SIR Modeling + COVID-19

Lesson Objectives

① Review (briefly) + continue extracting reaction rates from reaction equations

② Connect reaction rate modeling to real-world example (disease spread)

③ Compare how population behaviors/interventions impact rate parameters

④ Hypothesize how model changes in non-ideal cases (real world inequities)
  - ↳ important as we consider how to give advice/make decisions/policy + consider engineering ethics

Review

So far we have covered rate laws, which take the form of

$$r_A = f(\underbrace{T, C_i, P}, \text{etc})$$

↳ primarily care about these b/c they are impactful + tunable

So we can write

$$r_A = \underset{①}{f(T)} \cdot \underset{②}{f(C)}$$

where

① Temperature dependence → rate constant

② Concentration dependence

- ↳ derives from observations/lab (postulate from mechanism but only confirm w/ experiments)
- ↳ use mass action (most often Power law) to derive
- ↳ can be elementary reactions where
  - ① stoichiometric coefficients are identical to powers
  - ② involves a single reaction step

So for example

$$2A \xrightarrow{k} B$$

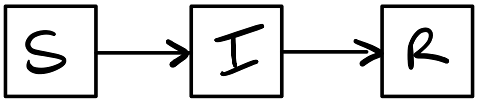
$$r_A = \frac{dC_A}{dt} = -2k\,C_A^2$$

$$r_B = \frac{dC_B}{dt} = kC_A^2$$

SIR Models

When modeling disease, SIR models are a common way to understand disease spread in a population

We have a system such that



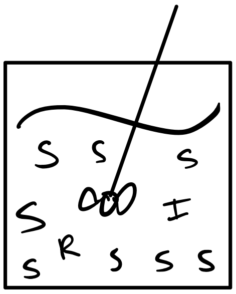
where

S = susceptible
I = infected
R = removed (ideally by recovery)

and each is in units of fraction of the population

$$S + I + R = 1$$

You can imagine the total population as a well-mixed batch reactor



where our initial fractions are

$S_i \approx$ close to 1

$I_i \approx$ very small fraction

$R_i \approx 0$

And you can imagine that the reactions governing disease spread can be described as

① $S + I \xrightarrow{\beta} I + I$

can be thought of as reactant

akin to auto-catalyst

where β - transmission rate constant

affected by ① average frequency of contact between individuals (since space is not accounted for otherwise)

② transmissibility of disease

② $$I \xrightarrow{\gamma} R$$

can be thought of as catalyst deactivation

where γ = recovery rate constant

① $\frac{1}{\gamma}$ = average duration of infection

So we can write reaction rates for each species w/ mass action kinetics

① Reaction rate of S

$$r_S = \frac{dS}{dt} = -\beta S I$$

② Reaction rate of I

$$r_I = \frac{dI}{dt} = \beta S I - \gamma I$$

③ Reaction rate of R

$$r_R = \frac{dR}{dt} = \gamma I$$

you can do a sanity check

we know that

$$S + I + R = 1$$

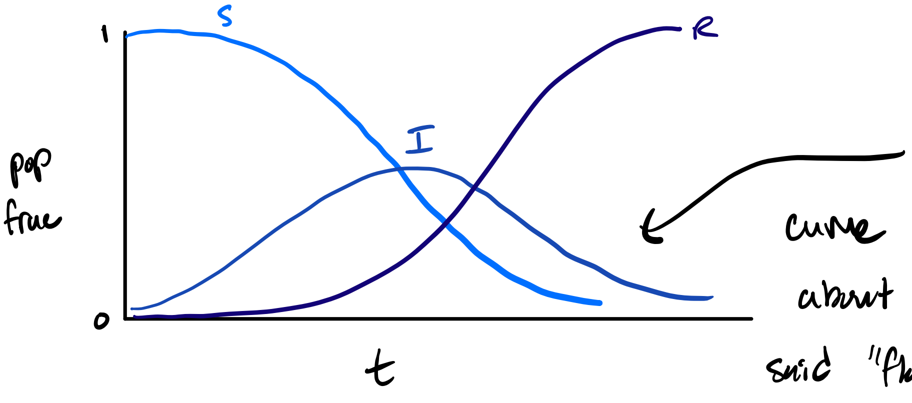
if you take the derivative

$$\frac{d\left[S + I + R\right]}{dt} = \frac{d(1)}{dt}$$

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$

$$(-\beta S I) + (\beta S I - \gamma I) + (\gamma I) = 0 \quad \checkmark$$

These fractions can be plotted over time + it typically looks like



this is the curve we talked about when we said "flatten the curve"

What assumptions does this model make? Poll Everywhere

- no birth or death
- well mixed / no spatial heterogeneity / everybody interacts at the same rate
- everybody gets sick with the same probability + recovers at the same rate
- no reinfection
- no inequities

Analysis

We can think about how much a disease will spread based on some analysis of these equations

What causes infections to rise?

↳ when $\uparrow \frac{dI}{dt}$

$$\frac{dI}{dt} = \beta S I - \gamma I$$

$$= I\left[\beta S - \gamma\right]$$

So if we assume $I \neq 0$

$\uparrow \frac{dI}{dt}$ if $\beta S - \gamma > 0$

$$\beta S > \gamma$$

$$\frac{\beta S}{\gamma} > 1$$

So infections will increase if

$$\frac{\beta S}{\gamma} > 1$$

This gives us the <u>replacement number</u> → expected # of people infected by a single infectious individual while they are infected

Each portion of the replacement number (r) has meaning

$r = \frac{\beta S}{\gamma}$ → produce βS new infections per unit time
→ be infectious for time $1/\gamma$

At time $t=0$, $S \approx 1$ and this gives us R_0 (R-naught)

↳ commonly used in media

$R_0 = \frac{\beta}{\gamma}$ low is better!

Example values: Seasonal flu = 1.3

Ebola = 1.8

COVID-19 = 4-10 (depending on variant)

Chicken pox = 10-12

Measles = 12-18

ACTIVITY → Think/Pair/share, each $\frac{1}{3}$rd of room gets one scenario

Let's think about this in the context of COVID

① How do $\beta + \gamma$ change between the original vs delta/omicron variant?

$\beta_{omicron} > \beta_{delta} > \beta_{original}$ → omicron + delta spread more easily

$\gamma_{omicron} > \gamma_{delta} > \gamma_{original}$ → (most people) recover more quickly

② How does having access to a vaccine change $\beta + \gamma$?

$\beta_{vax} < \beta_{no\ vax}$ ⇝ less spread

$\gamma_{vax} > \gamma_{no\ vax}$ ⇝ quicker recovery

③ How do social distancing + masking interventions affect $\beta + \gamma$?

$\beta_{\text{mask + SD}} < \beta_{\text{no mask or SD}}$ ⤳ less spread

γ is unaffected ⤳ won't change recovery rate

Inequities in COVID-19

In world-wide pandemics like COVID-19 pandemic, systemic inequities can cause dramatic downstream impacts on disease spread + human heath and survival

What might some inequities be that could have this impact?
- access to healthcare/treatment/vaccines
- occupation
  - ↳ remote vs essential worker or in-person job
- environmental conditions
  - ↳ marginalized communities experience more negative affects from pollution + climate change, + these can cause pre-existing conditions that make COVID more dangerous
- access to housing / type of housing
  - ↳ dictates exposure risk/frequency
- income
  - ↳ thus access to treatment + occupation ⇒ wealth gap
- pre-existing conditions
  - ↳ can be related to external inequities (enviromental injustice)

- discrimination
    - ↳ not sharing resources (like vaccines) equally beyond the US
    - ↳ different treatment by doctors for marginalized communities

During COVID-19, marginalized groups, such as racial + ethnic minorities faced increased risk of COVID-19 disease contraction, hospitalization, + death as a result of the compounded systemic inequities listed above (from CDC page). When developing disease interventions, preventions, + treatment (such as policy + analysis) it is important to consider those most vulnerable in a population.

The rate at which marginalized communities contracted or recovered from COVID-19 differs from that of those with more priviledge. Thus, understanding disease spread in a community(s) might not be as simple as modeling disease spread in a population that assumes all individuals are equally at risk or interacting, susceptible, + recover at the same rate

<u>ACTIVITY</u> - Brainstorm

How might the model/equations or model(s) change given the above?

- could have unique, interacting populations with different $\beta + \gamma$ values
- model w/more granular spatial distributions (global, country, state, country, city, communities)