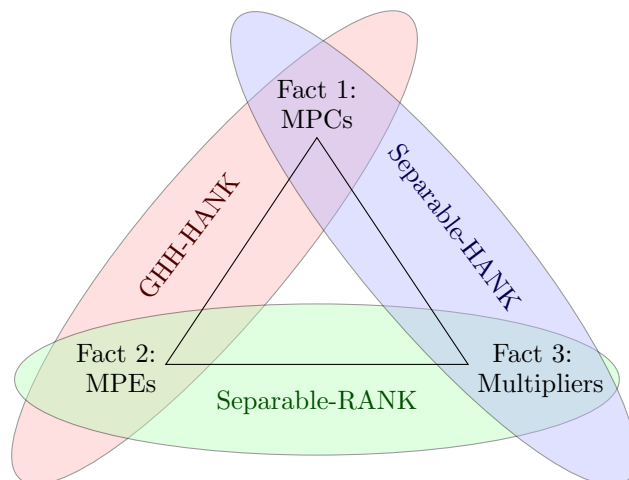
Figure 1: The trilemma

The most common framework for studying these multipliers is the New Keynesian model with a frictionless labor market, featuring sticky prices and flexible wages. As Monacelli and Perotti (2008) and Bilbiie (2011) have pointed out, introducing consumption-labor complementarity into this model leads to higher multipliers under accommodative policy. Building on their findings, we argue that when there is enough complementarity in the model to reconcile Facts 1 and 2, the resulting multiplier is high enough to violate Fact 3. This creates a *trilemma* for New Keynesian models: it is impossible to simultaneously match Facts 1, 2, and 3.

Figure 1 provides a graphical summary of this trilemma: it is possible to match any two of the three facts, but only at the cost of missing the third. First, with standard separable preferences, a heterogeneous-agent New Keynesian ("HANK") model with sticky prices and flexible wages can match MPCs and multipliers.[3] But this generates average MPEs that are much higher than in the data. For example, when we calibrate such a heterogeneous-agent model to feature a quarterly MPC of 0.25, the annual MPE is over 0.2—at least five times as large as the data. Alternatively, GHH preferences shut off wealth effects on labor supply, so that all agents have an MPE of 0. This allows a HANK model to match MPCs and MPEs simultaneously,[4] but at the cost of generating very high fiscal multipliers: our calibration implies an impact multiplier around 4.5, and a cumulative multiplier above 9. Finally, a representative-agent New Keynesian ("RANK") model with separable preferences can match fiscal multipliers and MPEs, but misses MPCs.

Our paper demonstrates that this trilemma is complete: even generalizing beyond the three models discussed above, there is no way to match all three facts at once in a New Keynesian model with a frictionless labor market. We conclude that solving the trilemma requires stepping outside of this class of models—in particular, adding frictions to the labor market. We provide our own

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[3]See McKay, Nakamura and Steinsson (2016) and Kaplan, Moll and Violante (2018) for prominent versions of such a model.

[4]This route is taken, for instance, by Bayer, Luetticke, Pham-Dao and Tjaden (2019), and was also taken by the original versions of Kaplan, Moll and Violante (2018) and Auclert (2019).

solution, based on sticky wages.

We establish the trilemma in two steps. First, we use consumer theory to show that, at the microeconomic level, MPCs and MPEs are related by a "complementarity index" (CI $\in [0, 1]$) that measures the strength of the complementarity between consumption and labor supply. Taking the elasticity of intertemporal substitution and the Frisch elasticity as given, lowering MPEs relative to MPCs requires raising CI: it is not possible to limit the wealth effect on labor supply without simultaneously raising the degree of complementarity between consumption and labor.

Second, using a combination of analytical results and numerical simulations, we show that it is the *same* complementarity index that, in New Keynesian models, regulates the size of the fiscal multiplier. The intuition is as follows. As a fiscal shock increases demand for output, wages must rise until enough labor is supplied in equilibrium to produce that output. When CI is positive, as households work more, they also want to consume more, and this additional consumption increases demand for output even further—prompting another increase in labor effort, and so on. The larger CI, the stronger this effect, and therefore the larger the multiplier. This explains the trilemma: the same force that reconciles Facts 1 and 2 makes it difficult to match Fact 3.

To cleanly isolate the importance of CI for the fiscal multiplier, we start by considering the representative-agent case. There, we show that the multiplier, if monetary policy maintains a constant real interest rate, is given by

$$\frac{dY_t}{dG_s} = \frac{1}{1 - (1 - \tau)\,\mathrm{CI}} \cdot \mathbf{1}_{s=t} \tag{1}$$

where $\tau < 1$ is the labor wedge. When preferences are separable, CI is 0, and one dollar of government spending increases output in the same period by one dollar: the constant-r fiscal multiplier is 1. This is the celebrated result from Woodford (2011). Under GHH preferences, CI is 1, and the constant-r fiscal multiplier is $\frac{1}{\tau}$. In standard parameterizations of the New Keynesian model, $\frac{1}{\tau}$ equals the elasticity of substitution between varieties ϵ_p, which is usually calibrated to be at least 5. This implies an equally high multiplier, far outside the range of Fact 3.

A sharp analytical result such as equation (1) cannot be obtained for general HANK models, which must be solved numerically. We therefore set up a quantitative HANK model with preferences that can generate an arbitrary complementarity index, and demonstrate that there does not exist any level of CI that can quantitatively reconcile Facts 1, 2 and 3. Specifically, we introduce preferences that we call "GHH-plus". These preferences span separable, GHH, and a continuum of cases in between, controlled by a single additional parameter $\alpha \in [0, 1]$.[5] For each α, we calibrate the model to hit an quarterly average MPC of 0.25, so that our model matches Fact 1 by construction. We then calculate the average MPEs and the fiscal multipliers that result from this parameterization. We show that there is no value of α that can also simultaneously hit Facts 2 and 3. This completes our formulation of the trilemma.

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[5]As we discuss in section 4.1, our preference specification is related to, but different from, Bilbiie (2020), who also introduces a specification that embeds separable and GHH as one parameter is varied.

The trilemma can be solved by introducing labor market frictions into a HANK model with separable preferences. In this paper, we do so via sticky wages, as in Auclert, Rognlie and Straub (2018). An alternative approach might be to introduce search frictions in the labor market, as in Gornemann, Kuester and Nakajima (2016) or Ravn and Sterk (2017).[6] In all of these models, workers are off their labor supply curves in the short run, so their effective MPEs are near 0, consistent with Fact 2. It is then possible to simultaneously hit MPCs (Fact 1) and multipliers (Fact 3).

A few other papers have argued that HANK models with flexible wages are difficult to reconcile with the data. Broer, Hansen, Krusell and Öberg (2020) focus on the implied countercyclicality of profits. Nekarda and Ramey (2020) argue that the implied cyclicality of marginal costs is rejected by the data. We add the trilemma to these voices against the flexible-wage model.[7]

The organization of the paper is as follows. In section 1 we provide a review of the literature supporting Facts 1–3. In section 2, we establish the relationship between MPCs, MPEs and the complementarity index in a standard consumer theory model. In section 3 we establish the relationship between the complementarity index and fiscal multipliers in RANK models. Finally, in section 4 we set up a HANK model with general preferences and show that there is no parameterization that can simultaneously match Facts 1–3. We provide our sticky-wage solution to the trilemma in section 5. The appendix collects proofs, additional model details and results, and explains our computational approach.

1 Evidence on MPCs, MPEs and multipliers

We begin by reviewing the empirical evidence that underlies Facts 1–3.

MPCs. We define the MPC as the level of the response of spending to a one-time, unexpected unit payment, in the period of the payment, and averaged across individuals. For example, the unweighted quarterly MPC is the spending response in the quarter of the payment, uniformly weighted across individuals. The literature has argued that this is a critical moment for discriminating across models, and that, as such, it should be used to calibrate macroeconomic models of consumption behavior.

The empirical literature on MPCs is vast. Kaplan and Violante (2014) summarize this literature and perform some of their own analysis on the 2001 US tax rebates. They find that the "collective evidence convincingly concludes that households spend approximately 25 percent of rebates on nondurables in the quarter that they are received." Alternative studies support similar magnitudes with alternative methods, and for alternative sources of income. This leads us to Fact 1:

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[6]This does not work for every model with search frictions, however. If there is an elastic search effort margin or an elastic intensive margin, MPEs can be too high to match Fact 2.

[7]Accordingly, we welcome the recent trend in the HANK literature of assuming sticky wages in addition to, or as a substitute for, sticky prices (e.g. Hagedorn, Manovskii and Mitman 2019, Alves, Kaplan, Moll and Violante 2019).

the unweighted quarterly MPC is about 0.25. We will calibrate our model, which has a quarterly frequency, to this target.

Some of the evidence on the MPC is at an annual frequency instead. The unweighted annual MPC in the literature is around 0.5. For example, the headline point estimate for the unweighted annual MPC in Fagereng, Holm and Natvik (2020) is 0.52, and the unweighted annual MPC from Jappelli and Pistaferri (2014) is 0.48.[8]

There also is some evidence on heterogeneity in MPCs, consistent with the predictions of standard heterogeneous-agent models. For example, in the papers discussed above, agents that have lower levels of liquid wealth or appear subject to financial constraints tend to have higher MPCs.

MPEs. We define the MPE as the negative of the level of the response of *earned income* to a one-time, unexpected unit payment, in the period of the payment, and averaged across individuals. Since the best estimates of MPEs require administrative earnings data that is only available annually, the MPE is often reported on an annual basis. While our definition makes the MPE the labor-market equivalent of the MPC, we note that this is a different concept from those traditionally used in the labor literature. This creates some difficulty in interpreting empirical estimates, as we now discuss.

One tradition in the labor literature is to define the MPE as a static object measuring how a one-time unexpected unit payment is split between consumption and earned income. These estimates correspond to, in our definition, MPE/(MPC + MPE)—a substantially larger number than the MPE we have defined.[9]

Another tradition is to measure the MPE out of permanent earnings changes, such as those caused by winning an annuity when playing the lottery. Classic papers in this literature, such as Imbens, Rubin and Sacerdote (2001), calculate the MPE as the change in earned income divided by the flow annuity payment. Because this object ignores intertemporal considerations—in particular, the fact that transfers in other periods can cause labor supply to fall this period—it again substantially overestimates the MPE as we define it.

There exist few credible, direct estimates of the MPE that we are interested in. The best available evidence comes from Cesarini, Lindqvist, Notowidigdo and Östling (2017), who measure the MPE out of small one-time lottery earnings in Sweden. Unlike most prior studies, they work with administrative earnings data, which mitigates measurement error and concerns about small samples and differential nonresponse. Moreover, they are able to control for the number of tickets bought, which makes the conditional random assignment assumption much more plausible. They report a precisely estimated MPE of 0.01, with a 95% confidence interval from 0.005 to 0.015.[10]

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[8]While the former use evidence from actual spending responses (to lottery earnings), the latter use answers to hypothetical survey questions. Parker and Souleles (2019) have found that these two methods of eliciting the MPC tend to yield similar answers.

[9]For example, Pencavel (1986) presents thirty estimates of the static MPE, which measure "the within-period division of an additional dollar of nonlabor income between the consumption of commodities and of leisure, [taking] savings decisions as being determined at a prior stage of the individual's allocation problem".

[10]The effect is persistent, so that the cumulative MPE is somewhat higher, at about 0.10. That cumulative number is

The other widely cited paper on MPEs, which uses lottery data from Massachusetts, is Imbens et al. (2001). As discussed above, their headline estimates, which range from 0.048 to 0.122, need to be adjusted downwards to match our definition: they report the average "MPE" out of the yearly payment from a 20-year annuity, while we are interested in the MPE from a one-time payment. The ratio between these two notions of MPE is model-dependent. In a permanent income model with no discounting and a unit interest rate, the Imbens et al. (2001) MPE would be 20 times higher than our MPE. As a lower bound on our MPE, therefore, we divide their lowest estimate by 20, which gives us 0.0024, or essentially zero. In a model with credit-constrained households, however, the two definitions are closer. For this case, we use our calibrated heterogeneous-agent model section 4, in which we calculate that the Imbens et al. (2001) MPE is 3.6 times higher than our MPE.[11] To obtain an upper bound on our MPE, we then take their highest estimate and divide by 3.6, delivering 0.034.

Combining this calculation with the Cesarini et al. (2017) results, and rounding up the upper bound, we formulate Fact 2 as: the annual MPE lies between 0 and 0.04.

Fiscal multipliers. Fiscal multipliers summarize the macroeconomic impact of government spending on output. The impact fiscal multiplier is the response of GDP to a unit change in aggregate government spending. The cumulative fiscal multiplier is the present discounted value of the response of GDP to a unit change in the present discounted value of government spending. It is well known in the literature that fiscal multipliers depend on the response of monetary policy. The headline numbers we focus on to compare model and data are for a certain type of accommodative monetary policy that maintains constant real interest rates. As we show explicitly in section 4.4, this type of monetary policy is less accommodative than the zero lower bound, but more accommodative than typical Taylor rules.

The empirical literature on fiscal multipliers is also vast. Ramey (2019) provides the most recent comprehensive review. She concludes that "the bulk of the estimates across the leading methods of estimation and samples lie in a surprisingly narrow range of 0.6 to 1", but that "The evidence for higher government spending multipliers during periods in which monetary policy is very accommodative, such as zero lower bound periods, is somewhat stronger. Recent time series estimates for the United States and Japan suggest that multipliers could be 1.5 or higher during those times." Ramey (2019) also makes a forceful argument that cumulative multipliers provide a more robust description of the data than impact multipliers (see also Mountford and Uhlig 2009). At times when policy is highly accommodative, Ramey (2011) suggests that "reasonable people" could defend an upper bound as high as 2.0. Overall, we take this summary of the evidence to support a range of fiscal multipliers between 0.6 to 2.0 when monetary policy is accommodative, for both impact and cumulative multipliers. This is our Fact 3.

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the counterpart of the cumulative MPC, which has been estimated to be near 1, e.g. in Fagereng et al. (2020).

[11]Specifically, we give agents in the separable heterogeneous-agent model of section 4 a 20-year annuity, and calculate the ratio of their earnings response in the first year to the MPE out of a one-time payment, which we find to be 3.6.

2 The MPC-MPE relationship

We consider a heterogeneous-agent model with frictionless labor supply, as in e.g. Aiyagari and McGrattan (1998). Households face idiosyncratic risk to their productivity e, which follows an exogenous Markov chain. A household with current idiosyncratic productivity e and assets a at time t, when the ex-post real interest rate is r_t, the effective wage is $w_t(e)$ and transfers are T_t, solves

$$
\begin{aligned}
V_t(a,e) &= \max_{c,n,a'} U(c,n) + \beta \mathbb{E}[V_{t+1}(a',e')|e] \\
\text{s.t.} \quad & c + a' = w_t(e)n + T_t + (1+r_t)a \\
& a' \geq \underline{a}
\end{aligned}
\tag{2}
$$

This model is the backbone of a recent literature that incorporates heterogeneity into the equilibrium analysis of monetary and fiscal policy. It nests the representative-agent case, in which e is a constant over time and $\underline{a} = -\infty$.

We allow for general $U(c,n)$ that satisfy standard conditions: continuous second derivatives, strict concavity, and Inada conditions that ensure an interior optimum. We will often refer to two particular cases. First, *separable* preferences take the form

$$
U(c,n) = u(c) - v(n).
$$

Second, following Greenwood et al. (1988), *GHH* preferences take the form

$$
U(c,n) = u(c - v(n)).
$$

In the solution to the dynamic programming problem (2), agents are on a first-order condition for consumption vs. labor at all times, irrespective of whether they are currently at the borrowing constraint. Consider an agent in state (a,e) facing the effective wage $\tilde{w} \equiv w_t(e)$ and a multiplier of $\lambda = \lambda_t(a,e)$ on his budget constraint (2), and choosing the bundle (c,n) of consumption and labor. This bundle must obey the first order conditions

$$
\begin{aligned}
U_c(c,n) &= \lambda \tag{3} \\
U_n(c,n) &= -\lambda \tilde{w} \tag{4}
\end{aligned}
$$

Let $c(\lambda, \tilde{w}), n(\lambda, \tilde{w})$ denote the solution to the system of equation (3) and (4).[12] We define three preference parameters locally given $(\lambda, \tilde{w})$. The elasticity of intertemporal substitution is defined

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[12]Given a solution for the multiplier $\lambda_t(a,e)$, the agent's policy functions for consumption and savings are then $c_t(a,e) = c(\lambda_t(a,e), w_t(e))$ and $n_t(a,e) = n(\lambda_t(a,e), w_t(e))$

in a standard way, as minus the elasticity of consumption to λ at fixed wage,

$$\text{EIS} \equiv -\frac{\partial \log c\left(\lambda, \tilde{w}\right)}{\partial \log \lambda}$$

and the Frisch elasticity of labor supply is similarly defined as the elasticity of labor to the wage at fixed λ,

$$\text{Frisch} \equiv \frac{\partial \log n\left(\lambda, \tilde{w}\right)}{\partial \log \tilde{w}}.$$

We also introduce the *complementarity index*, or CI for short:

$$\text{CI} \equiv \frac{\partial c\left(\lambda, \tilde{w}\right)}{\partial \tilde{w}} \bigg/ \tilde{w}\frac{\partial n\left(\lambda, \tilde{w}\right)}{\partial \tilde{w}}. \tag{5}$$

This is the marginal propensity to consume out of a change in earned income in response to $\tilde{w}$, holding λ fixed. The following lemma establishes some facts about CI.

Lemma 1. *For separable preferences, CI = 0. For GHH preferences, CI = 1. In general, CI = $\frac{U_{nc}/U_n}{U_{cc}/U_c}$.*

For separable preferences, CI $= 0$ because then λ uniquely determines c in (3). But when there is consumption-labor complementarity $U_{nc} > 0$, then CI > 0: higher labor effort n, holding $U_c = \lambda$ fixed, implies higher consumption c. The larger the response of c to n, the higher the complementarity index.

For GHH preferences, it is not an accident that CI $= 1$. Lemma 1 shows that in general, CI is the ratio of the semielasticity of U_n to c and the semielasticity of U_c to c. Since GHH preferences eliminate wealth effects on labor supply, a change in c must increase U_n and U_c proportionally, so that the first-order condition $U_n = -\tilde{w}U_c$ is undisturbed. In other words, the two semielasticities must be equal, so that their ratio CI is 1.

Now let us return to the more standard policy functions $c_t(a, e; T_t)$ and $n_t(a, e; T_t)$, making the dependence on the transfer T_t explicit. The *marginal propensity to consume* out of a transfer is

$$\text{MPC} \equiv \frac{\partial c_t(a, e; T_t)}{\partial T_t}$$

and the *marginal propensity to earn* is

$$\text{MPE} \equiv -\tilde{w}\frac{\partial n_t(a, e; T_t)}{\partial T_t}$$

With these definitions in hand, we prove the following relationship.

Proposition 1. *For any individual in state (a, e) at time t, we have*

$$\frac{\text{MPE}}{\text{MPC}} = \frac{\tilde{w}n}{c} \cdot \frac{\text{Frisch}}{\text{EIS}} \cdot (1 - \text{CI}) \tag{6}$$

Equation (6) relates, *at the individual level*, the ratio of MPC and MPE to the earned-income-to-consumption ratio and the three preference parameters EIS, Frisch and CI. Note that both MPC and MPE are complicated objects that depend on both the idiosyncratic state (a, e) and aggregate state t. But equation (6) shows that MPC is a sufficient statistic for MPE, and vice versa. The derivation of equation (6) requires only that conditions (3) and (4) be satisfied, so it characterizes behavior in a very broad class of models with flexible labor supply.

To understand the quantitative implications of (6), consider that in typical calibrations, Frisch $\simeq$ EIS, and asset and transfer income are small, so that $c \simeq \tilde{w}n$ for most agents. This implies that for these agents,

$$\text{MPE} \simeq (1 - \text{CI}) \cdot \text{MPC} \tag{7}$$

Suppose first that preferences are separable, so that CI is 0. Equation (7) then implies that for most agents, MPC and MPE are approximately the same. For example, constrained agents that do not save should have an MPC of 0.5 and an MPE of 0.5. As discussed in section 1, while there is some evidence that constrained agents have a high MPC, there is no good evidence supporting this kind of large MPE. More generally, equation (7) shows that the average MPE and the average MPC should be about the same. Hence, if preferences are separable, Facts 1 and 2 cannot hold simultaneously.

Indeed, equation (7) suggests that matching both facts requires raising the complementarity index towards 1. For instance, taking an annual MPE of 0.04 from the upper bound of Fact 2 and an annual MPC of 0.50 from Fact 1 requires CI to be at least $1 - 0.04/0.50 = 0.92$. GHH preferences, with their CI of 1, may therefore seem like a natural solution. Next we show that such high levels of CI create a challenge for Fact 3.

3 Consumption-labor complementarity and fiscal multipliers

In this section, we study a representative-agent model to gain insights into the relationship between CI and fiscal multipliers. While it is well known that the representative-agent model cannot match Fact 1, and is therefore not a candidate to solve the trilemma, the analytical formula we obtain from considering this case uncovers a force that is central in HANK models as well. Our result from this section complements existing results by Bilbiie (2011) and Monacelli and Perotti (2008) showing that consumption-labor complementarity can increase the fiscal multiplier in New Keynesian models.[13],[14]

To specify our RANK model, we embed the model of household behavior (2) in general equilibrium, under the assumptions of constant $e = 1$ and $\underline{a} = -\infty$. We further assume that the representative agent faces a constant marginal tax rate on labor earnings of τ^w, so that its post-tax-and-transfer labor earnings are $(1 - \tau^w)w_t n_t + T_t$. For simplicity, we write the model under

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[13]In contrast, complementarity generally decreases the multiplier in flexible-price models.

[14]Bilbiie (2009) and Eusepi and Preston (2015) also stress the importance of complementarities between consumption and labor for the aggregate effects of government spending.

perfect foresight, except for an unexpected shock at $t = 0$, and we drop expectations whenever there is no ambiguity.

The behavior of the representative household can be summarized by the Euler equation

$$U_c(c_t, n_t) = \beta(1 + r_t^e)U_c(c_{t+1}, n_{t+1}), \tag{8}$$

where r_t^e denotes the ex ante real interest rate that agents expect to hold between periods t and $t + 1$, and the labor supply equation, obtained by combining the intratemporal optimality conditions (3) and (4) and recognizing that $\tilde{w} = (1 - \tau^w)w$:

$$(1 - \tau^w)w_t = -\frac{U_n(c_t, n_t)}{U_c(c_t, n_t)}. \tag{9}$$

Aggregate consumption and labor are simply $C_t = c_t$ and $N_t = n_t$.

On the supply side, the model has a textbook New Keynesian structure. The final good is produced competitively as a CES aggregate of a continuum of intermediate goods with elasticity ϵ_p. Each intermediate good is produced by a monopolist using labor, according to a production function $f(N_t)$. These monopolists set their prices subject to isoelastic demand from the final good producer and quadratic (Rotemberg 1982) price adjustment costs. Their objective is to maximize the present value of profits, discounted at the ex-ante real interest rate r_t^e. Since all monopolists face the same profit maximization problem, there exists an equilibrium in which they choose the same price and produce the same quantity. This symmetric equilibrium is characterized by an aggregate production function

$$Y_t = f(N_t) \tag{10}$$

and a New Keynesian Phillips curve[15]

$$\log(1 + \pi_t) = \kappa_p\left(\frac{w_t}{f'(N_t)} - \frac{\epsilon_p - 1}{\epsilon_p}\right) + \frac{1}{1 + r_t^e}\frac{Y_{t+1}}{Y_t}\log(1 + \pi_{t+1}). \tag{11}$$

The combination of a labor tax and the monopoly power of intermediate goods producers leads, in general, to a non-zero *labor wedge*, which we define as[16]

$$\tau_t \equiv 1 + \frac{U_n(C_t, N_t)}{U_c(C_t, N_t)}\frac{1}{f'(N_t)}. \tag{12}$$

and which is strictly less than 1 under standard assumptions $U_n < 0$, $U_c > 0$, and $f' > 0$.

The fiscal authority purchases an exogenous amount G_t of the final good in every period, financed by lump-sum taxes at the margin. Since the model is Ricardian, the timing of these lump-sum taxes is irrelevant. We assume that the monetary authority sets the real rate directly,

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[15]As is well known, to first order, this Phillips curve is equivalent to the more common specification with Calvo pricing and discounting with the household's stochastic discount factor $\beta U_{c,t+1}/U_{c,t}$.

[16]A welfare-maximizing planner choosing allocations would set $-U_{n,t}/U_{n,c} = f'(N_t)$—that is, a zero labor wedge (Chari, Kehoe and McGrattan 2007).

as in Woodford (2011), and that the economy reverts to steady state once all shocks are past.[17]
Market clearing requires that

$$Y_t = C_t + G_t + \frac{\epsilon_p}{2\kappa_p} \log(1 + \pi_t)^2 Y_t. \tag{13}$$

Given perfect foresight after date 0, the model delivers a mapping between a sequence of government spending $\{G_t\}_{t \geq 0}$ and a path of output $\{Y_t\}_{t \geq 0}$. The next proposition characterizes the derivatives of this mapping—in other words, the model's fiscal multipliers.

Proposition 2. *Assume that monetary policy sets a constant real rate r. The marginal effect of a time-s government spending shock on time-t output is given by*

$$\frac{dY_t}{dG_s} = \frac{1}{1 - (1 - \tau)\text{CI}} \cdot \mathbf{1}_{s=t}. \tag{14}$$

Proposition 2 shows that fiscal multipliers in this case are static (output only responds in the period in which the government spends) and depend only on the steady-state labor wedge τ and complementarity index CI. In the special case of separable preferences (CI $= 0$), the multiplier is 1, as in Woodford (2011). By contrast, in the special case of GHH preferences (CI $= 1$), the multiplier is $1/\tau$. More generally, since $\tau < 1$, the multiplier is increasing in CI whenever it is positive.

Bilbiie (2011) previously derived an equivalent expression for the multiplier (14) under constant r and general preferences, which he stated using a certain set of elasticities of marginal utility and production (see his proposition 3). Our contribution is to instead write the fiscal multiplier in terms of τ and CI, clarifying that it is governed by the interaction between these two steady-state parameters.

Intuitively, (14) can be understood as follows. Increasing output by $dY = dG$ on the margin requires adding $dN = (1/f'(N))dY$ labor. By definition of CI, this increase in labor, when real interest rates and therefore marginal utility λ are held constant, increases consumption demand by $dC = \text{CI}\,\tilde{w}dN$. Since the after-tax wage $\tilde{w}$ equals $-U_n/U_c$ by (9), this increase in consumption demand is also equal to

$$dC = -\text{CI}\frac{U_n}{U_c}\frac{1}{f'(N)}dY = \text{CI}(1 - \tau)dY,$$

applying the definition of τ. That is, satisfying demand dG from the government increases demand from households by $\text{CI}(1 - \tau)dG$ in general equilibrium. This additional demand leads to a "second-round effect" of $\text{CI}^2(1 - \tau)^2 dG$, and so on. The process converges to the multiplier $1 + \text{CI}(1 - \tau) + \text{CI}^2(1 - \tau)^2 + \cdots = (1 - (1 - \tau)\text{CI})^{-1}$.

This argument is general, and only requires the definitions of CI and τ in terms of preferences and technology. In the model we have specified, however, the steady-state labor wedge τ takes the more specific form

$$\tau = 1 - (1 - \tau^w)\frac{\epsilon_p - 1}{\epsilon_p}. \tag{15}$$

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[17]This delivers determinacy in the representative-agent model with constant-r monetary policy.

Equation (15) shows the forces that shape the magnitude of the steady-state labor wedge in most New Keynesian models: the monopolistic distortion $\frac{\epsilon_p - 1}{\epsilon_p}$, and distortive taxation τ^w—which could be on either the employee side (as in this model), or on the firm side.

Recall that, in the case of GHH preferences, the fiscal multiplier is $1/\tau$. In typical calibrations, this will be a large number, because ϵ_p is calibrated to be high and τ^w is calibrated to be low. For instance, Nakamura and Steinsson (2014) report a constant-r multiplier of 7.00 under GHH preferences, reflecting their calibration choice of $\epsilon_p = 7$ and $\tau^w = 0$. Another typical calibration choice is to obtain an efficient steady state by offsetting monopolistic distortions with an employment subsidy of $\tau^w = -1/(\epsilon_p - 1)$. This yields $\tau = 0$ and a locally infinite multiplier. Clearly, none of these calibration choices can be reconciled with Fact 3.

More generally, equations (14) and (15) show why there is a tension in matching Facts 1, 2 and 3 jointly. The back-of-the-envelope calculation from section 2 suggested that CI ≥ 0.92 is necessary to match Facts 1 and 2. Meanwhile, equation (14) shows that to match Fact 3 with a multiplier below 2 in a RANK model, we need CI to be lower than $\frac{1-1/2}{1-\tau} = \frac{0.5}{1-\tau}$.

Even with conservative choices for a labor income tax of $\tau^w = 0.33$ (as in our model below) and $\epsilon = 7$, equation (15) then requires CI ≤ 0.88. And this bound is only likely to tighten as we move to flexible-labor HANK models—whose high MPCs tend to create higher multipliers—suggesting that these models will be unable to solve the trilemma. The next section verifies quantitatively that, indeed, no such solution exists.

4 The trilemma for HANK models

HANK models are famed for delivering Fact 1. One might hope that it is possible to parameterize these models to match Facts 2 and 3 as well. In this section, we demonstrate that this is not the case. In other words, we show that the trilemma has no solution in the space of New Keynesian models with a frictionless labor market.

To do this, we proceed as follows: we set up and calibrate a model that always features an average quarterly MPC of 0.25, thereby matching Fact 1 by construction. By varying the degree of complementarity in preferences, we then assess the quantitative tradeoff between matching Fact 2 and Fact 3. We show that it is impossible to match both.

4.1 HANK with GHH-plus preferences

Here we set up and parameterize our HANK model with flexible labor supply and an arbitrary degree of consumption-labor complementarity in preferences.

Households. Households solve the dynamic problem introduced in equation (2). Since we are solving the model numerically, we must now specify a functional form. We consider a flexible

felicity function with the following form:

$$U(c, n) = \frac{1}{1 - \sigma} \left(c - \varphi\alpha\frac{n^{1+\nu}}{1 + \nu} \right)^{1-\sigma} - \varphi(1 - \alpha)\frac{n^{1+\nu}}{1 + \nu} \tag{16}$$

We call these "GHH-plus preferences", since they nest isoelastic separable preferences (when $\alpha = 0$) and GHH preferences (when $\alpha = 1$). This choice is certainly not the only possible functional form for $U(c, n)$ with this property (see, for instance, Bilbiie 2020). But it has two additional properties that make it ideal for our purposes. First, it is convenient for computation: as we show in appendix D, U_c can be inverted to provide analytical expressions for c and n, which means that Carroll (2006)'s method of endogenous gridpoints can be applied. Second, as we show in appendix A.4, it delivers a simple expression for CI:

$$\text{CI}(e, a) = \frac{\alpha U_c(e, a)}{\alpha U_c(e, a) + 1 - \alpha} \in [0, 1]. \tag{17}$$

Equation (17) shows that CI is effectively parameterized by α, with CI $= 0$ for separable preferences and CI $= 1$ for GHH, as expected. In intermediate cases, CI is increasing in α but varies across households according to their marginal utilities of consumption.

Next, we follow the specification of the tax system in Auclert and Rognlie (2020) and let the effective wage and transfers be given by

$$w_t(e) = (1 - \tau_t^g)(1 - \gamma)w_t e, \tag{18}$$

$$T_t = (1 - \tau_t^g)\gamma w_t N_t. \tag{19}$$

Here, $\gamma \in [0, 1]$ governs the progressivity of the tax-and-transfer system, and τ_t^g is the overall tax rate on income. We calibrate both γ and τ^g to the data. This is important for two purposes. First, the literature has shown that the marginal incidence of taxes that the government levies to pay for its spending has a significant effect on fiscal multipliers in HANK models (e.g. Auclert et al. 2018, Hagedorn et al. 2019). For this marginal incidence to be realistic, we need an empirically plausible degree of tax progressivity. Second, as we showed in section 3, a positive marginal tax rate on labor increases the labor wedge, which in turn (at least in a representative agent model) is an important determinant of the fiscal multiplier whenever CI > 0. We therefore want to make sure that our results involve a plausible labor wedge.[18]

Asset markets. This economy has two assets in positive net supply: public debt in the form of one-period real bonds, and equity in intermediate goods firms. Since we assume certainty equivalence at the aggregate level, financial market equilibrium requires that these two assets offer the same return ex ante. Let p_t be the ex-dividend price of equity, and d_t be dividends. Then

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[18]With our tax system, the marginal tax rate on overall labor income in the steady state is $\tau^w = 1 - (1 - \tau^g)(1 - \gamma)$, and the labor wedge for every individual is $\tau = 1 - (1 - \tau^g)(1 - \gamma)\frac{\epsilon_p - 1}{\epsilon_p}$.

we have

$$1 + r_t^e = \mathbb{E}_t \left[\frac{d_{t+1} + p_{t+1}}{p_t} \right] \tag{20}$$

where, as in section 3, r_t^e denotes the real interest rate that agents expect to prevail between time t and time $t + 1$. Along perfect-foresight paths, the ex-post real interest rate r_t that enters agents' budget constraints in (2) is always equal to r_{t-1}^e. However, an unexpected aggregate shock at $t = 0$ may result in capital gains and imply $r_0 \neq r_{-1}^e$. Assuming that all households hold the same portfolio of bonds and equity, each of them experiences the same ex-post interest rate, equal to a weighted average of the realized return on equity and bonds:

$$1 + r_t = \underbrace{\frac{p_{t-1}}{p_{t-1} + B_{t-1}}}_{\text{equity share}} \frac{d_t + p_t}{p_{t-1}} + \underbrace{\frac{B_{t-1}}{p_{t-1} + B_{t-1}}}_{\text{bond share}} (1 + r_{t-1}^e) \tag{21}$$

where the last term in equation (21) reflects the fact that the return on government bonds at time t is always equal to r_{t-1}^e, since these are one-period real bonds.

Supply side. The supply side is the same as in section 3: the final good is produced competitively as a CES aggregate of intermediate goods with elasticity ϵ_p, and intermediate goods firms are monopolistically competitive with Rotemberg pricing.

We parameterize the production function for intermediate goods as:

$$f(N_t) = ZN_t - F. \tag{22}$$

The fixed cost F allows us to target firms' steady-state net profits, and therefore the value of their equity, independently of the elasticity ϵ_p. This is useful because the two play different roles in the model. The elasticity affects the value of the labor wedge and therefore the fiscal multiplier. Firm equity is important because it is an asset held by households and therefore determines the overall degree of liquidity in the economy, which in turn is a key determinant of MPCs.

Aggregate firm dividends are then

$$d_t = Y_t - w_t N_t - \frac{\epsilon_p}{2\kappa} \log(1 + \pi_t)^2 Y_t \tag{23}$$

where the last term is the price adjustment cost. By equation (20), the value of firms p_t is the present discounted value of their future dividends.

Government policy. The government purchases an exogenous amount of final goods G_t, runs the tax-and-transfer system, and issues one-period real debt B_t. Its overall budget constraint in period t is given by

$$B_t + \tau_t^g w_t N_t = (1 + r_{t-1}^e) B_{t-1} + G_t. \tag{24}$$

Following Auclert et al. (2018), we assume that the government follows a fiscal rule such that increases in spending relative to steady state are initially financed using increases in debt, which are then paid back at a rate of $1 - \rho_B$ each period,

$$B_t - B_{ss} = \rho_B(B_{t-1} - B_{ss} + G_t - G_{ss}) \tag{25}$$

where we call $\rho_B \in [0, 1)$ the "persistence" of debt, and a balanced-budget rule corresponds to $\rho_B = 0$. Given (25), the government adjusts the labor income tax rate τ_t^g so that (24) is satisfied at all times.

Just as in section 3, we specify the monetary policy rule to be $r_t^e \equiv r_{ss}$. We show explicitly in section 4.4 that this represents a moderate degree of accommodation, in between the zero lower bound and an active Taylor rule.

Finally, goods market clearing requires that

$$Y_t = C_t + G_t + \frac{\epsilon_p}{2\kappa_p}\log(1 + \pi_t)^2 Y_t. \tag{26}$$

By Walras's law, this is equivalent to asset market clearing $B_t + p_t = A_t$.

4.2 Calibration

As discussed at the top of this section, our goal is to assess the tradeoff between matching Fact 2 and Fact 3, conditional on matching Fact 1. Our calibration therefore varies α holding other key moments constant, including a quarterly average MPC of 0.25. It is well known that HANK models with a plausible supply of total assets cannot generate such large MPCs without additional features, such as illiquid assets or discount factor heterogeneity. We follow the latter route and use a simple form of permanent discount factor heterogeneity: we assume that one half of the population has discount factor β_1 and one half of the population has discount factor $\beta_2 < \beta_1$, and use (β_1, β_2) to jointly achieve our targets for the average MPC and the real interest rate r.

The model period is one quarter. For the most part, our calibration follows McKay et al. (2016), a prominent example of a HANK model with a canonical supply side, sticky prices, and flexible wages. We set the real interest rate r to an annualized value of 2% and the Frisch elasticity $1/\nu$ to 0.5. The elasticity of intertemporal substitution (EIS) varies across individuals, and we calibrate σ to target an average EIS of 0.5.[19] We choose the disutility of labor φ to normalize effective aggregate labor to 1, and aggregate labor productivity Z to normalize aggregate output to 1.

We assume that idiosyncratic labor productivity e follows an AR(1) process with a persistence of 0.966 as in McKay et al. (2016) and calibrate the variance of innovations to match the standard deviation of log gross earnings of 0.92 documented by Song, Price, Guvenen, Bloom and von Wachter (2019). We discretize the labor productivity process as a 25-point Markov chain. As in Auclert and Rognlie (2020), we set the ratio of bonds to annual GDP to 0.55, the steady-state labor

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[19]Appendix A.4 shows how to calculate the EIS for GHH-plus preferences.

Table 1: Fixed and calibrated parameters

parameter	name	value/target
$1/\nu$	Frisch elasticity	0.5
ρ_e	persistence of income process	0.966
τ^g	income tax level	0.191
γ	income tax progressivity	0.177
ϵ_p	elasticity of substitution	7
κ_p	NKPC slope	0.002
B	government bonds	$0.55 \cdot 4Y$
ρ_B	persistence of public debt	0.9
$\underline{a}$	borrowing constraint	0
β_1	upper discount factor	$r = 0.02/4$
β_2	lower discount factor	MPC $= 0.25$
$1/\sigma$	U_c curvature	average EIS $= 0.5$
φ	disutility of labor	$N = 1$
Z	aggregate labor productivity	$Y = 1$
σ_e	std of income shocks	Var $\left[\log(n_i e_i)\right] = 0.92^2$
F	fixed cost	$p = 0.85 \cdot 4Y$

The upper panel lists directly calibrated parameters. The lower panel lists parameters calibrated to match some target, whose values will depend on the choice of α.
Separable preferences: $(\beta_1, \beta_2) = (0.978, 0.923)$. GHH: $(\beta_1, \beta_2) = (0.987, 0.970)$.

tax to $\tau^g = 0.191$, and the progressivity parameter to $\gamma = 0.177$. This provides a fit comparable to the loglinear retention function of Heathcote, Storesletten and Violante (2017), but is slightly more convenient for our purposes because the marginal tax rate of agents is constant and equal to $(1 - \tau^g)(1 - \gamma)$. Together with a standard value for the elasticity of substitution, $\epsilon_p = 7$, these choices imply a steady-state labor wedge of $\tau = 0.43$, as in our back-of-the-envelope calculations from section 3.

Following McKay et al. (2016), we set the ratio of liquid assets to annual GDP to 1.4. In our model, this includes the value of firm equity as well as government bonds, and we calibrate the fixed cost F so that firm equity equals assets minus bonds, which is 0.85 times annual GDP. Together with the fiscal rule, these assumptions imply $G = 0.177$. We assume that households cannot borrow, $\underline{a} = 0$. Finally, we set the slope of the Phillips curve to $\kappa_p = 0.002$. This low value is irrelevant for our fiscal multiplier results under constant r, since it only affects the dynamics of inflation. We set it to avoid the explosively large multipliers with more flexible prices when nominal rates are constant (the so-called "paradox of flexibility"), which would only exacerbate the trilemma.

Table 1 summarizes our calibration. The upper panel contains the parameters we hold fixed, and the lower panel shows the parameters that we calibrate internally. Appendix D discusses our solution procedure.
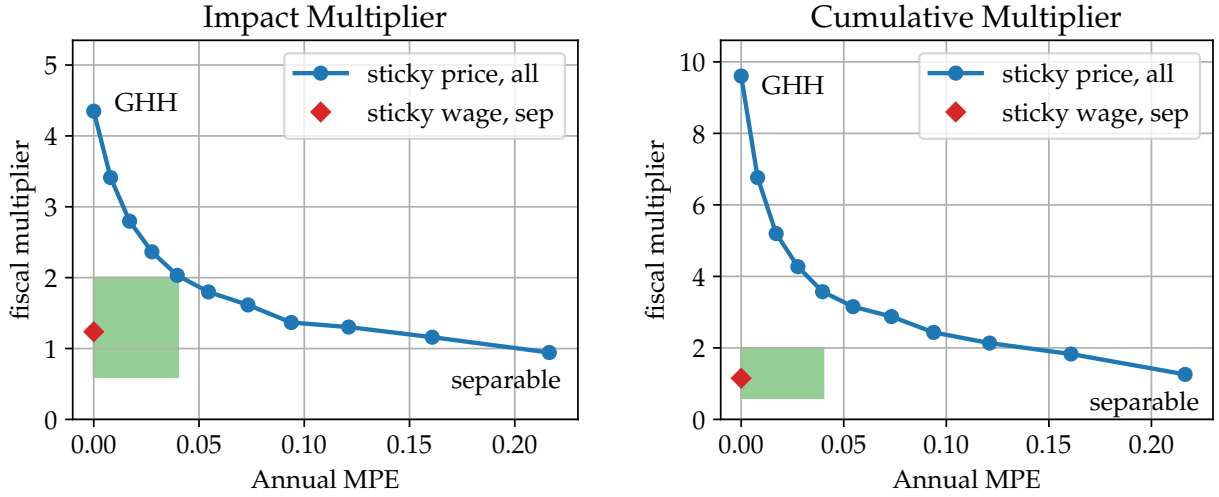
Figure 2: The tradeoff between fiscal multipliers and MPEs

4.3 The MPE-multiplier dilemma

Figure 2 presents our main quantitative result. The figure summarizes the model's performance in matching average annual MPEs (Fact 2) and fiscal multipliers (Fact 3) for the full range of possible values of the complementarity index. Each blue circle corresponds to the outcomes of a model with a different value of $\alpha \in [0,1]$. Recall that each of these models is calibrated to a quarterly MPC of 0.25, so by construction each of these circles matches Fact 1. The x axis plots the annual MPE, evaluating the quantitative ability of the model to match Fact 2. The y axis plots two measures of the fiscal multiplier, evaluating the quantitative ability of the model to match Fact 3. The green rectangle shows the range of values that are acceptable according to our facts. Clearly, none of the models gets close quantitatively, at least if we judge using cumulative multipliers on the right panel: models that have plausible values for the MPE have far too large fiscal multipliers, and vice versa. It takes high complementarity to match Fact 2, but low complementarity to match Fact 3. This is the trilemma for New Keynesian models. We now provide more detailed context for this conclusion, and discuss how it relates to proposition 1 and proposition 2.

The x-axis in both panels of figure 2 shows the average annual MPE generated by the model.[20] The leftmost point corresponds to $\alpha = \text{CI} = 1$ (GHH preferences), and α then declines as we move to the right, with the rightmost point corresponding to separable preferences ($\alpha = \text{CI} = 0$). The magnitudes on this axis reflect the aggregate implications of proposition 1. Recall that the Frisch elasticity and the average EIS are calibrated to be the same number. While agents have different ratios of earned income to consumption, the average ratio is $(1 - \tau^g)(1 - \gamma)WN/C = 0.66/0.82 = 0.80$. We therefore expect the quarterly MPE to be roughly $0.80 \times (1 - \text{CI}) \times 0.25 = (1 - \text{CI}) \times 0.20$. Indeed, the quarterly MPE for the separable model is close to this value, though slightly lower at

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[20]This is computed as the (negative of) the sum of the responses of earned income in the first four quarters to a one-time unit lump-sum transfer to all agents in quarter 0.

0.16, and the annual MPE is 0.23, far outside the acceptable range from Fact 2.[21] Only high values of α, near GHH, fall inside the acceptable range: it takes high complementarity to match Fact 2.

The y-axis shows the fiscal multiplier in response to an AR(1) shock to government spending with a persistence of 0.7, as in Auclert et al. (2018). On the left panel, we plot the impact multiplier dY_0/dG_0. On the right panel, we plot cumulative multipliers, defined as $\frac{\sum_t (1+r)^{-t} dY_t}{\sum_t (1+r)^{-t} dG_t}$. Figure 3 shows the impulse responses of consumption corresponding to each of these points. As in the representative-agent case of proposition 2, the multiplier is rising in CI, but now high MPCs provide further amplification, especially when measured cumulatively (Auclert et al. 2018). For instance, under GHH preferences, proposition 2 implies a constant-r multiplier of $1/\tau = 2.32$, both on impact and cumulatively, but here the cumulative multiplier is almost 10. Only low values of α, near separable, fall inside the acceptable range: it takes low complementarity to match Fact 3.[22]

The calibration with $\alpha = 0.6$ features an annual MPE of 0.04 and an impact fiscal multiplier of 2.03. This is just at the boundary of the acceptable range. However, it also features a cumulative fiscal multiplier of 3.57, far outside the acceptable range. In brief, there is no sticky-price, flexible-wage model that can solve the trilemma.

4.4 Alternative monetary and fiscal policies

In this section, we consider alternative monetary and fiscal policies. Since none of these policies will change the calibrated steady state, MPCs and MPEs are unaffected, and we focus on fiscal multipliers.

The left panel of table 2 displays cumulative fiscal multipliers for different levels of complementarity $\alpha \in \{0, 1/2, 1\}$ under three monetary policy rules: an active Taylor rule $i_t = r + \phi \pi_t$ with a coefficient on inflation of $\phi = 1.25$, our benchmark with constant r, and a simulation with constant nominal interest rates for 3 years, followed by reversion to an active Taylor rule. Relative to constant r, an active Taylor rule pushes down the multiplier, bringing it closer to its flexible-price level, which is 0 for GHH and less than 1 for separable preferences. With constant nominal interest rates, on the other hand, the multiplier increases even further, consistent with Christiano, Eichenbaum and Rebelo (2011). This form of accommodative policy therefore makes the trilemma even worse. Constant r, meanwhile, is the intermediate case—neither active nor highly accommodative.

The right panel of table 2 displays cumulative fiscal multipliers under three fiscal policy rules: balanced budget with $\rho_B = 0$, our baseline with $\rho_B = 0.9$, and very persistent debt $\rho_B = 0.95$. The table shows an intriguing property of the sticky price model: in contrast to typical HANK models

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[21]The representative-agent calculation 0.20 overstates the quarterly MPE when CI = 0 because of the negative covariance between MPCs and earned income that the model generates. The annual MPE is only around 50% higher than the quarterly MPE because there is a large share of impatient agents at or very close to their borrowing constraint.

[22]For the separable sticky-price case, the impact (though not the cumulative) multiplier is slightly lower in HANK than RANK, due to peculiar distributional forces discussed in appendix E. In general, we have found that these forces make the separable calibration somewhat fragile.

Table 2: Cumulative Fiscal Multipliers with Alternative Monetary and Fiscal Policies

	Monetary			Fiscal		
	Taylor rule	constant-r	3-year peg	$\rho_B = 0$	$\rho_B = 0.9$	$\rho_B = 0.95$
Separable	1.09	1.26	3.19	1.26	1.26	1.26
$\alpha = 0.5$	2.44	3.15	4.27	3.15	3.15	3.15
GHH	6.76	9.60	13.25	9.60	9.60	9.60
Sticky Wage	1.11	1.15	1.24	0.92	1.15	1.57

(Auclert et al. 2018), the timing of fiscal deficits is irrelevant for the multiplier. As we establish formally in appendix B.1, this reflects an unusual form of "Ricardian equivalence", which is due to the interaction of our tax system with tradable firm equity: both the path of after-tax wages $(1 - \tau_t^g)(1 - \gamma)w_t$ and the total value of assets are unaffected to first order by the path of τ_t^g that the government uses to finance its spending. Given that there is some empirical evidence that deficit financing tends to raise the fiscal multiplier, this provides another argument against the sticky-price, flexible-wage model. It also implies that conditional on spending, no path for bond financing can ever lower the multiplier and solve the trilemma.

5 A solution to the trilemma

To solve the trilemma, we propose to push workers off their labor supply curves in the short run, imposing nominal wage stickiness and demand-determined labor in the spirit of Erceg, Henderson and Levin (2000). This does not avoid the problem of high multipliers when CI is high: indeed, as appendix A.3 makes clear, the multiplier result in proposition 2 is identical under sticky wages.[23] It does, however, imply that MPE $= 0$, which allows us to choose low CI and obtain multipliers consistent with Fact 3 without contradicting Fact 2. For simplicity, in our calibration we will choose separable preferences, CI $= 0$.

5.1 Model setup

We introduce sticky wages by borrowing the microfoundation of the wage Phillips curve from Auclert et al. (2018), and adapting it to the tax system in (18)–(19).[24] Households solve a version of the problem in (2), but they now take their hours n as given. This makes their MPE equal to 0 by construction, and the model is therefore immediately consistent with Fact 2.

Aggregate hours are determined by a labor union, who sets wages on behalf of households subject to satisfying the aggregate demand for labor from final goods firms. In appendix B.2, we

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[23]The only difference is that we must use CI $= \frac{U_{nc}/U_n}{U_{cc}/U_c}$ from lemma 1 to define CI directly in terms of utility, since definition (5) no longer makes sense when workers are not on their labor supply curves. This CI can be interpreted as the MPC out of earnings from demand-determined labor, divided by the wage markdown.

[24]The tax code affects marginal incentives to work, and therefore the wage-setting problem of labor unions.

show that the maximization problem of these unions yields a wage Phillips curve of

$$\log(1 + \pi_t^w) = \kappa_w \left[N_t U_N(C_t^*, N_t^*) - w_t N_t (1 - \tau_t)(1 - \gamma) \left(\frac{\epsilon_w - 1}{\epsilon_w} \right) U_C(C_t^*, N_t^*) \right]$$
$$+ \beta \log(1 + \pi_{t+1}^w), \tag{27}$$

where π_t^w denotes nominal wage inflation, $U_N(C_t^*, N_t^*)$ is the average marginal disutility of labor and $U_C(C_t^*, N_t^*)$ is the productivity-weighted average marginal utility of consumption. In the representative-agent counterpart of the model, these are simply the U_N and U_C of the representative agent, and therefore (27) coincides with the formulation in Erceg et al. (2000).

We assume that prices are flexible ($\kappa_p = \infty$), so that intermediate goods producers always charge their desired markup. Hence, the real wage is constant at $w_t = Z \left(\frac{\epsilon_p - 1}{\epsilon_p} \right)$, and price and wage inflation coincide, $\pi_t^w = \pi_t$. Finally, since wage adjustment incurs a cost in terms of utils and not goods, goods market clearing becomes simply $Y_t = C_t + G_t$. This completes the formulation of the sticky-wage model.

5.2 Results

We calibrate the model as in section 4.2. We assume separable preferences ($\alpha = 0$), and calibrate the model to deliver Fact 1. Figure 2 shows that the model can then deliver Facts 2 and 3 simultaneously. The MPE is 0 by construction. The impact fiscal multiplier is 1.24 and the cumulative fiscal multiplier is 1.15, well within the range of Fact 3. The red line of figure 3 shows that the impulse responses of private consumption feature a moderate degree of crowding in for about 20 quarters.

Table 2 shows that the sticky wage model is also well-behaved with respect to alternative assumptions about monetary and fiscal policy. Alternative monetary rules affect the multiplier moderately. Alternative fiscal policy rules affect the multiplier to a greater extent, with more back-loaded fiscal policy generating larger fiscal multipliers. Under all of these assumptions, however, multipliers remain within the range allowed by Fact 3. Hence, the sticky wage model provides a solution to the trilemma.

There are other reasons to prefer the sticky-wage, flexible-price model to its sticky-price, flexible-wage counterpart. First, as appendix figure 4 shows, the equity price is mildly procyclical instead of strongly countercyclical.[25] This gives the model a chance of fitting both the cyclicality of the price-cost margin (e.g. Nekarda and Ramey 2020) and the response of equity prices to government spending shocks. It also avoids the large redistribution effects associated with firm ownership, which Broer et al. (2020) show to be problematic, and which drive peculiar heterogeneity in impulse responses in our own flexible-wage model, as appendix E discusses in detail.

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[25]To obtain a simple benchmark HANK model, one can dispense with intermediate goods-producing sector and thus economy-wide profits altogether. This simplifies the asset structure of the economy without affecting the dynamics of the model much.
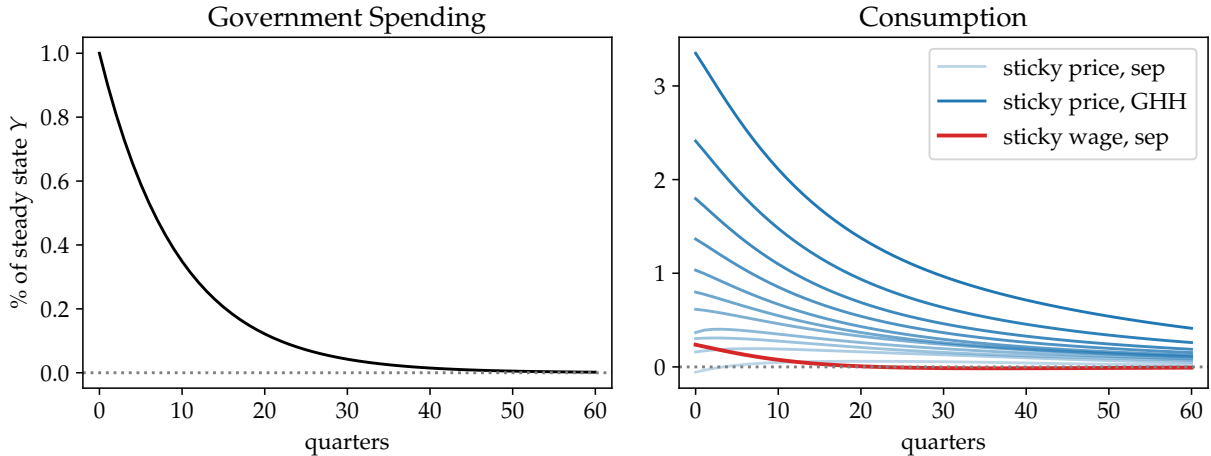
Figure 3: Government Spending and the Response of Private Consumption

Each blue line corresponds to a different value of CI, going from $\alpha = 0$ (light) to $\alpha = 1$ (dark) in steps of 0.1.

6 Conclusion

New Keynesian models with flexible wages and sticky prices face a trilemma: they cannot simultaneously match high MPCs, low MPEs, and fiscal multipliers that are moderate under accommodative monetary policy. Our proposed solution is to take workers off their short-run labor supply curves. This directly implies low MPEs, allowing us to use separable preferences, which can then match high MPCs and moderate fiscal multipliers. We achieve this with sticky wages in a HANK model. Although we keep this model intentionally simple, it could be extended with common quantitative features like sticky prices and investment and still solve the trilemma. An alternative route might be to add search frictions, as in Gornemann et al. (2016) and Ravn and Sterk (2017).

While there is no good way to solve the trilemma without adding labor market frictions, some calibrations come closer than others. Preferences with an intermediate degree of labor-consumption complementarity, between separable and GHH, achieve the best tradeoff of MPEs and multipliers. For preferences with complementarity, a high labor wedge can also bring the multiplier down to a more reasonable level, and in these cases we suggest that researchers report the labor wedge. Finally, if monetary policy is active (with a high Taylor rule coefficient), rather than accommodative as in fact 3, the model's predictions are closer to neoclassical. This sidesteps the trilemma, but only as long as active policy is in force.

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A   Proofs

A.1   Proof of Lemma 1

Proof. Applying the implicit function theorem to (3) yields

$$U_{cc}\frac{\partial c(\lambda,\tilde{w})}{\partial \tilde{w}} + U_{cn}\frac{\partial n(\lambda,\tilde{w})}{\partial \tilde{w}} = 0.$$

Rearrange to get

$$\frac{\partial c(\lambda,\tilde{w})}{\partial \tilde{w}} \bigg/ \frac{\partial n(\lambda,\tilde{w})}{\partial \tilde{w}} = -\frac{U_{cn}}{U_{cc}}$$

and then divide by the ratio of (4) and (3), $\tilde{w} = -U_n/U_c$, to obtain the desired result

$$\text{CI} = \frac{\partial c(\lambda,\tilde{w})}{\partial \tilde{w}} \bigg/ \tilde{w}\frac{\partial n(\lambda,\tilde{w})}{\partial \tilde{w}} = \frac{U_{nc}/U_n}{U_{cc}/U_c}. \tag{28}$$

For separable preferences, $U_{nc} = 0$, and it is clear that $\text{CI} = 0$. For GHH preferences, $U(c,n) = u(c - v(n))$, and so $U_n = u'(c - v(n))v'(n)$ and $U_c = u'(c - v(n))$ and $U_{nc} = u''(c - v(n))v'(n)$ and $U_{cc} = u''(c - v(n))$. Hence

$$\frac{U_{nc}}{U_n} = \frac{U_{cc}}{U_c} = \frac{u''(c - v(n))}{u'(c - v(n))}$$

which shows that $\text{CI} = 1$. □

A.2   Proof of Proposition 1

Proof. Log-linearizing (3)-(4) yields a linear system

$$\begin{bmatrix} \frac{cU_{cc}}{U_c} & \frac{nU_{cn}}{U_c} \\ \frac{cU_{nc}}{U_n} & \frac{nU_{nn}}{U_n} \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{n} \end{bmatrix} = \begin{bmatrix} \hat{\lambda} \\ \hat{\lambda} + \hat{w} \end{bmatrix}$$

The 2×2 matrix inverse formula implies that the solution is

$$\begin{bmatrix} \hat{c} \\ \hat{n} \end{bmatrix} = \frac{U_c U_n}{cn} \frac{1}{U_{cc}U_{nn} - U_{cn}^2} \begin{bmatrix} \frac{nU_{nn}}{U_n} & -\frac{U_{cn}n}{U_c} \\ \frac{-cU_{nc}}{U_n} & \frac{cU_{cc}}{U_c} \end{bmatrix} \begin{bmatrix} \hat{\lambda} \\ \hat{\lambda} + \hat{w} \end{bmatrix}$$

It follows that

$$\frac{\frac{\partial \log n(\lambda,w)}{\partial \log \lambda}}{\frac{\partial \log n(\lambda,w)}{\partial \log w}} = \frac{\frac{cU_{cc}}{U_c} - \frac{cU_{nc}}{U_n}}{\frac{cU_{cc}}{U_c}} = 1 - \frac{U_{nc}/U_n}{U_{cc}/U_c} = 1 - \text{CI}$$

and hence, using the definitions of EIS and Frisch in the main text,

$$\frac{\frac{\partial \log n(\lambda,w)}{\partial \log \lambda}}{\frac{\partial \log c(\lambda,w)}{\partial \log \lambda}} = \frac{\frac{\partial \log n(\lambda,w)}{\partial \log w} \frac{\partial \log n(\lambda,w)}{\partial \log \lambda}}{\frac{\partial \log n(\lambda,w)}{\partial \log w} \frac{\partial \log c(\lambda,w)}{\partial \log \lambda}} = -\frac{\text{Frisch}}{\text{EIS}}(1 - \text{CI}).$$

The effect of a one-time transfer is therefore

$$\frac{\text{MPE}}{\text{MPC}} = \frac{-w\frac{dn}{dT}}{\frac{dc}{dT}} = \frac{-wn\frac{\partial \log n(\lambda,w)}{\partial \log \lambda}\frac{d \log \lambda}{dT}}{c\frac{\partial \log c(\lambda,w)}{\partial \log \lambda}\frac{d \log \lambda}{dT}} = \frac{wn}{c}\frac{\text{Frisch}}{\text{EIS}}(1 - \text{CI}).$$

$\square$

A.3 Proof of Proposition 2

Proof. Totally differentiating (3) and substituting the linearized market clearing condition $dC_t = dY_t - dG_t$ (from which the second-order price adjustment cost drops out) gives

$$U_{cc}(dY_t - dG_t) + U_{cn}dN_t = d\lambda_t.$$

Now differentiate the production function (10) and use the definition of the labor wedge to get

$$dN_t = \frac{1}{f'}dY_t = -(1 - \tau)\frac{U_c}{U_n}dY_t$$

Combine these two equations to get

$$dY_t = \frac{1}{1 - (1 - \tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}}\left(dG_t + \frac{1}{U_{cc}}d\lambda_t\right). \tag{29}$$

where we recognize $\text{CI} = \frac{U_{cn}/U_n}{U_{cc}/U_c}$ in the denominator.

All that is left is to express λ_t as a function of the exogenous interest rates. Iterating the Euler equation (8) forward until some T beyond which there are no more shocks, and we have returned to steady state, gives

$$\lambda_t = \beta^{T-t}\left(\prod_{j=0}^{T-t-1}(1 + r^e_{t+j})\right) U_c$$

$$\log \lambda_t = (T - t)\log \beta + \log U_c + \sum_{j=0}^{T-t-1}\log(1 + r^e_{t+j})$$

$$d \log \lambda_t = \sum_{j=0}^{T-t-1} d \log(1 + r^e_{t+j})$$

and so we have solved for output:

$$dY_t = \frac{1}{1 - (1-\tau)\,\mathrm{CI}} \left[dG_t + \frac{U_c}{U_{cc}} \sum_{j=0}^{T-t-1} d\log(1 + r_{t+j}^e) \right].$$ (30)

□

Note that this argument only requires constant real interest rates and the linearized market clearing condition, not our specific sticky-price, flexible-wage general equilibrium. In particular, it works equally well if there are sticky wages instead of, or in addition to, sticky prices, assuming that we directly define CI as $\frac{U_{cn}/U_n}{U_{cc}/U_c}$ (since (5) no longer makes sense when households are not on their labor supply curves).

A.4 Properties of GHH-plus preferences

Recall that GHH-plus preferences are of the form

$$U(c,n) = \frac{1}{1-\sigma} \left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu} \right)^{1-\sigma} - \varphi(1-\alpha)\frac{n^{1+\nu}}{1+\nu}.$$

The partials of this function are

$$U_c(c,n) = \left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu} \right)^{-\sigma},$$

$$U_n(c,n) = -\varphi n^\nu \left[\alpha U_c(c,n) + 1 - \alpha \right],$$

$$U_{cc}(c,n) = -\sigma \left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu} \right)^{-\sigma-1},$$

$$U_{cn}(c,n) = -\varphi\alpha n^\nu U_{cc}(c,n).$$

The complementarity index thus equals

$$\mathrm{CI} = \frac{U_c}{U_n}\frac{U_{nc}}{U_{cc}} = \frac{U_c}{\varphi n^\nu \left[\alpha U_c + 1 - \alpha\right]}\frac{\varphi\alpha n^\nu U_{cc}}{U_{cc}} = \frac{\alpha U_c}{\alpha U_c + 1 - \alpha}.$$ (31)

That is, CI can take any value in $[0,1]$ depending on the choice of $\alpha \in [0,1]$. We recognize the special cases from lemma 1: for separable preferences, $\alpha = 0 = \mathrm{CI}$, and for GHH preferences, $\alpha = 1 = \mathrm{CI}$.

Next we derive Frisch and EIS. Starting from the labor supply equation (3)(4) we get

$$w\lambda = \varphi n^{\nu} \left(\alpha\lambda + 1 - \alpha \right),$$

$$n(\lambda, w) = \left(\frac{w}{\varphi} \frac{\lambda}{\alpha\lambda + 1 - \alpha} \right)^{\frac{1}{\nu}},$$

$$\log n(\lambda, w) = \frac{1}{\nu} \left[\log w - \log \varphi + \log \lambda - \log(\alpha\lambda + 1 - \alpha) \right],$$

which shows that Frisch is still parameterized directly by ν:

$$\text{Frisch} = \frac{\partial \log n(\lambda, w)}{\partial \log w} = \frac{1}{\nu}. \tag{32}$$

EIS does not have such a simple form, but we can compute it recursively as follows:

$$\frac{\partial n(\lambda, w)}{\partial \log \lambda} = \frac{n(\lambda, w)}{\nu} \left(\frac{1 - \alpha}{\alpha\lambda + 1 - \alpha} \right),$$

$$\frac{\partial c(\lambda, w)}{\partial \log \lambda} = \phi \alpha n(\lambda, w)^{\nu} \left(\frac{\partial n(\lambda, w)}{\partial \log \lambda} \right) - \frac{1}{\sigma} \lambda^{-\frac{1}{\sigma}},$$

$$\text{EIS} = -\frac{\partial \log c(\lambda, w)}{\partial \log \lambda} = -\frac{1}{c(\lambda, w)} \frac{\partial c(\lambda, w)}{\partial \log \lambda}. \tag{33}$$

B Additional results

B.1 Proof of "Ricardian equivalence" in the sticky-price model

We will prove a special case of Ricardian equivalence that applies to first order in the model. The key idea is that in equilibrium, in whatever period the government raises taxes to pay for spending, firms will raise pretax wages by the same amount, so that the after-tax wage is unchanged, and indeed consumption, labor, and assets are unchanged for every household in every period.

For this to happen, it is essential that the firm equity is discounted using the same interest rate paid on government debt. This way, when the government decreases the debt from $t + 1$ onward by raising taxes at time t, there is an exactly offsetting increase in the valuation of firms, because the loss from raising wages at time t is no longer included in the time $t + 1$ equity price.

Proof. First, note that ex-post return r_t is fixed in all periods except 0:

$$1 + r_t = \begin{cases} \frac{p_{ss}}{p_{ss} + B_{ss}} \frac{d_0 + p_0}{p_{ss}} + \frac{B_{ss}}{p_{ss} + B_{ss}} (1 + r_{ss}) & \text{for } t = 0 \\ 1 + r_{ss} & \text{for } t \geq 1 \end{cases} \tag{34}$$

Therefore, the only endogenous inputs to the households' problem are the paths of after-tax wages and transfers, and the cum dividend price of equity in period 0. In particular, the aggregate labor

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supply function can be written as

$$N_t = \mathcal{N}_t\left(\{(1 - \tau_s^g)w_s, T_s\}_{s \geq 0}, d_0 + p_0\right).$$

Second, iterating the pricing equation (52) forward and using the definition of dividends (51) (ignoring second-order adjustment costs) and the production function (49) yields

$$
\begin{aligned}
d_0 + p_0 &= \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} d_t \\
&= \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (Y_t - w_t N_t) \\
&= \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} \left[(Z - w_t)N_t - F\right],
\end{aligned}
\tag{35}
$$

which shows that ex-post return depends only on the present values of $\{N_t\}_{t \geq 0}$ and $\{w_t N_t\}_{t \geq 0}$. Third, iterating the government budget constraint (55) forward yields

$$(1 + r_{ss})B_{-1} + \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} G_t = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} \tau_t^g w_t N_t. \tag{36}$$

Let the sequences $\{\tau_t^{g,*}, N_t^*, w_t^*, T_t^*\}_{t \geq 0}$ be part of an equilibrium. Now consider an alternative path of taxes that is part of an equilibrium $\{\tau_t^g, N_t, w_t, T_t\}_{t \geq 0}$. We're going to guess and verify that $N_t = N_t^*$ and $(1 - \tau_t^g)w_t = (1 - \tau_t^{g,*})w_t^*$ for all $t \geq 0$. That is, the pre-tax wage offsets any change in taxes and the real outcomes are the same.

The conjecture immediately implies that

$$(1 - \tau_t^g)w_t N_t = (1 - \tau_t^{g,*})w_t^* N_t^*, \qquad \forall t \geq 0$$

and hence

$$T_t = \gamma(1 - \tau_t^g)w_t N_t = \gamma(1 - \tau_t^{g,*})w_t^* N_t^* = T_t^*, \qquad \forall t \geq 0. \tag{37}$$

Now take present discounted values and use the fact that both allocations satisfy (36) to see that the present value of wage costs is the same:

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t}(1 - \tau_t^g)w_t N_t = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t}(1 - \tau_t^{g,*})w_t^* N_t^*,$$

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} w_t N_t = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} w_t^* N_t^*.$$

It follows, by the conjecture $N_t = N_t^*$, that the present value of pre-tax wages is also the same:

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} w_t = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} w_t^*.$$

It follows from (35) that

$$p_0 + d_0 = p_0^* + d_0^*. \tag{38}$$

We have shown that all inputs of the aggregate labor supply function are the same in both allocations, hence verified $N_t = N_t^*$. It follows that $Y_t = Y_t^*$ and $C_t = C_t^*$ and $A_t = A_t^*$. This means that goods market clearing holds period by period, up to the price-adjustment cost which is a second order term. Asset market clearing then follows from Walras's law. This means that $B_t + p_t = B_t^* + p_t^*$ for all t even though $B_t \neq B_t^*$ and $p_t \neq p_t^*$ in general. $\qquad\square$

B.2 Derivation of the wage Phillips curve

Here, we adapt the derivation of the wage Phillips curve from Auclert et al. (2018) to take into account the specification of the tax system in (18)–(19). Although our calibration features separable preferences, here we allow for general nonseparable preferences.

Let household i supply a continuum of labor services n_{ikt} which are imperfect substitutes. We assume that there is a union for each type of labor k. These unions set a common wage per efficiency unit w_{kt}, and their members have to supply labor demanded at that wage. Labor demand has the usual isoelastic form with elasticity ϵ_w. The unions' objective is to maximize the average welfare of their members, taking their consumption choice as given.

At any time t, union k sets its real wage w_{kt} to maximize

$$\sum_{\tau \geq 0} \beta^{t+\tau} \left[\int U(c_{it+\tau}, n_{it+\tau}) di - \frac{\epsilon_w}{2\kappa_w} \log \left(\frac{w_{kt+\tau}}{w_{kt+\tau-1}} (1 + \pi_t) \right)^2 \right]$$

subject to the demand curve

$$N_{kt} = \left(\frac{w_{kt}}{w_t} \right)^{-\epsilon_w} N_t.$$

Using symmetry as appropriate, the FOC may be written as

$$0 = \int U_c(c_{it}, n_{it}) \frac{\partial c_{it}}{\partial w_{kt}} + U_n(c_{it}, n_{it}) \frac{\partial n_{it}}{\partial w_{kt}} di - \frac{\epsilon_w}{\kappa_w} \frac{\log(1 + \pi_t^w)}{w_t} + \frac{\beta \epsilon_w}{\kappa_w} \frac{\log(1 + \pi_{t+1}^w)}{w_t}. \tag{39}$$

Next, we express the partial derivatives of consumption and labor. Household i's total hours are

$$n_{it} = \int N_{kt} dk = \int \left(\frac{w_{kt}}{w_t} \right)^{-\epsilon_w} N_t dk$$

and therefore

$$\frac{\partial n_{it}}{\partial w_{kt}} = -\epsilon_w \int \frac{N_{kt}}{w_{kt}} dk = -\epsilon_w \frac{N_t}{w_t} \tag{40}$$

By the envelope theorem, the marginal effect on consumption is equal to the marginal effect

on income. Household income is

$$z_{it} = (1 - \tau_t)(1 - \gamma)w_t e_{it} n_{it} + T_t$$

$$= (1 - \tau_t)(1 - \gamma)e_{it} \int w_{kt} n_{ikt} dk + T_t$$

$$= (1 - \tau_t)(1 - \gamma)e_{it} N_t \int w_{kt} \left(\frac{w_{kt}}{w_t} \right)^{-\epsilon_w} dk + T_t$$

and thus

$$\frac{\partial z_{it}}{\partial w_{kt}} = N_t(1 - \epsilon_w)(1 - \tau_t)(1 - \gamma)e_{it} \tag{41}$$

Substituting (40) and (41) into the FOC gives

$$\log(1 + \pi_t^w) = \kappa_w \left[N_t \int -U_n(c_{it}, n_{it}) di \right.$$
$$\left. - w_t N_t(1 - \tau_t)(1 - \gamma) \left(\frac{\epsilon_w - 1}{\epsilon_w} \right) \int e_{it} U_c(c_{it}, n_{it}) di \right] \tag{42}$$
$$+ \beta \log(1 + \pi_{t+1}^w)$$

This means that household heterogeneity affects wage setting via two cross-sectional moments: the average marginal disutility of labor and the productivity-weighted average marginal utility of consumption. To get a more compact expression, let virtual aggregate consumption and labor (C_t^*, N_t^*) be defined implicitly by

$$U_N(C_t^*, N_t^*) = \int -U_n(c_{it}, n_{it}) di, \tag{43}$$

$$U_C(C_t^*, N_t^*) = \int e_{it} U_c(c_{it}, n_{it}) di. \tag{44}$$

We can then write the wage Phillips curve as

$$\log(1 + \pi_t^w) = \kappa_w \left[N_t U_N(C_t^*, N_t^*) - w_t N_t(1 - \tau_t)(1 - \gamma) \left(\frac{\epsilon_w - 1}{\epsilon_w} \right) U_C(C_t^*, N_t^*) \right] \tag{45}$$
$$+ \beta \log(1 + \pi_{t+1}^w).$$

which is equation (27) in the main text.

C Summary of model equations

C.1 Sticky-price HANK

This is the benchmark model we use to generate the blue circles in figure 2.

- Households:

The Bellman equation

$$V_t(e_{it}, a_{it-1}) = \max_{c_{it}, n_{it}, a_{it}} \frac{1}{1-\sigma} \left(c_{it} - \varphi\alpha \frac{n_{it}^{1+\nu}}{1+\nu} \right)^{1-\sigma} - \varphi(1-\alpha)\frac{n_{it}^{1+\nu}}{1+\nu} + \beta\mathbb{E}_t \left[V_{t+1}(e_{it+1}, a_{it}) \right]$$

$$\text{s.t.} \quad c_{it} + a_{it} = (1+r_t)a_{it-1} + (1-\gamma)(1-\tau_t^g)w_t e_{it} n_{it} + T_t$$

$$a_{it} \geq \underline{a}$$

gives aggregate consumption, asset demand, and labor supply:

$$C_t = \mathcal{C}_t\left(\{r_s, r_s^e, \tau_s, w_s, T_s\}\right) = \int c_{it} di, \tag{46}$$

$$A_t = \mathcal{A}_t\left(\{r_s, r_s^e, \tau_s, w_s, T_s\}\right) = \int a_{it} di, \tag{47}$$

$$N_t = \mathcal{N}_t\left(\{r_s, r_s^e, \tau_s, w_s, T_s\}\right) = \int e_{it} n_{it} di. \tag{48}$$

- Production:

$$Y_t = ZN_t - F \tag{49}$$

- Phillips curve:

$$\log(1+\pi_t) = \kappa_p \left(\frac{w_t}{Z} - \frac{\epsilon_p - 1}{\epsilon_p} \right) + \frac{1}{1+r_t^e} \frac{Y_{t+1}}{Y_t} \log(1+\pi_{t+1}). \tag{50}$$

- Dividends:

$$d_t = Y_t - w_t N_t - \frac{\epsilon_p}{2\kappa_p} \log(1+\pi_t)^2 Y_t \tag{51}$$

- Equity price:

$$p_t = \frac{d_{t+1} + p_{t+1}}{1 + r_t^e} \tag{52}$$

- Realized return on portfolio:

$$1 + r_t = \frac{p_{t-1}}{p_{t-1} + B_{t-1}} \frac{d_t + p_t}{p_{t-1}} + \frac{B_{t-1}}{p_{t-1} + B_{t-1}}(1 + r_{t-1}^e) \tag{53}$$

- Monetary policy:

$$r_t^e \equiv r_{ss} \tag{54}$$

- Government budget:

$$B_t + \tau_t w_t N_t = (1 + r_{t-1}^e)B_{t-1} + G_t. \tag{55}$$

- Public debt policy:
$$B_t - B_{ss} = \rho_B(B_{t-1} - B_{ss} + G_t - G_{ss}) \tag{56}$$

- Lump-sum transfer:
$$T_t = \gamma(1 - \tau_t)w_t N_t \tag{57}$$

- Market clearing:
$$Y_t = C_t + G_t + \frac{\epsilon_p}{2\kappa_p}\log(1 + \pi_t)^2 Y_t \tag{58}$$
$$A_t = B_t + p_t \tag{59}$$

C.2 Sticky-wage HANK

This is the benchmark model we use to generate the dark red diamond on figure 2. Here we list only the equations that differ from the sticky-price model in appendix C.1.

- Households:

  The Bellman equation
  $$
  \begin{aligned}
  V_t(e_{it}, a_{it-1}) &= \max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[V_{t+1}(e_{it+1}, a_{it})\right] \\
  \text{s.t.} \quad & c_{it} + a_{it} = (1 + r_t)a_{it-1} + (1 - \gamma)(1 - \tau_t)w_t N_t e_{it} + T_t \\
  & a_{it} \geq \underline{a}
  \end{aligned}
  $$

  gives aggregate consumption, asset demand, and marginal utility of consumption:
  $$C_t = \mathcal{C}_t\left(\{r_s, r_s^e, \tau_s, w_s, N_s, T_s\}\right) = \int c_{it} di, \tag{60}$$
  $$A_t = \mathcal{A}_t\left(\{r_s, r_s^e, \tau_s, w_s, N_s, T_s\}\right) = \int a_{it} di, \tag{61}$$
  $$U_t^c = \mathcal{N}_t\left(\{r_s, r_s^e, \tau_s, w_s, N_s, T_s\}\right) = \int e_{it}c_{it}^{-\sigma} di. \tag{62}$$

- Phillips curve: replace (50) with
  $$\log(1 + \pi_t) = \kappa_w\left[\varphi N_t^{1+\nu} - (1 - \gamma)(1 - \tau_t)w_t N_t\left(\frac{\epsilon_w - 1}{\epsilon_w}\right)U_t^c\right] + \beta\log(1 + \pi_{t+1}) \tag{63}$$

- Price setting:
  $$\frac{w_t}{Z} = \frac{\epsilon_p - 1}{\epsilon_p} \tag{64}$$

- Dividends: replace (51) with

$$d_t = Y_t - w_t N_t \tag{65}$$

- Market clearing: replace (58) with

$$Y_t = C_t + G_t \tag{66}$$

C.3 Extensions

Here we briefly describe the models used to construct table 2.

- Taylor rule: replace (54) with

$$i_t = r_{ss} + \phi \pi_t \tag{67}$$

$$1 + r_t^e = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{68}$$

- Nominal peg: replace (67) with

$$i_t = \begin{cases} r_{ss} & \text{for } t = 0, 1, \dots 11 \\ r_{ss} + \phi \pi_t & \text{for } t \geq 12 \end{cases} \tag{69}$$

D Computational appendix

Endogenous gridpoints method for GHH-plus preferences. Recall that GHH-plus preferences are

$$U(c, n) = \frac{1}{1 - \sigma} \left(c - \varphi \alpha \frac{n^{1+v}}{1+v} \right)^{1-\sigma} - \varphi(1 - \alpha) \frac{n^{1+v}}{1+v}$$

and the first order conditions are

$$U_c(c_{it}, n_{it}) = \lambda_{it} + \beta(1 + r_t^e) \mathbb{E}_t U_c(c_{it+1}, n_{it+1}) \tag{70}$$

$$-U_n(c_{it}, n_{it}) = w_t(e_{it}) U_c(c_{it}, n_{it}) \tag{71}$$

$$\lambda_{it}(a_{it} - \underline{a}) = 0 \tag{72}$$

$$c_{it} + a_{it} = (1 + r_t) a_{it-1} + w_t(e_{it}) n_{it} + T_t \tag{73}$$

The core of the algorithm is a backward iteration of U_c on a discrete grid $\mathcal{G} = (e, a)$. Let Π denote the Markov transition matrix of the income state e. In this section only, we're going to use primes to refer to the next period. Thus, U_c' denotes marginal utility next period and (e', a') refers to the grid next period.

1. <u>Initial guess</u> $U_c'(e', a')$, marginal utility tomorrow on tomorrow's grid.

2. _$e' \to e$_ For unconstrained agents (70) implies that

$$U_c(e, a') = \beta(1 + r^e)\Pi U_c'(e', a').$$

Then use the labor supply equation (71) to obtain

$$n(e, a') = \left(\frac{w(e)}{\varphi} \frac{U_c(e, a')}{\alpha U_c(e, a') + 1 - \alpha} \right)^{\frac{1}{\nu}},$$

$$c(e, a') = U_c(e, a')^{-\sigma} + \varphi\alpha \frac{n(e, a')^{1+\nu}}{1 + \nu}.$$

3. _$a' \to a$_ the budget constraint (73) defines a mapping $a(e, a')$:

$$c(e, a') + a' - w(e)n(e, a') = (1 + r)a + T_t$$

which we can use to obtain $c(e, a)$ and $n(e, a)$ by linear interpolation. Then

$$a'(e, a) = (1 + r)a + T + w(e)n(e, a) - c(e, a).$$

4. Borrowing constraint Let $\mathcal{B} \subset \mathcal{G}$ be the set of gridpoints where $a'(e, a) < \underline{a}$. At these points only, $c(e, a)$ and $n(e, a)$ obtained in step 3 are invalid. The right values solve

$$c(e, a), n(e, a) = \arg\max_{c, n} U(c, n) \quad \text{s.t. } c = (1 + r)a + w(e)n + T$$

a static optimization problem that's independent of the current guess of U_c' and thus can be precomputed when solving for the steady state.

5. Update guess

$$U_c(e, a) = \left(c(e, a) - \varphi\alpha \frac{n(e, a)^{1+\nu}}{1 + \nu} \right)^{-\sigma}.$$

General equilibrium. We solve for impulse responses directly in sequence space as explained in Auclert, Bardóczy, Rognlie and Straub (2019) using the toolkit published alongside that paper, available at `https://github.com/shade-econ/sequence-jacobian`. Here we sketch the method for the benchmark sticky-price model.

There are 14 numbered equilibrium conditions in appendix C.1. Either goods market clearing or asset market clearing is redundant by Walras's law. That is, the we have a system of $13 \times T$ nonlinear equations that characterize the time paths—truncated at some large T[26]—of endogenous variables $\mathbf{U} = \{C_t, A_t, N_t, r_t, r_t^e, \tau_t, w_t, T_t, Y_t, \pi_t, d_t, p_t, B_t\}_{t=0}^{T-1}$ conditional on a path of the exogenous

---

[26]The choice of the truncation horizon is dictated by how long it takes for the economy to approximately return to the steady state. We found that $T = 200$ is sufficient in all cases we consider in the paper.

variable $\mathbf{G} = \{G_t\}_{t=0}^{T-1}$. We solve the system $\mathbf{H}(\mathbf{U}, \mathbf{G}) = 0$ in two steps. First, we reduce the set of unknowns to $\{\pi_t, w_t, N_t\}_{t=0}^{T-1}$ by taking advantage of the limited dependency among the equilibrium conditions. Second, we compute the Jacobians of $\mathbf{H}$ around the steady state. The Jacobians are sufficient to compute aggregate impulse responses.

E   Additional impulse responses under separable preferences

Figure 4 contrasts the set of impulse responses for our sticky-price and sticky-wage models with separable preferences. Interestingly, although the cumulative multipliers are similar in the two cases, the dynamics are quite different: the consumption response in the sticky-price case starts negative and then becomes persistently positive, while the consumption response in the sticky-wage case more closely mirrors the trajectory of government spending.

The non-monotonic aggregate consumption response reflects some of the peculiar properties of the sticky-price model. As the middle two rows of figure 4 show, this model features both a very large decline in equity prices and a large increase in wages (which, given our fiscal rule, funds an increase in transfers). Since patient households own most of the equity, they suffer a negative wealth effect, and as figure 5 shows, cut their consumption while increasing hours. Impatient households, on the other hand, experience a large positive wealth effect from wages and transfers, and increase their consumption while cutting hours. These two very large and opposite-signed effects on both consumption and hours—which seem empirically implausible—net out to a much smaller cumulative increase in consumption. (Note that although these extremely heterogeneous responses show up in our model in patient vs. impatient agents, all that is really needed is wealth heterogeneity.) The consumption impulse response starts negative because the MPC out of the large date-0 decline in equity wealth initially dominates.

In contrast, the sticky-price impulse responses in figure 6 display a much more plausible degree of heterogeneity.
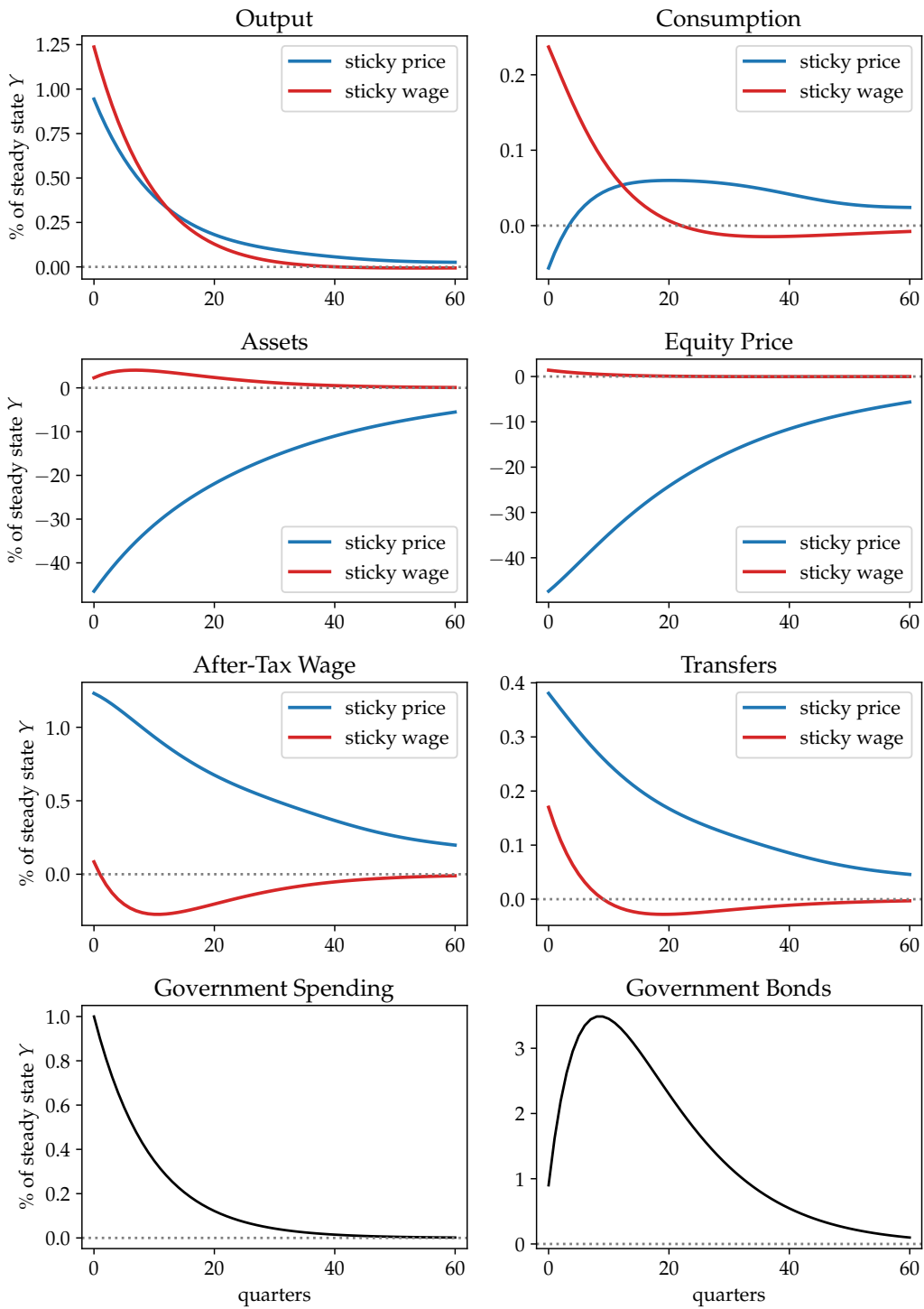
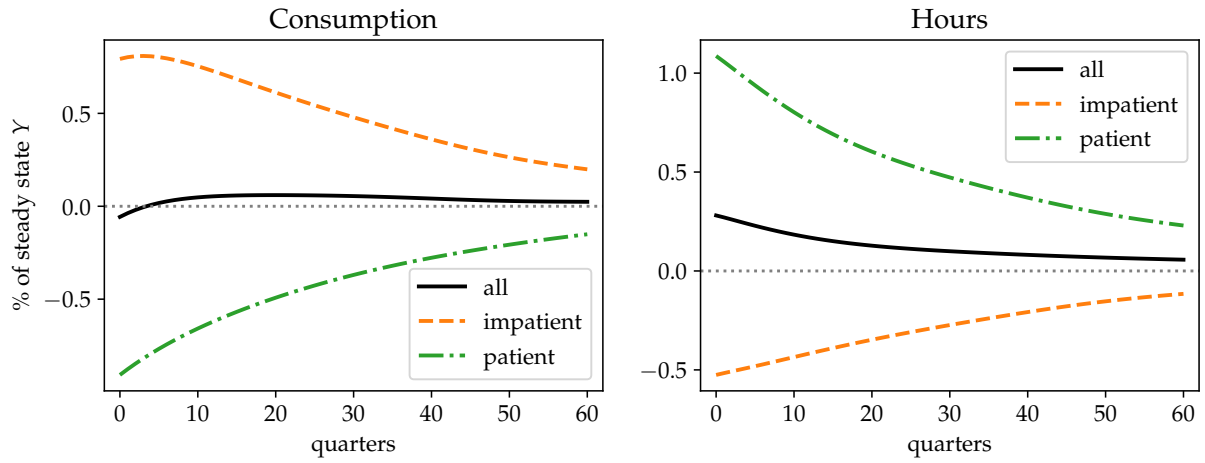Figure 4: Impulse Responses with Separable Preferences

Figure 5: Disaggregated Responses in Sticky-Price Model with Separable Preferences
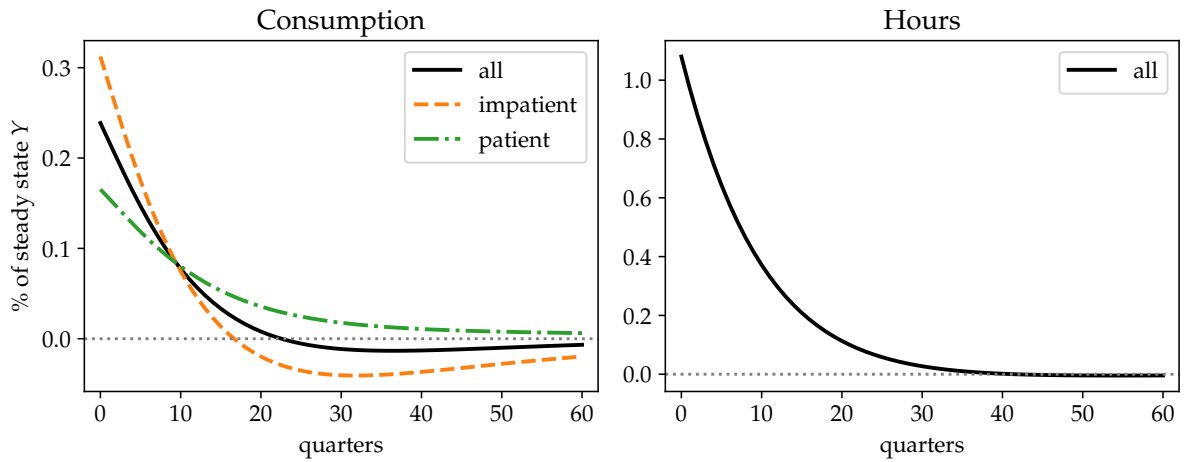

Figure 6: Disaggregated Responses in Sticky-Wage Model with Separable Preferences